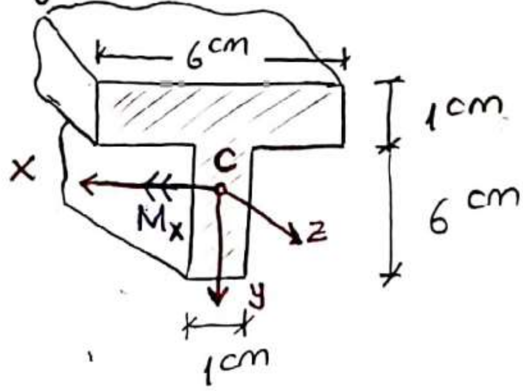
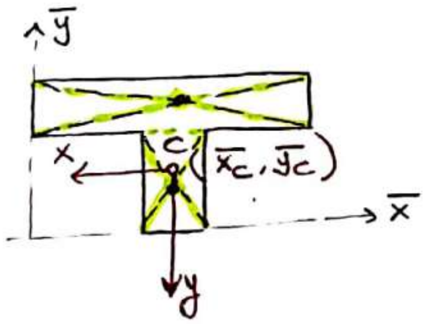
Example: The beam has a cross-sectional area in the shape of "T" section. Determine the maximum allowable internal moment M_x i that can be applied to the beam and sketch the bending stress distribution over the cross-section. The beam is made of material having an allowable tensile and compressive stress of $(\sigma_{allow})_t = (\sigma_{allow})_c = 14\ kN/cm^2$, respectively.



The cross-sectional area is symmetric about "y" axis that is perpendicular to the neutral axis on which the moment "M" acts along.

In order to calculate moment of inertia of the cross-sectional area about the neutral axis (x), the centeroid of the cross-section must be determined with respect to $(\bar{x}, \bar{y})$ axis.



$$\bar{x}_c = 3^{cm}$$

$$\bar{y}_c = \frac{\Sigma(y_i A_i)}{\Sigma A_i}$$

$$\bar{y}_c = \frac{(6.1).(6,5) + (6.1)(3)}{6+6} = 4{,}75\,cm$$

$$I_x = \frac{bh^3}{12} + Area(y)^2$$

$$I_x = \frac{6.1^3}{12} + (6.1).(-1{,}75)^2 + \frac{1.6^3}{12} + (6.1)(1{,}75)^2$$

$$I_x = 55{,}25\ cm^4 //$$

In order to determine the maximum allowable internal moment M., safety condition must be written as follows:

$$\sigma_{max} = \frac{M_{max}}{I_x} \cdot y_{max} \le (\sigma_{allow})_t \quad \text{--(1)}$$

$$|\sigma_{min}| = \left| \frac{M_{max}}{I_x} \cdot y_{min} \right| \leq (\sigma_{allow})_c \quad \text{--} (2)$$

$$\left.\begin{aligned} y_{max} &= 4{,}75 \text{ cm} \\ y_{min} &= -2{,}25 \text{ cm} \end{aligned}\right\} \quad (\sigma_{allow})_c = (\sigma_{allow})_t \; ; \; y_{max} > |y_{min}|$$

So evaluation of (1)st equation will be enough for the calculation of M_{max}.

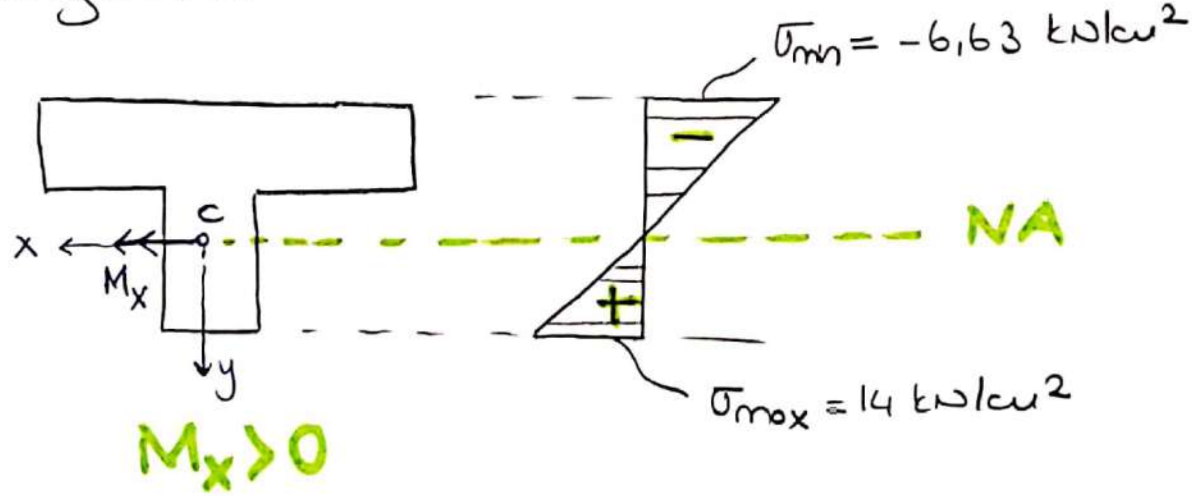
$$\sigma_{max} = \frac{M_{max}}{55{,}25 \text{ cm}^4} (4{,}75 \text{ cm}) \leq 14 \text{ kN/cm}^2$$

