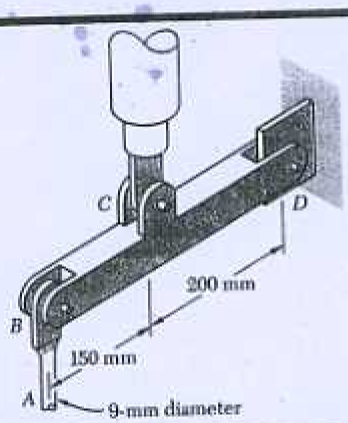
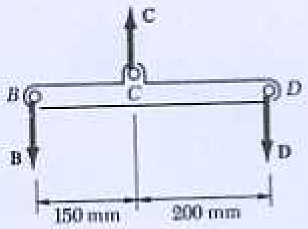
SAMPLE PROBLEM 1.3



The rigid beam BCD is attached by bolts to a control rod at B, to a hydraulic cylinder at C, and to a fixed support at D. The diameters of the bolts used are: $d_B = d_D = 8$ mm, $d_C = 12$ mm. Each bolt acts in double shear and is made from a steel for which the ultimate shearing stress is $\tau_U = 300$ MPa. The 9-mm-diameter control rod AB is made of a steel for which the ultimate tensile stress is $\sigma_U = 450$ MPa. If the minimum factor of safety is to be 3.0 for the entire unit, determine the largest upward force which may be applied by the hydraulic cylinder at C.



Solution. The factor of safety with respect to failure must be 3.0 or more in each of the three bolts and in the control rod. These four independent criteria will be considered separately.

Free Body: Beam BCD. We first determine the force at C in terms of the force at B and in terms of the force at D.

$+\circlearrowleft\Sigma M_D = 0$: $\quad B(350 \text{ mm}) - C(200 \text{ mm}) = 0 \qquad C = 1.750\,B \qquad (1)$

$+\circlearrowleft\Sigma M_B = 0$: $\quad -D(350 \text{ mm}) + C(150 \text{ mm}) = 0 \qquad C = 2.33\,D \qquad (2)$

Control Rod. For a factor of safety of 3.0 we have

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450 \text{ MPa}}{3.0} = 150 \text{ MPa}$$

The allowable force in the control rod is

$$B = \sigma_{all}(A) = (150 \text{ MPa})\tfrac{1}{4}\pi(9 \text{ mm})^2 = 9.54 \text{ kN}$$

Using Eq. (1) we find the largest permitted value of C:

$$C = 1.750\,B = 1.750(9.54 \text{ kN}) \qquad C = 16.70 \text{ kN} \blacktriangleleft$$

Bolt at B. $\tau_{all} = \tau_U/F.S. = (300 \text{ MPa})/3 = 100$ MPa. Since the bolt is in double shear, the allowable value of force B is

$$B = \tau_{all}(2A) = (100 \text{ MPa})\frac{2\pi}{4}(8 \text{ mm})^2 = 10.05 \text{ kN}$$

From Eq. (1): $\quad C = 1.750\,B = 1.750(10.05 \text{ kN}) \qquad C = 17.59 \text{ kN} \blacktriangleleft$

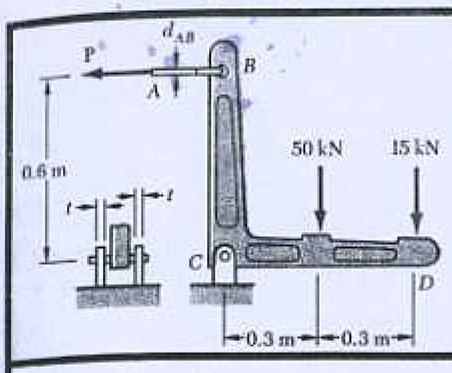
Bolt at D. Since this bolt is the same as bolt B, the allowable force is $D = B = 10.05$ kN. Using Eq. (2):

$$C = 2.33\,D = 2.33(10.05 \text{ kN}) \qquad C = 23.4 \text{ kN} \blacktriangleleft$$

Bolt at C. We again have $\tau_{all} = 100$ MPa and write

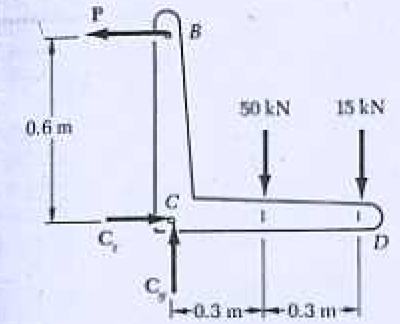
$$C = \tau_{all}(2A) = (100 \text{ MPa})\frac{2\pi}{4}(12 \text{ mm})^2 \qquad C = 22.6 \text{ kN} \blacktriangleleft$$

