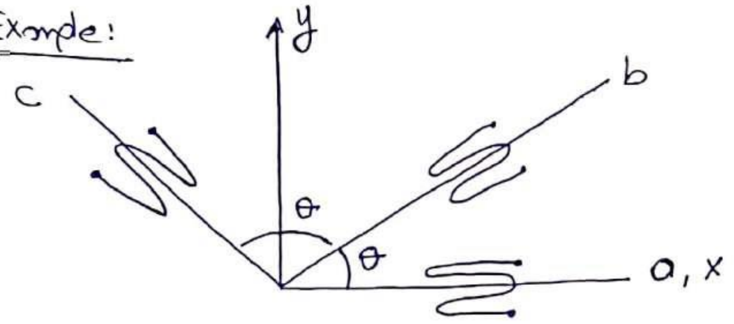
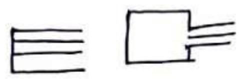
Example:





$\varepsilon_x, \varepsilon_y, \gamma_{xy} = ?$

The state of plane-strain at point is measured using the strain rosette shown in figure. Strain rosettes are arranged in 60° pattern. Determine;

(a) the in-plane principal strains at the point and the directions in which they act

(b) Show the results on the Mohr's circle.

$\theta = 60°$

$\varepsilon_a = 4 \times 10^{-6}$

$\varepsilon_b = -2 \times 10^{-6}$

$\varepsilon_c = 6,25 \times 10^{-6}$

The readings from the papers are:

$\theta = 0°, \quad \varepsilon_a = \varepsilon_\xi = 4 \times 10^{-6} = \varepsilon_x$

$\theta = 60°, \quad \varepsilon_b = \varepsilon_\xi = -2 \times 10^{-6}$

$\theta = 120°, \quad \varepsilon_c = \varepsilon_\xi = 6,25 \times 10^{-6}$

$$\varepsilon_\xi = \varepsilon_x \cdot \cos^2\theta + \varepsilon_y \cdot \sin^2\theta + \gamma_{xy} \cdot \sin\theta \cos\theta$$

$$-2 \times 10^{-6} = 4 \times 10^{-6} \left(\frac{1}{2}\right)^2 + \varepsilon_y \cdot \left(\frac{\sqrt{3}}{2}\right)^2 + \gamma_{xy}\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$6,25 \times 10^{-6} = 4 \times 10^{-6} \left(-\frac{1}{2}\right)^2 + \varepsilon_y \left(\frac{\sqrt{3}}{2}\right)^2 + \gamma_{xy}\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$\varepsilon_y = 1,5 \times 10^{-6}$ //

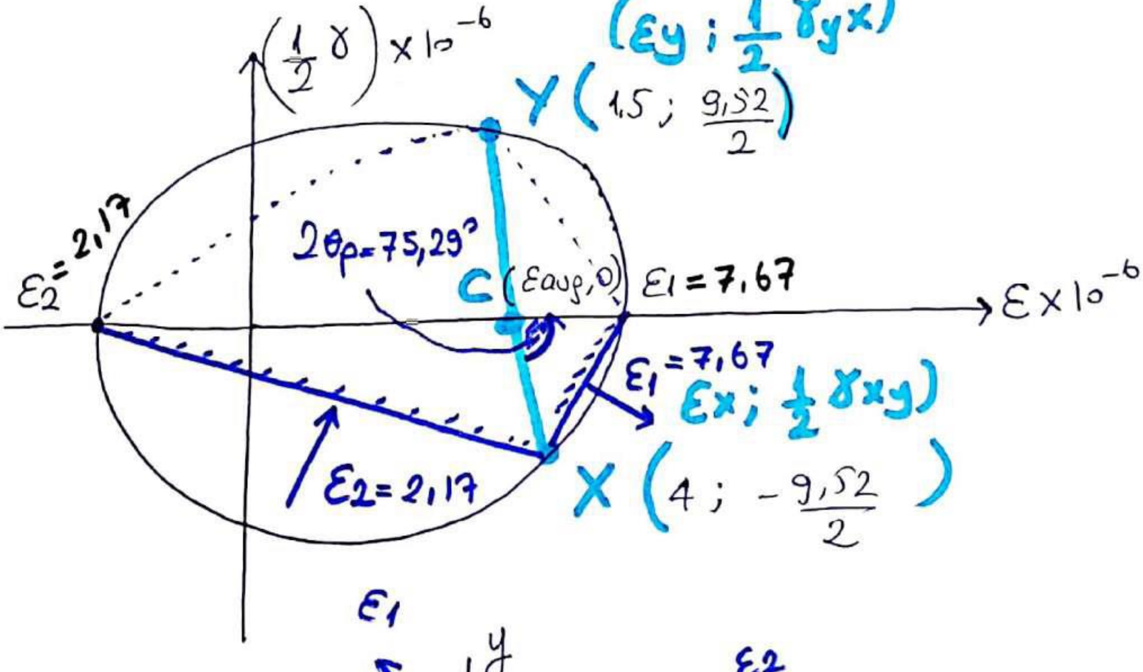
$\gamma_{xy} = -9,52 \times 10^{-6}$ rad //