$$M_{max} \leq 162{,}8 \text{ kNcm}$$

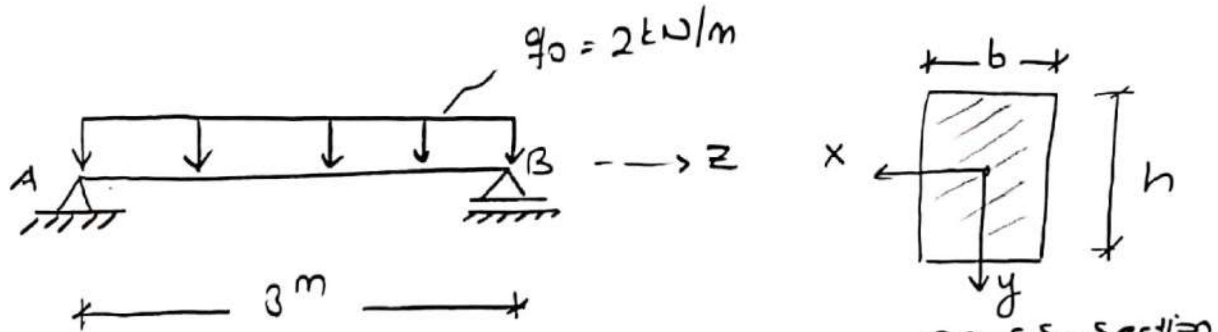
$$\sigma_{max} = \frac{M_{max}}{I_x} y_{max} = \frac{162{,}8}{55{,}25} (4{,}75) = 14 \text{ kN/cm}^2$$

$$\sigma_{min} = \frac{M_{max}}{I_x} y_{min} = \frac{162{,}8}{55{,}25} (-2{,}25) = -6{,}63 \text{ kN/cm}^2$$

Bending-stress distribution:



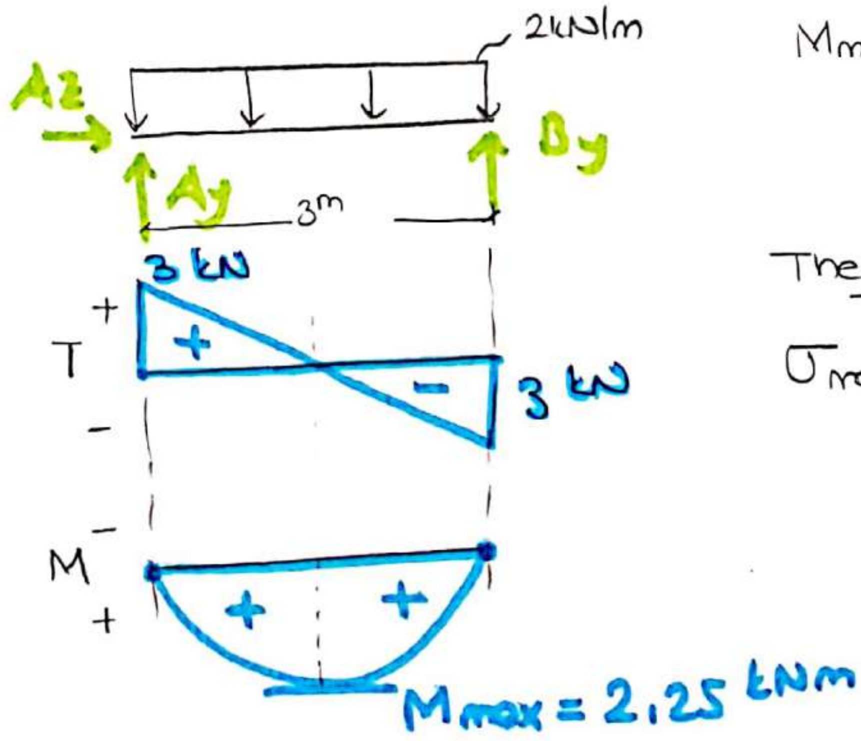
Example: The simply supported beam in Figure has the rectangular cross-sectional area. The allowable tensile and compressive stress of the material are $(\sigma_{allow})_t = (\sigma_{allow})_c = 10$ MPa, respectively. Determine the required width "b" and height "h" of the beam that will support the loading shown if the proportion $b = 2h/3$ is maintained.



cross-section

There is symmetry with respect to x and y axis. So no need to calculate the centroid for the member's cross-sectional area. The neutral axis is also the horizontal centroidal axis for the cross-section.

