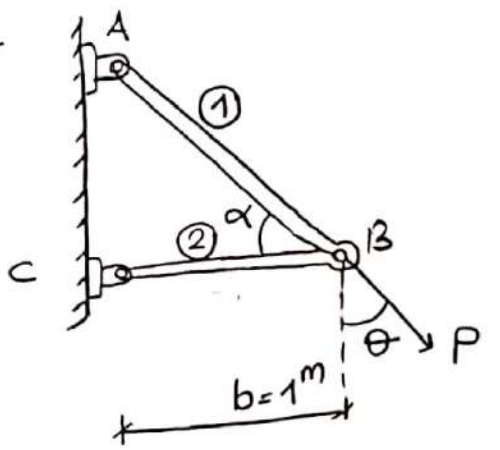
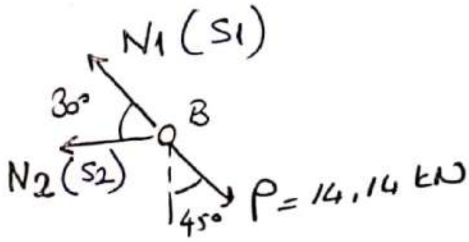
Example:



$\theta = 45°$
$\alpha = 30°$

The linkage is made of two pin connected steel members (1 and 2) having a cross-sectional areas of 10 cm² and 5 cm², respectively. Determine the horizontal and vertical displacement of B point.

$E = 210\ GPa$
$P = 14{,}14\ kN$



$\Sigma F_x = 0$

$P.\sin 45 = S_2 + S_1.\cos 30$

$\boxed{10 = S_2 + S_1.0{,}866}$ --- (1)

$S_2 \ (N_2) \cong -7{,}32\ kN$

$\Sigma F_y = 0$

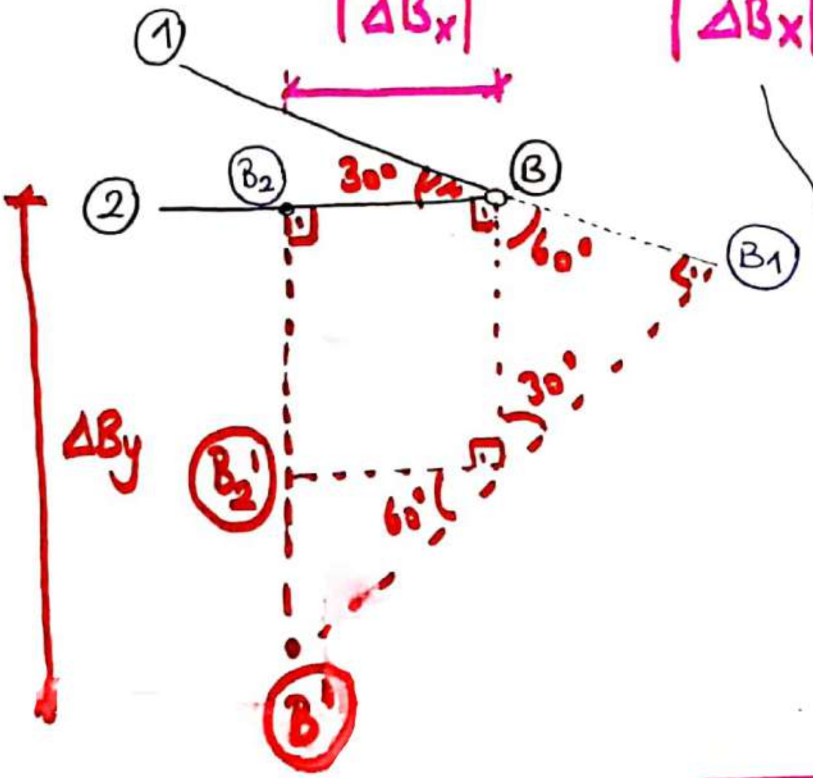
$S_1.\sin 30 = P.\cos 45$ -- (2)

$S_1 = 20\ kN$ (N_1)

$$\Delta L = \frac{N_i L_i}{E_i A_i} \Rightarrow$$

$$\Delta L_1 = \frac{S_1 (b/\cos 30)}{E(A_1)} = \frac{(20\times10^3)^N (1{,}155^m)}{(210\times10^9)(0{,}001)} \cong 0{,}11\ mm$$

$$\Delta L_2 = \frac{S_2(b)}{E A_2} = \frac{(-7{,}32\times10^3)(1)}{(210\times10^9)\,0{,}0005} = -0{,}07 \text{ mm}$$



