**Growth**

We defined technology as the relation between output and inputs which consist of labor and capital. As we have also discussed, this functional relation has a profound impact on how income is distributed among workers, capital owners, and entrepreneurs who earn wage, rent, and profit income.

This section analyzes a different theme: the growth of income. In particular, we shall focus on the mechanism of long-run growth of income per capita. Figure 1 shows the logarithm of GDP per capita in Turkey from 1951 to 2011. The growth rate of GDP per capita can be easily computed by the slope of the trend line (Why?)

This gives us 3% average growth rate for GDP per capita in Turkey over 60 years. Due to this growth rate, the average income in Turkey increased 5.5 times. To see how growth rate is crucial to our lives, let us ask what would happen if the growth rate was 2% instead of 3%. Then income would increase only 3.3 times. This means we would need 25 years more to reach our incomes that we actually have today. Or, in other words, your parents’ incomes today would be more or less equal to their parent’s actual incomes 25 years ago.

As for the growth rate in the Western world, see the growth rates of income per capita, population given in the table below:



Over the last century average income increased more than 7 fold in the Western World. But the average income increased less than 3 times between 1820 and 1900. The difference stems from 1.24% growth rate in the 19th century while the growth rate is 1.94% during the 20th century. It must be clear now that the mechanism that determines the growth rate of income per capita is very crucial. Now let us see the most basic and simple mechanism of economic growth from an economic point of view. Our theoretical model is the famous Solow’s growth model.

**Solow’s Model**

Assume the production technology is

where *Y* is output (GDP) and *K* is capital and *A* is information (knowledge) per worker and *L* is employment, which is equal to the number of workers. Therefore, gives us the total amount of knowledge that workers provide in production. It is noteworthy that is called “effective labor” as gives the efficacy of each worker.

So this production technology is of Cobb-Douglas type where inputs are and . Although this is a Cobb-Douglas technology, the discussions of this section neatly generalizes to any constant returns to scale technology.

As usual, assume . In order to introduce time, write for output, for capital, for information and for labor at time .

**Rates of Change**

Assume that everyone works, and inelastically supplies a unit of labor. This means is also the population. Therefore, income per capita (average income) is

Hence, by definition, the growth rate of is

**IMPORTANT:** In order to see how you can compute the growth rate of a variable, see the previous note which explain the mathematics of growth.

**Exercise\*:** The growth rate of GDP is 3% in Turkey in 2015. But the population growth rate is 1.35% in the same year. Then what is the growth rate of GDP per capita (GDP/Population) in Turkey in 2015?

**Exercise\*:** The growth rate of GDP is expected to be 1% in Turkey in 2016. But the population growth rate will still be 1.5%. What will be the growth rate of GDP per capita (GDP/Population) in Turkey in 2015?

Of course, the way in which *K* and *A* and *L* grow should be specified to understand how grows. Their growth rates will automatically tell us the growth rate of average income, . First let us see the growth rate of employment . To make things simple, assume employment is equal to the population, and sssume that the growth rate of the population is constant. This means

Note that is the rate of change and is the growth rate of population.

**Example:** in Turkey. This means 1.35% annual population growth rate. If population is 80 million people today, the rate of change in population in 1 year will be

This is the rate of change in population. Hence next year, the population in Turkey will be 80+1.2=81.2 million people.

Now let us see the growth rate of knowledge . Assuming knowledge constantly grows at a fixed rate the rate of change in knowledge is

Finally, let us define the rate of change in capital . Individuals save from their income which is invested in capital. The share of saving in output is . Therefore, gives total savings. We assume is a constant.

**Example:** 22% of GDP in Turkey goes to savings on average which means in Turkey.

However, capital also naturally becomes useless over time, a process we call depreciation. The amount of depreciated capital is where is a fixed rate of depreciation.

**Example:** If today and , corresponding to 4% depreciation rate, then total amount of depreciation in capital is , which is the amount of capital that becomes useless on its own.

Since the rate of change in capital is savings minus depreciation, we get

**Example:** As in the examples above, assume , , , and then saving is

and the rate of total amount of depreciation in capital is

 which is the amount of capital that becomes useless on its own. In sum

is the increase in capital (rate of change in capital) in this example.

**Exercise\*:** Assume , , , and . Then what is the growth rate of capital?

**Long-run**

In this section, we discuss and in the long run. In economics, long-run means a sufficiently long period so that variables of interest grow at constant rates. Therefore, the long-run in the present model is defined as follows.

**Definition:** Long-run is when the growth rates of and and and are constant.

So the growth rates of and can temporarily change to adopt the economy in the short-run. But in the long-run, the adjustment ends and their growth rates are permanently constant. Based on this definition, we can answer how fast average income (GDP per capita) grows in the long-run. This is very important because it will tell us the permanent growth rate of our incomes.

**Proposition:** In the long-run

**Proof:** The growth rate of is

By definition, is constant in the long-run. Then, since and are constants, we can infer that is also constant in the long-run. But the ratio of two variables is constant if their growth rates are the same. Therefore, in the long-run

So theoretically we expect in the long. This can be checked against the data. The result is very surprising for Turkey. See the graph below.

The actual values are and . Now let us see arguably the most important result in this section.

**Exercise\*:** If the growth rate of capital is 5%, and the saving rate is 10%, and the depreciation rate of capital is 10%, what is the average productivity of capital?

**Thereom:** In the long-run

**Proof:** We know that

and

These imply

in the long-run. But . Deduce that

This means the growth rate of income per capita in the long-run is essentially equal to the growth rate of knowledge. There is no way of becoming richer in the long-run other than expanding existing knowledge.

**Exercise\*:** The 1999 Izmit earthquake occurred on the 17th of August, and destroyed thousands of buildings, factories, infrastructure, and killed thousands of people. How would this catastrophe affect the growth of GDP, and the GDP per capita in the long-term according to the Solow’s model of growth?

**Exercise\*:** Suppose that the population growth is 2%, and the growth of GDP per capita is 3%. What is the growth of capital according to the Solow’s model?

**Exercise\*:** Suppose that the government decides to support massive construction projects. How would this affect the long-term growth according to the Solow’s model?

Interestingly, we can use the fact that to compute the long-run income per capita.

**Thereom:** In the long-run

**Proof:** Since it follows that

This means we need to find the value of in the long-run to calculate the average income in the long-run. To find this value, recall that

Therefore,

But in the long-run which means

in the long-run. But means

This is equivalent to

In other words,

Conclude that

in the long-run.

What have we seen?

1. Long-run growth of GDP is equal to the long-run growth of capital.
2. Long-run growth of GDP per capita is equal to the long-run growth of knowledge.
3. Losng-run GDP per capita does not depend on initial level of income.