In the beginning, the critical section under the effect of maximum bending moment must be determined.



$M_{max} = 2{,}25$ kNm (at the middle point of the beam span)

The required condition is;

$$\sigma_{max} = \frac{M_{max}}{I_x} \cdot y_{max} \leq \sigma_{allow}$$

$y_{max} = h/2$; $I_x = \frac{bh^3}{12} = \frac{2h}{3} \cdot \frac{h^3}{12} = h^4/18$

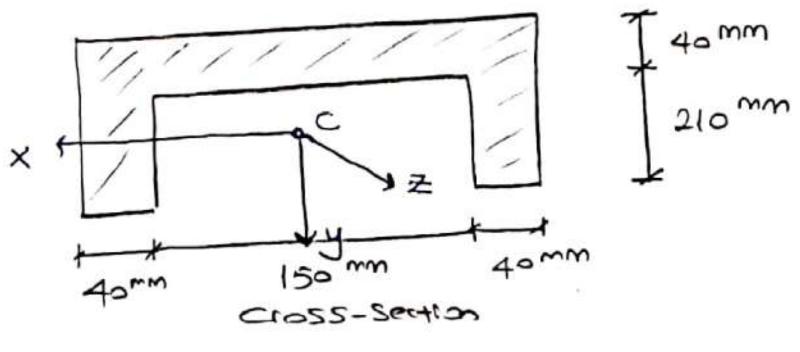
$(b = 2h/3)$.

$$\sigma_{max} = \frac{2{,}25 \text{ kNm}}{\frac{h^4}{18}} \left(h/2 \right) \leq 10^4 \text{ kN/m}^2$$

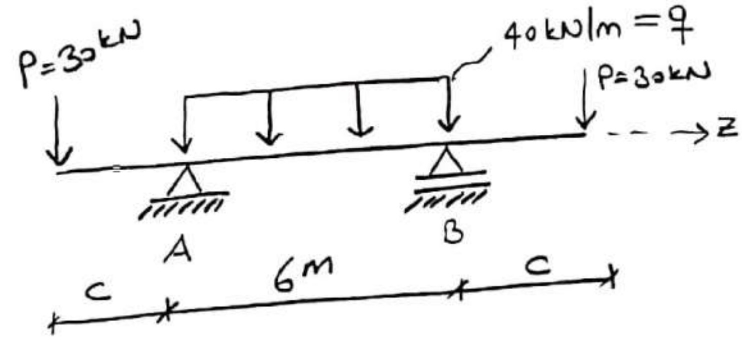
$$h \geq \underline{\underline{12{,}7 \text{ cm}}}$$