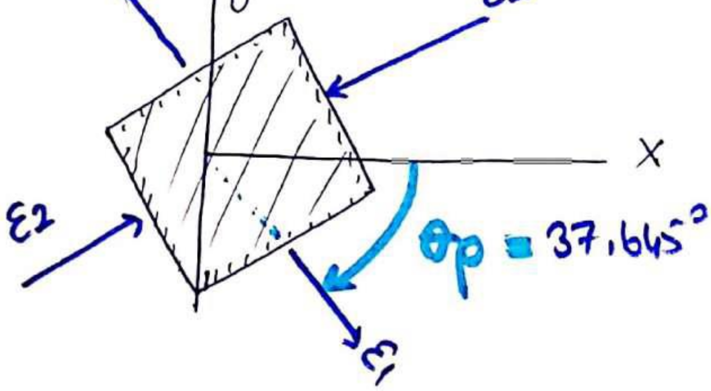
$$\varepsilon_{1,2} = \frac{1}{2}\left(\varepsilon x + \varepsilon y\right) \pm \sqrt{\left[\frac{1}{2}\left(\varepsilon x - \varepsilon y\right)\right]^2 + \left(\frac{1}{2}\gamma xy\right)^2}$$

$$\left.\begin{array}{l} \varepsilon_1 \cong 7,67 \times 10^{-6} \\ \varepsilon_2 \cong -2,17 \times 10^{-6} \end{array}\right\} \varepsilon_1 > \varepsilon_2$$

$$\tan(2\theta_p) = \frac{\gamma xy}{\varepsilon x - \varepsilon y} = \frac{-9,52 \times 10^{-6}}{(2,5 \times 10^{-6})}$$

$$2\theta_p = -75,29^\circ //$$





Example: Components of the linear strain tensor ε at a point with respect to the (x, y, z) coordinate system are given as:

$$[\varepsilon] = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 5 & -2 \\ 0 & -2 & 8 \end{bmatrix} \times 10^{-5}$$

a) Find the principal strains and the unit vectors along the principal directions.

$$I_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z = (10+5+8)10^{-5} = 23 \times 10^{-5}$$

$$I_2 = \varepsilon_x \varepsilon_y + \varepsilon_x \varepsilon_z + \varepsilon_y \varepsilon_z - \left(\frac{1}{2}\gamma_{xy}\right)^2 - \left(\frac{1}{2}\gamma_{xz}\right)^2 - \left(\frac{1}{2}\gamma_{yz}\right)^2$$

$$= \left[(10.5) + (10.8) + (5.8) - (-2)^2\right] 10^{-10}$$

$$= 166 \times 10^{-10}$$

$$I_3 = \det[\varepsilon]^T = \begin{vmatrix} 10 & 0 & 0 \\ 0 & 5 & -2 \\ 0 & -2 & 8 \end{vmatrix} \times 10^{-5}$$

$$= 36 \times 10^{-14}$$

$$\varepsilon^3 - I_1 \varepsilon^2 + I_2 \varepsilon - I_3 = 0$$

$$\left.\begin{aligned} \varepsilon_1 &= 10 \times 10^{-5} \\ \varepsilon_2 &= 9 \times 10^{-5} \\ \varepsilon_3 &= 4 \times 10^{-5} \end{aligned}\right\}$$