$|\Delta B_x| = \Delta L_2 = \overline{BB_2} = 0{,}07$ mm

$\overline{BB_1} = \Delta L_1 = 0{,}11$ mm

horizontal disp. of B point

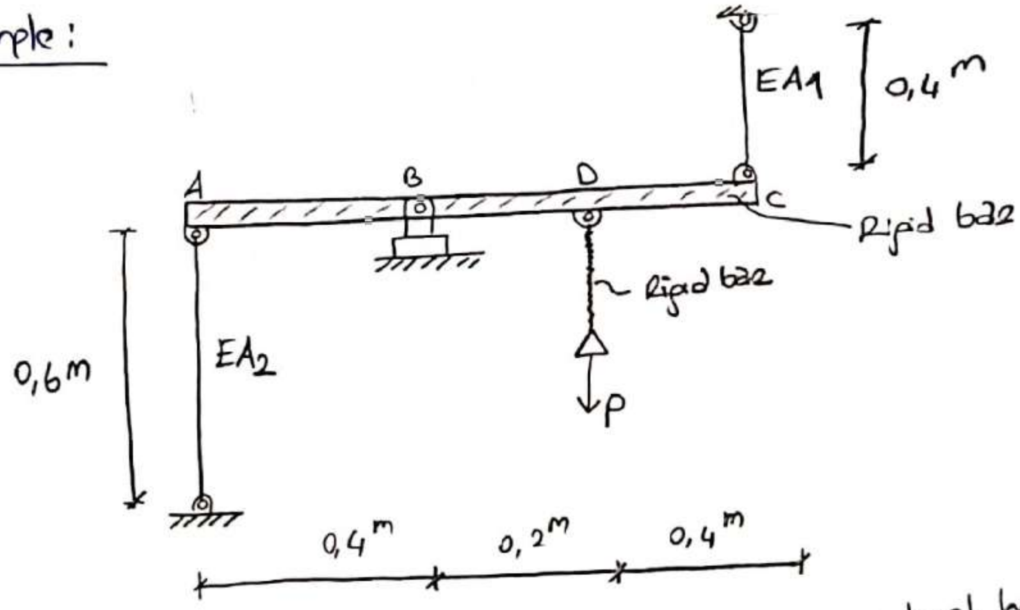
$$\Delta B_y = \overline{B_2' B'} + \overline{B_2 B_2'}$$

$$\overline{B_2 B_2'} \Rightarrow \cos 60 = \frac{\overline{BB_1}}{\overline{B_2 B_2'}} = 0{,}22 \text{ mm}$$

$$\overline{B' B_2'} \Rightarrow \tan 60 = \frac{\overline{B' B_2'}}{|\Delta B_x|} = 0{,}12 \text{ mm}$$

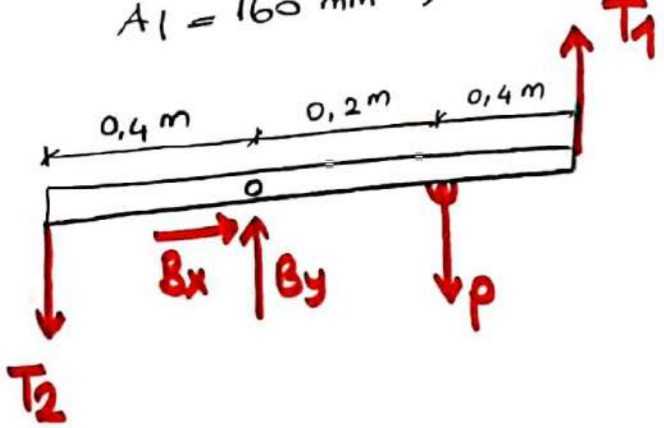
$$\Delta B_y = 0{,}22 + 0{,}12 = 0{,}34 \text{ mm} \;//$$ → vertical disp. of B point

Example:



The rigid and massless AC bar is kept horizontally by two elastic cables and one hinged support. If the allowable stress σ_{allow} for the cables is = 140 MPa, determine the magnitude of "P" that can be carried in safety. **