$$b = \frac{2h}{3} \geq \underline{\underline{8{,}5 \text{ cm}}}$$

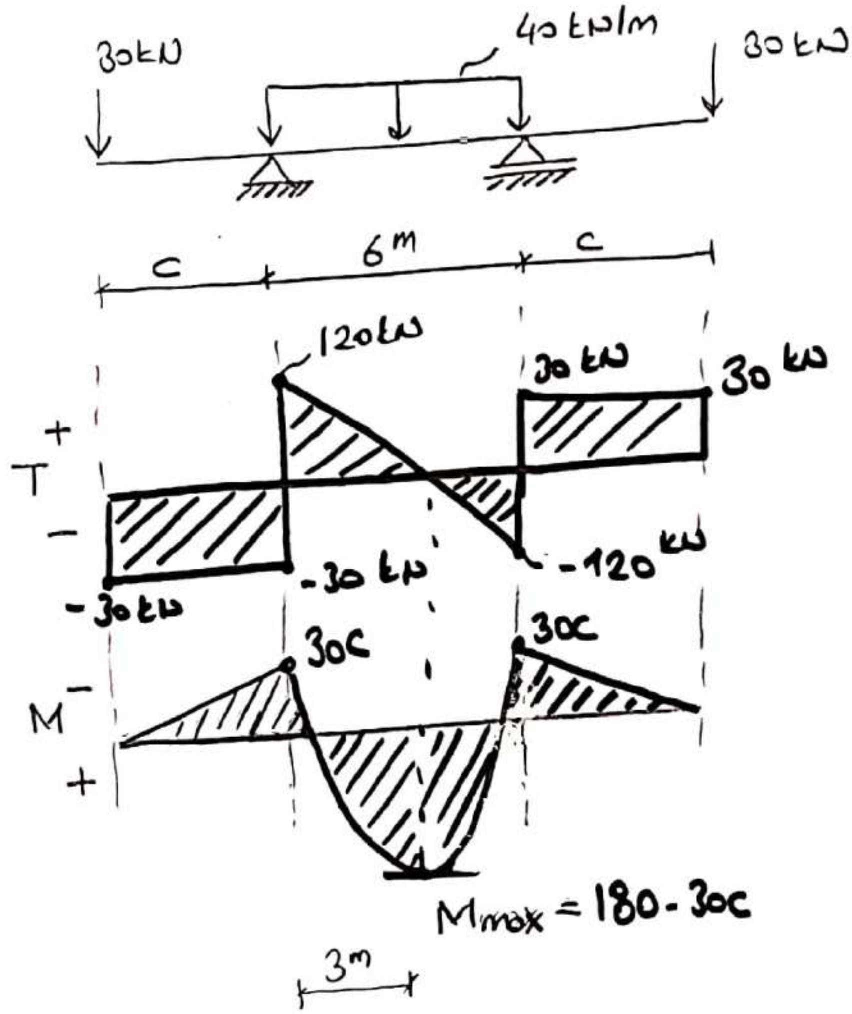
<u>Example</u>! The beam has a cross-sectional area in the shape of a channel. Determine the required overhanging length "c" of the beam when subjected to the loading shown in Figure. The allowable tensile and compressive stress for the beam are $(\sigma_{allow})_t = 50$ MPa, and $(\sigma_{allow})_c = 80$ MPa, respectively.



Cross-Section



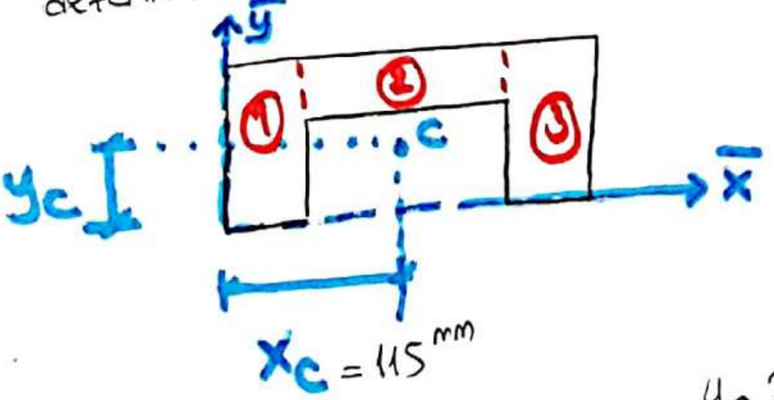
In the beginning, the bending moment diagram must be drawn to determine the most critical sections.



$M_{max} = 180 - 30c$ [kNm] at middle of the beam span

$M_{min} = -30c$ [kNm] at supports

Both moments must be checked for the safety. The cross-section is symmetric with respect to y axis, so y_c distance must be determined. x_c is equal to 115 mm with respect to left corner.



$$y_c = \frac{\sum y_i A_i}{\sum A_i}$$

$$y_c = \frac{2(40 \cdot 250 \cdot 125) + (150 \cdot 40 \cdot 230)}{2(40 \cdot 250) + (150 \cdot 40)}$$

$y_c \cong 15\,cm \cong 150\,mm$

For the calculation of I_x ;

$$I_x = \frac{bh^3}{12} + \text{Area}\,(y \text{ distance})^2$$

$$I_x = 2\left[\left(\frac{40 \cdot 250^3}{12}\right) + 40 \cdot 250 \cdot 25^2\right] + \frac{150 \cdot 40^3}{12} + (150 \cdot 40)(-80)^2$$

$$I_x = 15587 \text{ cm}^4$$

$$\sigma_{max} \leq (\sigma_{allow})_t$$

$$\sigma_{min} \leq (\sigma_{allow})_c$$

For bending moment at support ; calculation of "c" $\boxed{\text{distance value}}$

$$\sigma_z = \frac{M_{min}}{I_x}\, y \Longrightarrow \begin{array}{l} y_1 = -10 \text{ cm} \rightarrow \sigma_{max} \\ y_2 = 15 \text{ cm} \rightarrow \sigma_{min} \end{array}$$

$$\frac{(-30c)^{kNm}}{1{,}5587 \times 10^{-4}\, m^4}\,(-0{,}1^{m}) \leq \underbrace{50 \times 10^3 \ kN/m^2}_{(\sigma_{allow})_t}$$