$\varepsilon_1 > \varepsilon_2 > \varepsilon_3$

The roots are principal strains.

$$\begin{bmatrix} 10-10 & 0 & 0 \\ 0 & 5-10 & -2 \\ 0 & -2 & 8-10 \end{bmatrix} \times 10^{-5} \begin{Bmatrix} \bar{n}_{x1} \\ \bar{n}_{y1} \\ \bar{n}_{z1} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$-5\,\bar{n}_{y1} - 2\,\bar{n}_{z1} = 0$$
$$-2\,\bar{n}_{y1} - 2\,\bar{n}_{z1} = 0$$
$$\bar{n}_{x1}^2 + \bar{n}_{y1}^2 + \bar{n}_{z1}^2 = 1$$

$$\bar{n}_{y1} = 0 \;;\; \theta_{y1} = 90^\circ$$
$$\bar{n}_{z1} = 0 \;;\; \theta_{z1} = 90^\circ$$
$$\bar{n}_{x1} = 1 \;;\; \theta_{x1} = 0^\circ$$

$$\begin{bmatrix} 10-9 & 0 & 0 \\ 0 & 5-9 & -2 \\ 0 & -2 & 8-9 \end{bmatrix} \times 10^{-5} \begin{Bmatrix} \bar{n}_{x2} \\ \bar{n}_{y2} \\ \bar{n}_{z2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

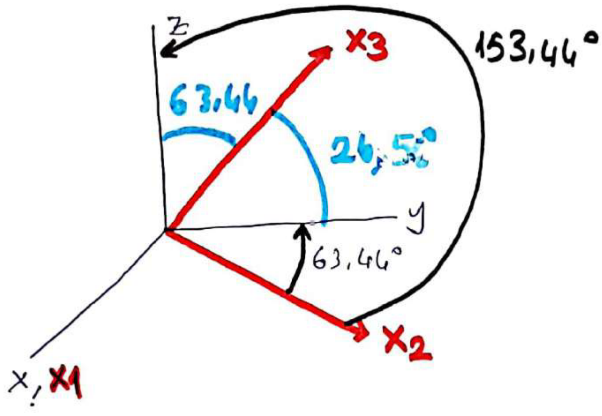
$$\bar{n}_{x2} = 0$$
$$\bar{n}_{y2} = \pm 1/\sqrt{5}$$
$$\bar{n}_{z3} = \mp 2/\sqrt{5}$$

$$\theta_{x2} = 90^\circ$$
$$\theta_{y2} = \underline{\underline{63{,}44^\circ}} \;;\; 116{,}56^\circ$$
$$\theta_{z2} = \underline{\underline{153{,}44^\circ}} \;;\; 26{,}56^\circ$$

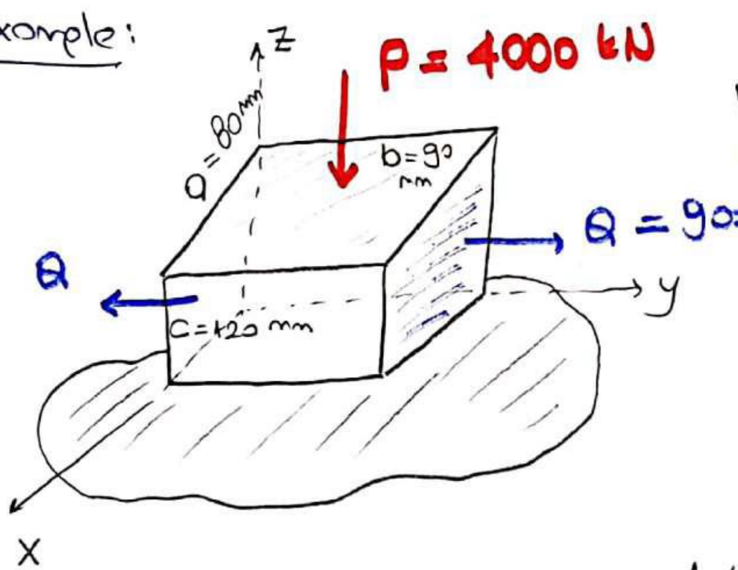
$$\begin{bmatrix} 10-4 & 0 & 0 \\ 0 & 5-4 & -2 \\ 0 & -2 & 8-4 \end{bmatrix} \times 10^{-5} \begin{Bmatrix} \bar{n}_{x3} \\ \bar{n}_{y3} \\ \bar{n}_{z3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\bar{n}_{x3} = 0$$
$$\bar{n}_{y3} = \pm 2/\sqrt{5}$$
$$\bar{n}_{z3} = \pm 1/\sqrt{5}$$