$A_1 = 160\ mm^2$, $A_2 = 240\ mm^2$

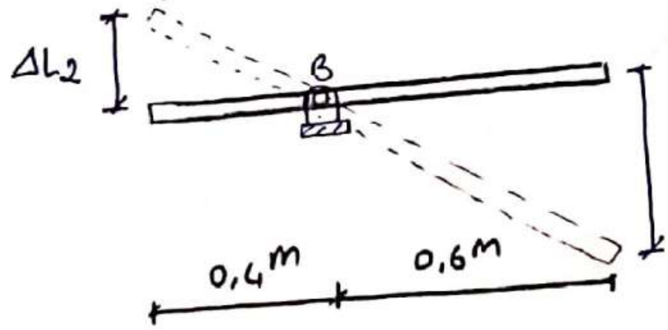


$+\circlearrowleft \sum M_B = 0$

$$\boxed{P \cdot 0{,}2 = T_2(0{,}4) + T_1(0{,}6)} \quad \text{--- (A)}$$

Compatibility relation.

$$\frac{\Delta L_1}{0{,}6} = \frac{\Delta L_2}{0{,}4}$$



$$\Delta L_1 = \frac{T_1 \cdot L_1}{EA_1}$$

$$\Delta L_2 = \frac{T_2 \cdot L_2}{EA_2}$$

$$\boxed{\frac{T_1 \cdot L_1}{EA_1 \cdot 0.6} = \frac{T_2 \cdot L_2}{EA_2\, 0.4}} \quad --- \text{ (B)}$$

$$0{,}4 \cdot \cancel{E}\,(240\ mm^2) = 0{,}6\,\cancel{E}\,(160\ mm^2) \cdot \frac{T_2 \cdot 600\ mm}{T_1 \cdot 400\ mm}$$

$$\boxed{96\,T_1 = 144\,T_2}$$

$$96\left[\frac{0{,}2P - 0{,}4\,T_2}{0{,}6}\right] = 144\,T_2$$

comes from Eq (A)

$$\boxed{T_2 = 2P/13}$$

$$\boxed{T_1 = 3P/13}$$

$$\sigma_{max} \leq \sigma_{allow} \quad ; \quad FS = \frac{\sigma_{max} = \sigma_{fail}}{\sigma_{allow}}$$

$$\sigma_1 = \frac{T_1}{A_1} \quad ; \quad \sigma_2 = \frac{T_2}{A_2}$$

$$\sigma_1 = \frac{3P/13}{(160\ mm^2)} \leq 140\ \underbrace{N/mm^2}_{\sigma_{allow}}$$

$$\boxed{P_1 \lesssim 97{,}1\ kN}$$

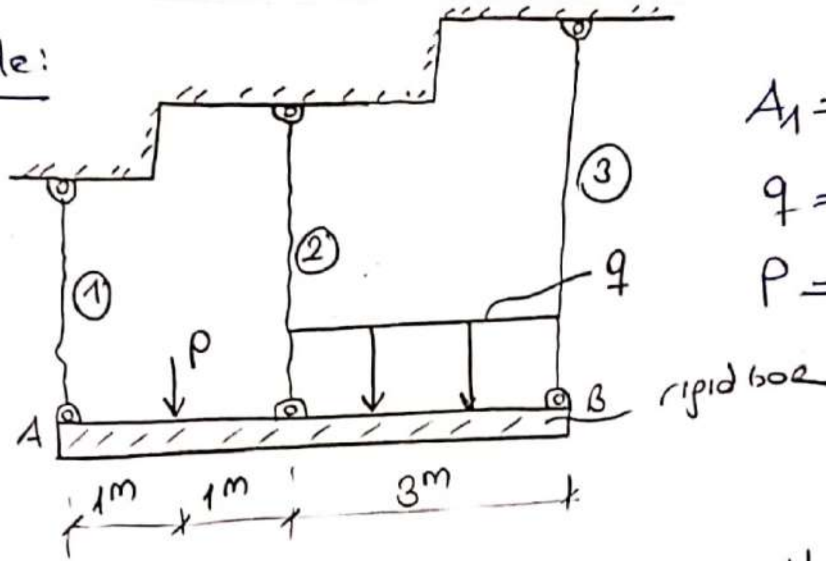
$$\sigma_2 = \frac{2P}{13(240\ mm^2)} \leq 140$$

$$\boxed{P_2 \lesssim 218{,}4\ kN}$$

$$\boxed{P = P_{max} \leq 97{,}1\ kN}$$

(6)

Example:



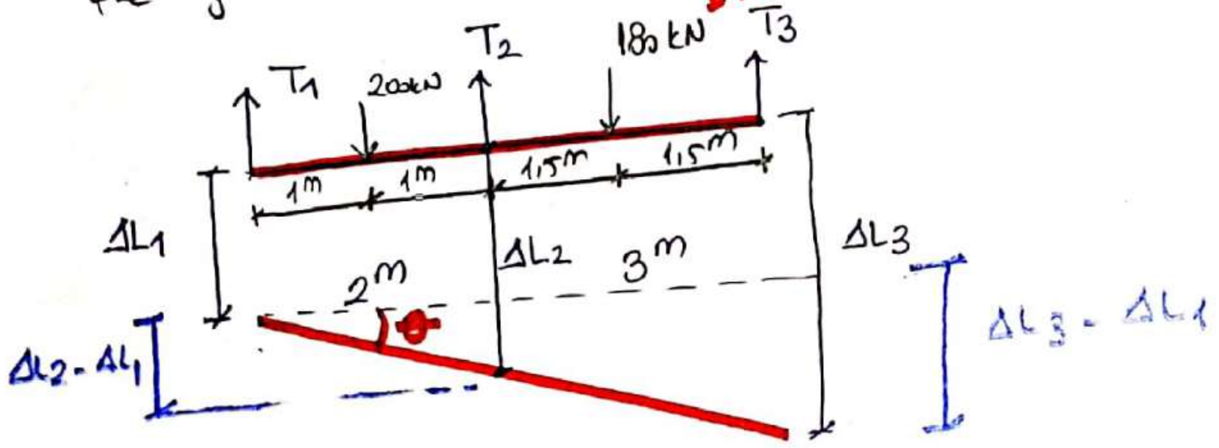
$A_1 = A$, $A_2 = 1{,}5A$, $A_3 = 2A$

$q = 60\ kN/m$

$P = 200\ kN$

The rigid bar is originally horizontal and it is assumed that it has no weight. This bar is supported by 3 cables each having different cross sectional areas and $E = 200 GPa$. $\sigma_{allow} = 140\ MPa$

Determine the minimum cross-sectional area A with regard to structural system safety and determine the slight rotation of the rigid AB bar after loading.



$\overset{+}{\uparrow} \sum F_y = 0$

$$\boxed{T_1 + T_2 + T_3 = 380\ kN} \quad \text{-- (1)}$$

$+\circlearrowleft \sum M_A = 0$

$$T_2 \cdot 2 + T_3 \cdot 5 - P \cdot 1 - q \cdot 3 \cdot 3{,}5 = 0$$

$$\boxed{2T_2 + 5T_3 = 830\ kN} \quad \text{-- (2)}$$

we've 3 unknowns.

Compatibility Rel.

$$\underbrace{\frac{\Delta L_2 - \Delta L_1}{2} = \frac{\Delta L_3 - \Delta L_1}{5}}_{(3)}$$

$$\Delta L_1 = \frac{T_1 . L_1}{EA} \quad ; \quad \Delta L_2 = \frac{T_2 . L_2}{E(1,5A)} \quad ; \quad \Delta L_3 = \frac{T_3 . L_3}{E(2A)}$$

$$\frac{1}{2}\left(\frac{T_2 . L_2}{1,5EA} - \frac{T_1 . L_1}{EA}\right) = \frac{1}{5}\left(\frac{T_3 . L_3}{2AE} - \frac{T_1 L_1}{EA}\right)$$

$$5T_2 - 3T_3 = 3T_1 \quad \text{--- (3)}$$

$$\text{Eq (1)} \Rightarrow T_2 + T_3 = 380 - T_1$$

$$\boxed{T_2 = 142,5 \text{ kN}}$$

$$T_3 = 103 \text{ kN}$$

$$T_1 = 128,5 \text{ kN}$$

$$\# \boxed{\sigma_{max} \leq \sigma_{allow}} \#$$

$$\sigma_1 = \frac{T_1}{A_1} = \frac{128500 \text{N}}{A} \leq \sigma_{allow} = 140 \times 10^6 \text{ N/m}^2$$

$$A_1 \geq 918 \text{ mm}^2$$

$$\sigma_2 = \frac{142500}{1,5A} \leq \sigma_{allow}$$

$$A_2 \geq 679 \text{ mm}^2$$

$$\sigma_3 = \frac{103000}{2A} \leq \sigma_{allow}$$

$$A_3 \geq 390 \text{ mm}^2$$

$$A \geq 918 \text{ mm}^2$$

$$A = 918 \text{ mm}^2$$

$$\Delta L_1 = \frac{(128{,}5 \times 10^3)(2)^N}{(918 \times 10^{-6})(200 \times 10^9)} = 1{,}4 \times 10^{-3} m$$