$$c \leq 2{,}59^{m}$$

$$\frac{(-30c)^{kNm}}{1{,}5587 \times 10^{-4}\, m^4}\,(0{,}15^{m}) \leq \underbrace{-80 \times 10^3 \ kN/m^2}_{(\sigma_{allow})_c}$$

$$c \leq 2{,}77^{m}$$

For bending moment at beam span ; calculation of "c" distance value

$$\sigma_2 = \frac{M_{max}}{I_x} \cdot y \Rightarrow \begin{array}{l} y_1 = -10\ cm \rightarrow \sigma_{min} \\ y_2 = 15\ cm \rightarrow \sigma_{max} \end{array}$$

$$\frac{180-30c}{1{,}5587 \times 10^{-4}} (-0{,}1) \leq -80 \times 10^3$$

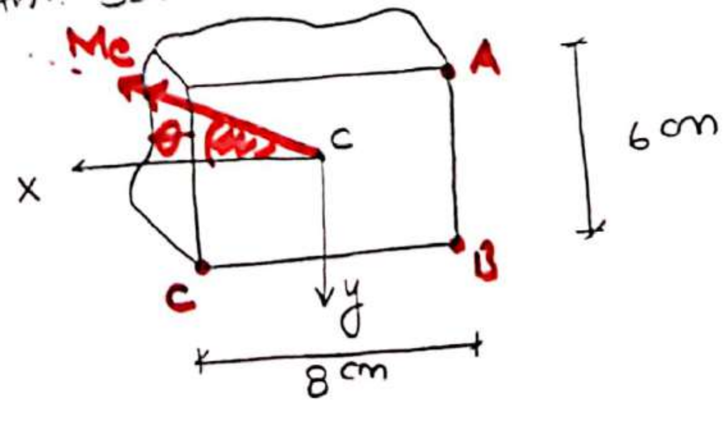
$$c \leq 1{,}84^m$$

$$\frac{180-30c}{1{,}5587 \times 10^{-4}} (0{,}15) \leq 50 \times 10^3$$

$$c \leq 4{,}26^m$$

Compare c values and determine the suitable one ; $\underline{\underline{c \leq 1{,}84^m}}$

Example: The member has a rectangular cross section and is subjected to a bending moment of M_e = 5 kNm. (a) Specify the orientation of the neutral axis, (b) sketch the bending stress distribution and (c) determine the normal stress developed at corners A, B and C of the section. Set $\theta = 30°$



(a) Due to symmetry with respect to x and y axes, no need to determine the centroid of the cross-section. Moments of inertia of the cross-sectional area about the x and y axis must be determined, respectively.

$$I_x = \frac{bh^3}{12} = \frac{8.6^3}{12} = 144\ cm^4$$

$$I_y = \frac{hb^3}{12} = \frac{6.8^3}{12} = 256\ cm^4$$

$M_e = 500$ kNcm ; Moment should be resolved into components directed along the principal axes.

$$M_x = M_e \cdot \cos 30 = 433\ kNcm$$
$$M_y = -M_e \cdot \sin 30 = -250\ kNcm$$

$$\sigma_z = \frac{M_x}{I_x} \cdot y - \frac{M_y}{I_y} \cdot x$$

$$\sigma_z = \frac{433}{144} y - \frac{(-250)}{256} x$$

$$\sigma_z = 3y + 0{,}977x \quad [kN/cm^2] \quad \text{-- (A)}$$

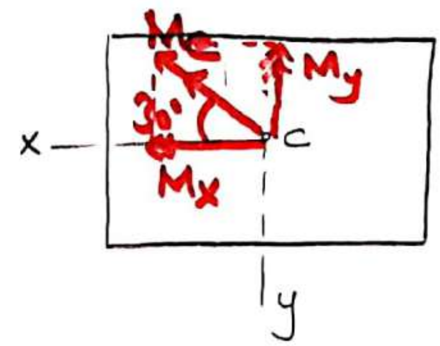
To specify the orientation of the neutral axis; $\boxed{\sigma_z = 0}$