$$\theta_{x3} = 0^\circ$$
$$\theta_{y3} = 153{,}44^\circ \;;\; \underline{\underline{26{,}56^\circ}}$$
$$\theta_{z3} = \underline{\underline{63{,}44^\circ}} \;;\; 116{,}56^\circ$$



Example:



A specimen of cast-iron has under the effect of P and Q forces. The initial lengths of the specimen are written. The modulus of elasticity of the material is E = 120 GPa. and Poisson's ratio $\nu = 0,3$.

$E = 120\,GPa$ (modulus of elasticity of the material)

$\nu = 0,3$ (Poisson's ratio of the material)

Determine the stress and strain components relative to x, y, and z axes.

$A_z = a.b = 80 \times 90 = 7200\ mm^2$

$A_y = a.c = 80 \times 120 = 9600\ mm^2$

Normal stresses:

$$\sigma_z = \frac{P}{A_z} = -\frac{4000 \times 10^3\ N}{7200\ mm^2} = -556\ MPa$$

$$\sigma_y = \frac{Q}{A_y} = \frac{900 \times 10^3\ N}{9600\ mm^2} \cong 94\ MPa$$

$$\sigma_x = 0$$

Strain values:

$$\begin{cases} \varepsilon_x = \dfrac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \\ \varepsilon_y = \dfrac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \\ \varepsilon_z = \dfrac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] \end{cases}$$

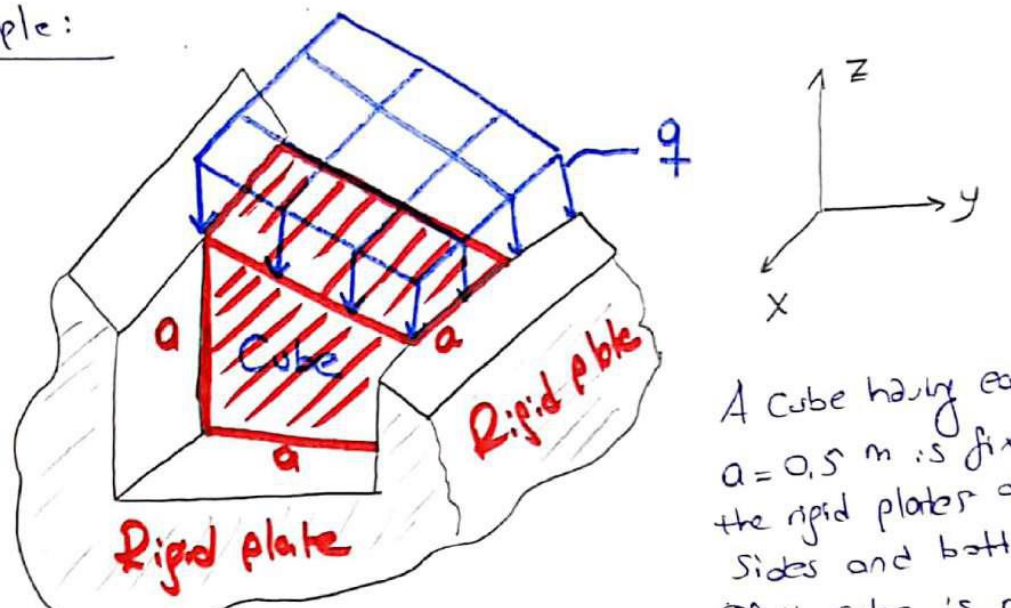
$$\varepsilon_x = \frac{1}{120\times10^3 \text{ MPa}}\left[0 - 0{,}3\,(94 - 556)\right] \cong 1155\times10^{-6}$$