Summary. We have found separately four maximum allowable values of the force C. In order to satisfy all these criteria we must choose the *smallest* value, namely: $\qquad C = 16.70 \text{ kN} \blacktriangleleft$



SAMPLE PROBLEM 1.2

Two forces are applied to the bracket BCD as shown. (*a*) Knowing that the control rod AB is to be made of a steel having an ultimate normal stress of 600 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.3. (*b*) The pin at C is to be made of a steel having an ultimate shearing stress of 350 MPa. Determine the diameter of the pin C for which the factor of safety with respect to shear will also be 3.3. (*c*) Determine the required thickness of the bracket supports at C knowing that the allowable bearing stress of the steel used is 300 MPa.



Free Body: Entire Bracket. The reaction at C is represented by its components $\mathbf{C}_x$ and $\mathbf{C}_y$.

$$+\circlearrowleft \Sigma M_C = 0: \quad P(0.6\text{ m}) - (50\text{ kN})(0.3\text{ m}) - (15\text{ kN})(0.6\text{ m}) = 0 \qquad P = 40\text{ kN}$$

$$\Sigma F_x = 0: \quad C_x = 40\text{ kN}$$

$$\Sigma F_y = 0: \quad C_y = 65\text{ kN} \qquad C = \sqrt{C_x^2 + C_y^2} = 76.3\text{ kN}$$

a. **Control Rod *AB*.** Since the factor of safety is to be 3.3, the allowable stress is

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{600\text{ MPa}}{3.3} = 181.8\text{ MPa}$$

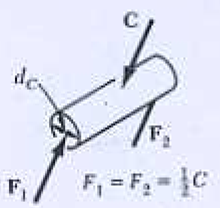
For $P = 40$ kN the cross-sectional area required is

$$A_{\text{req}} = \frac{P}{\sigma_{\text{all}}} = \frac{40\text{ kN}}{181.8\text{ MPa}} = 220 \times 10^{-6}\text{ m}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_{AB}^2 = 220 \times 10^{-6}\text{ m}^2 \qquad d_{AB} = 16.74\text{ mm} \quad ◀$$

b. **Shear in Pin *C*.** For a factor of safety of 3.3, we have

$$\tau_{\text{all}} = \frac{\tau_U}{F.S.} = \frac{350\text{ MPa}}{3.3} = 106.1\text{ MPa}$$



Since the pin is in double shear, we write

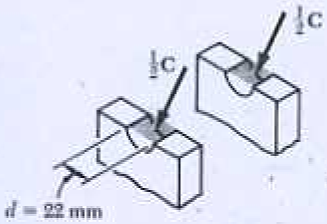
$$A_{\text{req}} = \frac{C/2}{\tau_{\text{all}}} = \frac{(76.3\text{ kN})/2}{106.1\text{ MPa}} = 360\text{ mm}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_C^2 = 360\text{ mm}^2$$

$$d_C = 21.4\text{ mm} \qquad \text{Use: } d_C = 22\text{ mm} \quad ◀$$

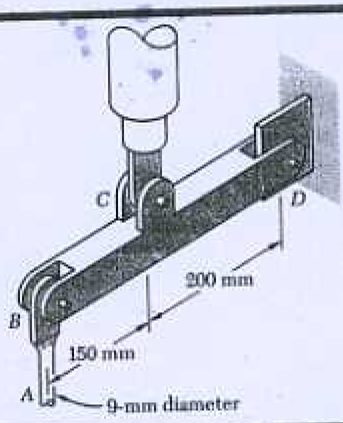
The next larger size pin available is of 22-mm diameter and should be used.

c. **Bearing at *C*.** Using $d = 22$ mm, the nominal bearing area of each bracket is $22t$. Since the force carried by each bracket is $C/2$ and the allowable bearing stress is 300 MPa, we write



$$A_{\text{req}} = \frac{C/2}{\sigma_{\text{all}}} = \frac{(76.3\text{ kN})/2}{300\text{ MPa}} = 127.2\text{ mm}^2$$

Thus $22t = 127.2 \qquad t = 5.78$ mm $\qquad$ Use: $t = 6$ mm ◀



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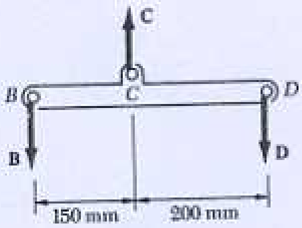

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