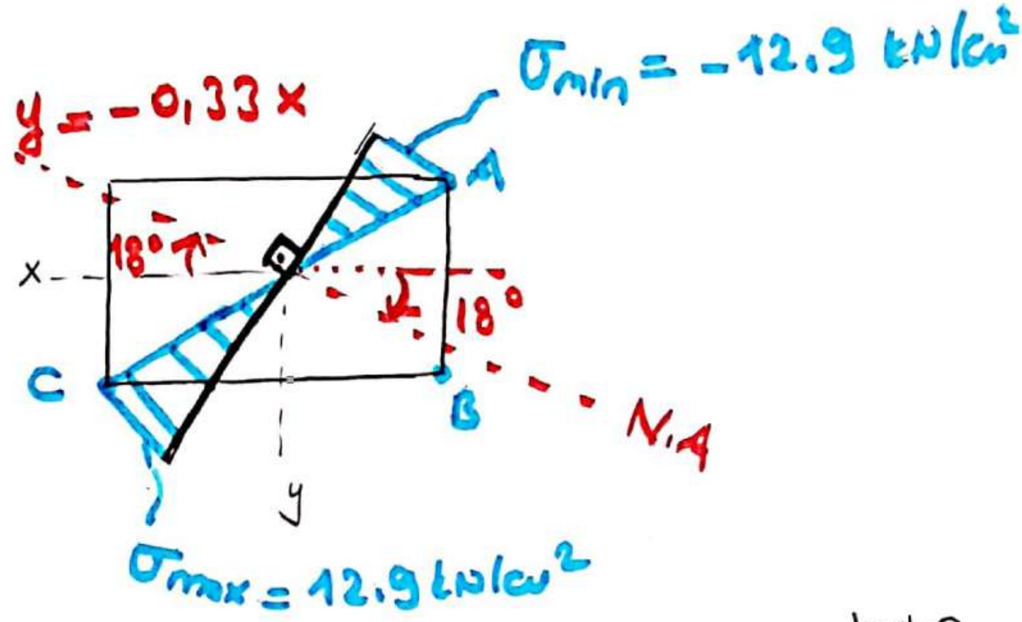
$$3y = -0{,}977x$$
$$y \cong -0{,}33x$$

$\tan\alpha = y/x = -0{,}33$

$\alpha \cong -18°$

(b)





Normal stress distribution will be drawn along a line that is perpendicular to NA.

$C(4\text{ cm}, 3\text{ cm})$
$A(-4\text{ cm}, -3\text{ cm})$ } The maximum and minimum stresses values at distances farthest from the neutral axis must be determined

$\sigma_{min} = \sigma_A$
$\sigma_{max} = \sigma_C$ } use coordinates of A and C points to calculate σ_{max}, σ_{min} by considering Eq (A)

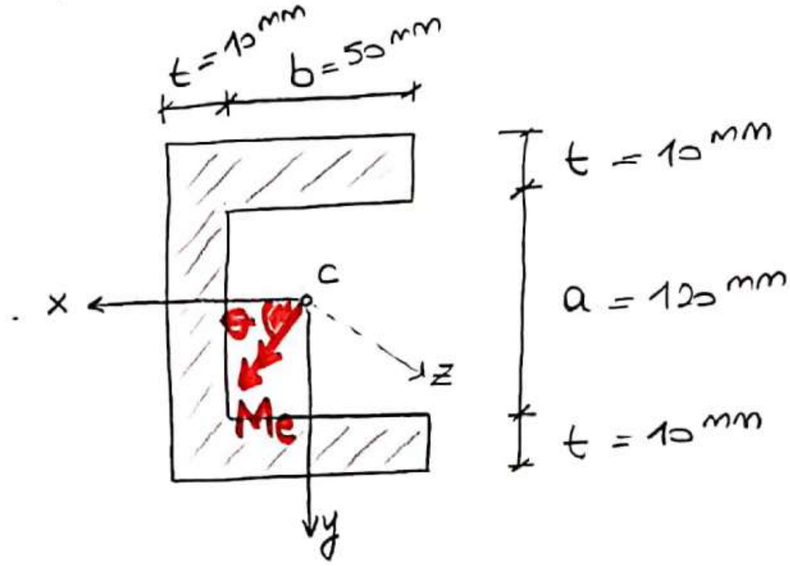
(c)

$$\sigma_{max} = \sigma_z \Big|_{\substack{x=4\\y=3}} = 3(3) + 0{,}977(4) = 12{,}9 \text{ kN/cm}^2$$

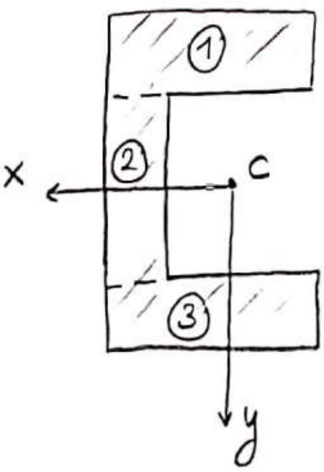
$$\sigma_{min} = \sigma_z \Big|_{\substack{x=-4\\y=-3}} = 3(-3) + 0{,}977(-4) = -12{,}9 \text{ kN/cm}^2$$

$$\sigma_B = \sigma_z \Big|_{\substack{x=-4\\y=3}} = 3(3) + 0{,}977(-4) \cong 5{,}1 \text{ kN/cm}^2$$

Example: If the beam is made from a material having an allowable tensile and compressive stress of $(\sigma_{allow})_t = (\sigma_{allow})_c = 0,14\ kN/mm^2$, determine the maximum allowable internal moment M_e that can be applied to the beam. Set $\tan\theta = 3/4$.