$$\varepsilon_y = \frac{1}{120\times10^3 \text{ MPa}}\left[94 - 0{,}3\,(0 + 94)\right] \cong 2173\times10^{-6}$$

$$\varepsilon_z = \frac{1}{120\times10^3 \text{ MPa}}\left[-556 - 0{,}3\,(0 + 94)\right] = -4868\times10^{-6}$$

in diameter

Example:



A cube having each side $a = 0,5$ m is fixed to the rigid plates at its two sides and bottom surface. If the cube is made from an elastic material $E = 20$ GPa and $\nu = 0,2$. Determine .....

$a = 0,5$ m

$E = 20$ GPa

$\nu = 0,2$

Determine the change in side lengths of cube that are parallel to x and z axes, respectively if the cube is under the effect of compressive stress $q = 60$ MPa.

$\sigma_z = 60$ MPa (be careful it's compressive stress)

$\varepsilon_y = 0$

$\sigma_x = 0$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x^{\nearrow 0} + \sigma_z)\right]$$

$$0 = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z)\right] \; ; \; \boxed{\sigma_y = \nu\,\sigma_z} \cdots (1)$$

$$\sigma_y = 0,2(-60) = -12 \text{ MPa}$$

$$\varepsilon_x = \frac{1}{E}\left[\overset{\nearrow 0}{\sigma_x} - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_x = \frac{-\nu\,\sigma_z}{E}\left[1+\nu\right] = \frac{-0{,}2\,(-\overset{\text{MPa}}{60})}{20\times10^3\,\text{MPa}}\,(1+0{,}2)$$

$$\underline{\underline{\varepsilon_x = 7{,}2\times10^{-4}}}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\overset{\nearrow 0}{\sigma_x} + \overset{\nu\sigma_z}{\sigma_y}\right)\right]$$

$$\varepsilon_z = \frac{\sigma_z}{E}\left[1-\nu^2\right] = \frac{-60\text{ MPa}}{20\times10^3\text{ MPa}}\left[1-0{,}2^2\right]$$

$$\varepsilon_z = \underline{\underline{-2{,}88\times10^{-3}}}$$

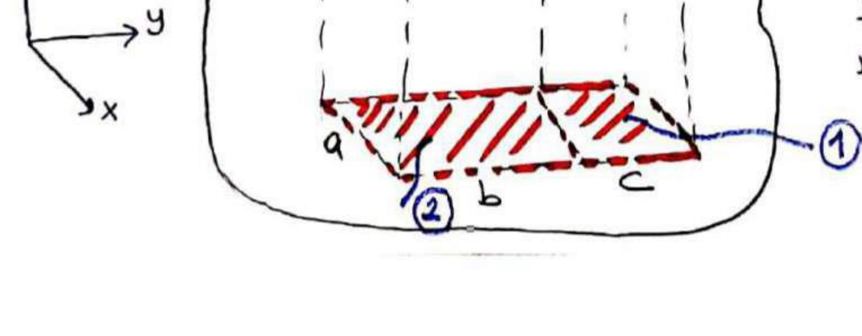
$$\boxed{\varepsilon = \frac{\Delta L}{L}}$$

$$\Delta a_x = \varepsilon_x \cdot a = (7{,}2\times10^{-4})\,(500^{\text{mm}}) = 0{,}36\text{ mm}$$

$$\Delta a_z = \varepsilon_z \cdot a = (-2{,}88\times10^{-3})\,(500) = -1{,}44\text{ mm}$$

<u>Example:</u>

$q = 120$ MPa (compressive stress).



$a = b = 30$ cm

$c = h = 40$ cm.