$y_c = 70\ mm$
x_c must be determined.



$A_1 = A_3 = 600\ mm^2$

$A_2 = 1200\ mm^2$

$A = A_1 + A_2 + A_3 = 2400\ mm^2$

$$x_c = \frac{\sum_{i=1}^{3} x_i A_i}{\sum A_i} = \frac{2[10\cdot 60\cdot 30] + (120\cdot 10\cdot 5)}{2400}$$

$$x_c = \underline{\underline{17,5\ mm}}$$

$$I_x = 2\left[\frac{60\cdot 10^3}{12} + (60\cdot 10)(65)^2\right] + \frac{10\cdot 120^3}{12} + \overbrace{(10\cdot \cancel{120})(0^2)}^{0}$$

$$I_x = 652\times 10^4\ mm^4$$

$$I_y = 2\left[\frac{10\cdot 60^3}{12} + (10\cdot 60)(-12,5)^2\right] + \frac{120\cdot 10^3}{12} + (120\cdot 10)(12,5)^2$$

$$I_y = 74,5\times 10^4\ mm^4$$

$I_{xy} = 0 \implies$ Important $\rightarrow$ For this case, the normal stress distribution formula;

$$M_x = M_e \cdot \cos\theta \rightarrow 4/5$$
$$M_y = M_e \cdot \sin\theta \rightarrow 3/5$$

$$\sigma_z = \frac{M_x}{I_x} \cdot y - \frac{M_y}{I_y} \cdot x$$

$$\sigma_z = \frac{M_e \cdot 4}{5 \cdot 652 \times 10^4} y - \frac{M_e \cdot 3}{5 \cdot 745 \times 10^4} x$$

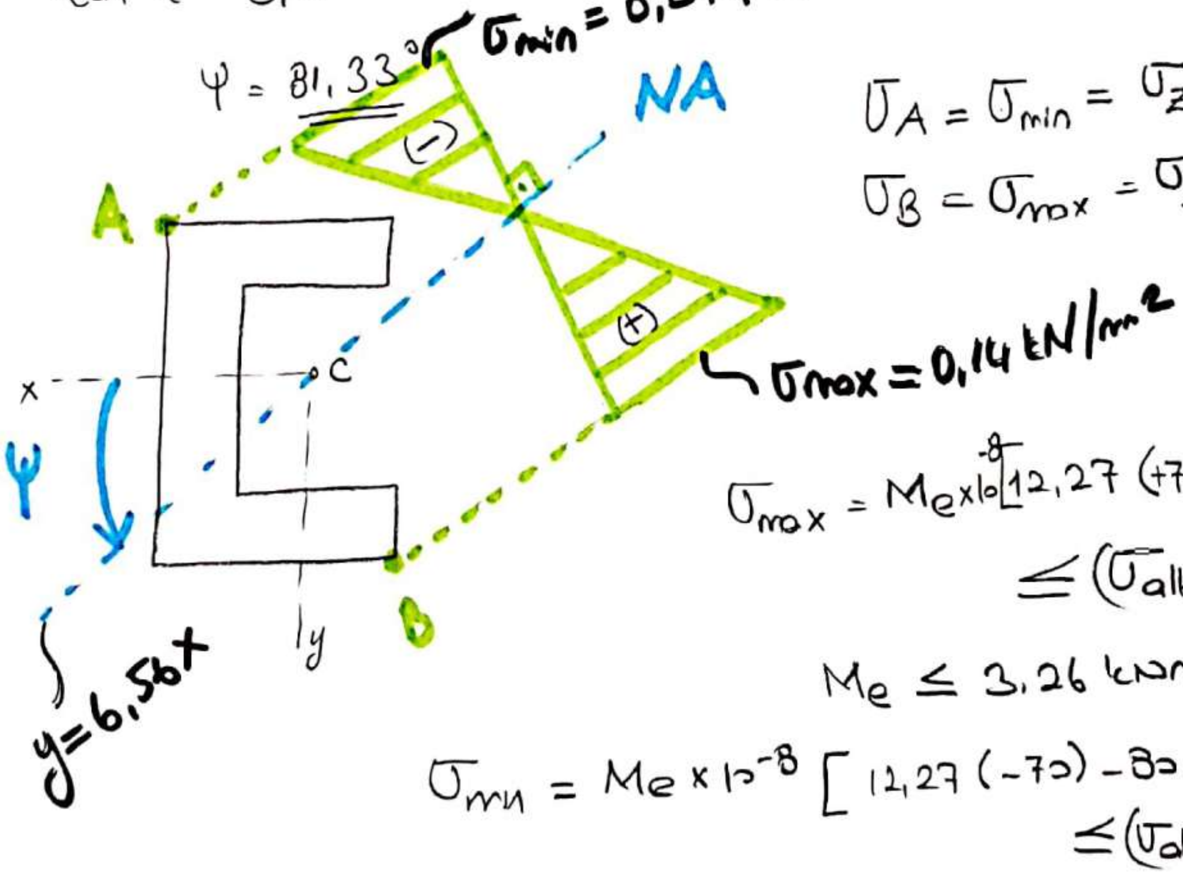
$$\sigma_z = M_e \, (12{,}27\, y - 80{,}53\, x)\, 10^{-8} \quad \ldots \text{A}$$