$E_1 = 120$ GPa

$E_2 = 180$ GPa

$\nu_1 = 0{,}2$

$\nu_2 = 0{,}3$

$\sigma_{z2} = -q = -120$ MPa

$\sigma_{z1} = 0$

Two blocks made from different elastic materials are fixed to each other as well as the rigid hollow. The dimensions of these blocks are given. The modulus of elasticity of the materials are given. If the top surface of the 2nd block is subjected to compressive stress $q = 120$ MPa, determine the change in their lengths and the strains and stresses in the blocks.

The relation for the strains in the y direction;

$$b.\, \varepsilon_{y2} + c\, \varepsilon_{y1} = 0$$

$$\varepsilon_{x1} = \varepsilon_{x2} = 0$$

$$\sigma_{y1} = \sigma_{y2}$$

$$\varepsilon_{x1} = \frac{1}{E_1}\left[\sigma_{x1} - v_1(\sigma_{y1} + \sigma_{z1})\right] \quad \cdots (1)$$

$$\varepsilon_{x2} = \frac{1}{E_2}\left[\sigma_{x2} - v_2(\sigma_{y2} + \sigma_{z2})\right] \quad \cdots (2)$$

$$\varepsilon_{y1} = \frac{1}{E_1}\left[\sigma_{y1} - v_1(\sigma_{x1} + \sigma_{z1})\right] \quad \cdots (3)$$

$$\varepsilon_{y2} = \frac{1}{E_2}\left[\sigma_{y2} - v_2(\sigma_{x2} + \sigma_{z2})\right] \quad \cdots (4)$$

$$\varepsilon_{z1} = \frac{1}{E_1}\left[\sigma_{z1} - v_1(\sigma_{x1} + \sigma_{y1})\right] \quad \cdots (5)$$

$$\varepsilon_{z2} = \frac{1}{E_2}\left[\sigma_{z2} - v_2(\sigma_{x2} + \sigma_{y2})\right] \quad \cdots (6)$$

Eq (1) $\Rightarrow \sigma_{x1} = v_1 \sigma_{y1}$; $(\sigma_{z1} = 0;\ \varepsilon_{x1} = 0)$

Eq (2) $\Rightarrow \sigma_{x2} = v_2(\sigma_{y2} - q)$; $(\varepsilon_{x2} = 0;\ \sigma_{z2} = -q)$

Eq (3) $\Rightarrow$ substitute $(\sigma_{z1} = 0)$; $\varepsilon_{y1} = \frac{1}{E_1}\left[\sigma_{y1} - v_1 \sigma_{x1}\right]$

Eq (4) Substitute $(\sigma_{z2} = -q)$; $\varepsilon_{y2} = \frac{1}{E_2}\left[\sigma_{y_2} - \nu_2(\sigma_{x_2} - q)\right]$

Eq (5) Substitute $(\sigma_{z1} = 0)$; $\varepsilon_{z1} = \frac{1}{E_1}\left[-\nu_1(\sigma_{x1} + \sigma_{y1})\right]$

Eq (6) Substitute $(\sigma_{z2} = -q)$; $\varepsilon_{z2} = \frac{1}{E_2}\left[-q - \nu_2(\sigma_{x2} + \sigma_{y2})\right]$

$\sigma_{y2} = \sigma_{y1} \cong -16,54$ MPa

$\sigma_{x1} \cong -3,31$ MPa

$\sigma_{x2} \cong -40,86$ MPa

$\varepsilon_{y1} = -132 \times 10^{-6}$

$\varepsilon_{z1} \cong 33 \times 10^{-6}$

$\varepsilon_{y2} = 176 \times 10^{-6}$

$\varepsilon_{z2} \cong -571 \times 10^{-6}$

$\left.\begin{array}{l} \Delta h_1 = \varepsilon_{z1} \cdot h \\ \Delta h_2 = \varepsilon_{z2} \cdot h \end{array}\right\} \begin{array}{l} \Rightarrow \cong 0,013 \text{ mm} \\ \Rightarrow \cong -0,228 \text{ mm.} \end{array}$