$\sigma_z = 0$; (NA $\leftarrow$ orientation of the)

$$12{,}27\, y - 80{,}53\, x = 0$$
$$\boxed{y = 6{,}56\, x}$$ .. Equation of NA

$\tan\psi = y/x = 6{,}56$ (slope of NA line)

$\psi = 81{,}33°$



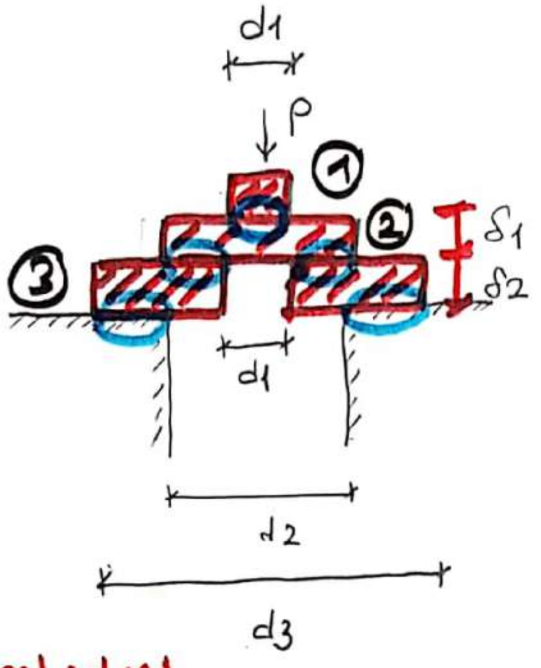
$$\sigma_A = \sigma_{min} = \sigma_z\,(17{,}5,\ -70)$$
$$\sigma_B = \sigma_{max} = \sigma_z\,(-42{,}5,\ 70)$$

$$\sigma_{max} = M_e \times 10^{-8} [12{,}27\,(+70) - 80{,}53\,(-42{,}5)] \leq (\sigma_{allow})_t = 0{,}14 \text{ kN/mm}^2$$

$$M_e \leq 3{,}26 \text{ kNm} \leq 3260 \text{ kNmm}$$

$$\sigma_{min} = M_e \times 10^{-8} [12{,}27\,(-70) - 80{,}53\,(17{,}5)] \leq (\sigma_{allow})_c = 0{,}14 \text{ kN/mm}^2$$

HW:



$P = 250$ kN

$\tau_{allow} = 80$ MPa

$\sigma_{allow} = 180$ MPa

the (red colored / hatched) parts

Shear: For the 2^{nd} : $(\pi \cdot \delta_1 \cdot d_1)(\tau_{allow})$

$\hookrightarrow P \leq (\pi \cdot d_1 \cdot \delta_1)\, \tau_{allow}$

$$\frac{250}{\pi \cdot \delta_1 \cdot d_1} \leq 8 \quad \cdots (1)$$

For the 3^{rd} : $P \leq (\pi \cdot d_2 \cdot \delta_2)\, \tau_{allow}$

$$\frac{250}{\pi \cdot \delta_2 \cdot d_2} \leq 8 \quad \cdots (2)$$

blue parts

Crushing: 1^{st} : $\dfrac{250}{\pi \cdot d_1^2/4} \leq 18$; $d_1 \geqslant 4{,}21$ cm (3)

2^{nd} : $\dfrac{250}{\pi (d_2^2 - d_1^2)/4} \leq 18$ $d_2 \geqslant 5{,}95$ cm (4)

3^{rd}: $\dfrac{250}{\dfrac{\pi(d_3^2 - d_2^2)}{4}} \leq 18$; $d_3 \geq 7{,}29$ cm (5)

$$\left.\begin{aligned} \delta_1 &\geq 2.37 \text{ cm} \\ \delta_2 &\geq 1.67 \text{ cm} \end{aligned}\right\} \text{From (1) and (2)}$$