$$\Delta L_1 = 1.4 \text{ mm}$$

$$\Delta L_2 = 1{,}55 \text{ mm}$$

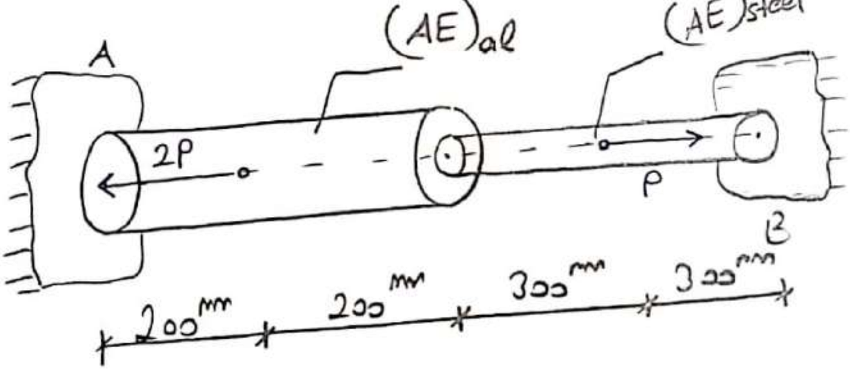
$$\Delta L_3 = 1{,}78 \text{ mm}$$

$$\theta = \tan^{-1}\left[\frac{\Delta L_3 - \Delta L_1 \ (\text{mm})}{5000 \ (\text{mm})}\right]$$

$$\theta = 0{,}00435^\circ$$

Example:

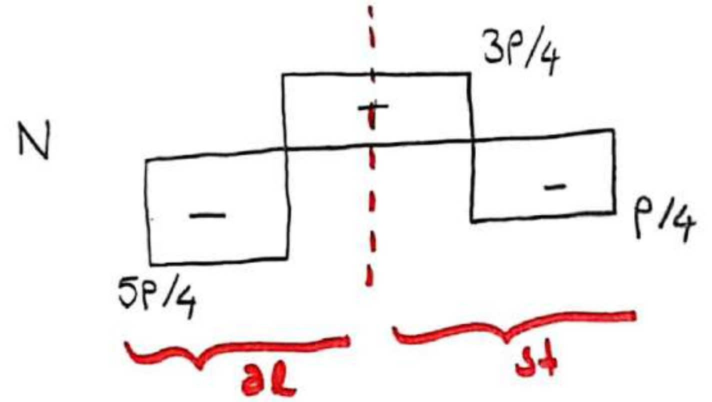
HW

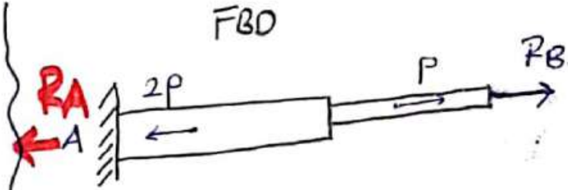


	Aluminum	Steel
E	70 GPa	210 GPa
A	2000 mm²	1000 mm²
σ_{allow}	60 MPa	100 MPa

The rod is fixed connected at its ends A and B and is subjected to axial loads. The rod is made of two different materials, aluminum and steel. Determine the support reactions and draw the normal force diagram. In addition, determine the maximum value of the P load that the rod can safely carry.

HW – Example: $\xleftarrow{R_A = -5P/4} \qquad \longrightarrow R_B = -P/4$



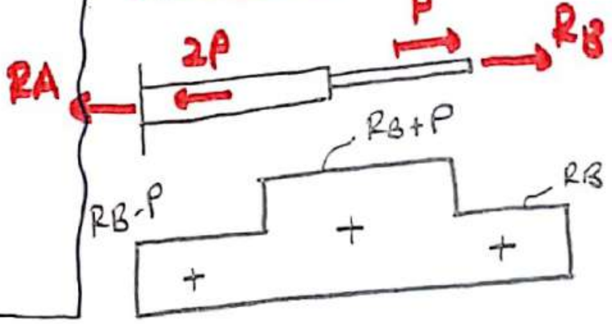


$\underline{\Sigma F_z = 0}$ Eq. Eqn.

$R_A = R_B + P - 2P$

$R_A = R_B - P$

$$\left.\begin{aligned} P_s &\le 133 \text{ kN} \\ P_a &\le 96 \text{ kN} \end{aligned}\right\} \quad P \le 96 \text{ kN}$$



$\boxed{\Delta L = 0}$ ... compatibility relation

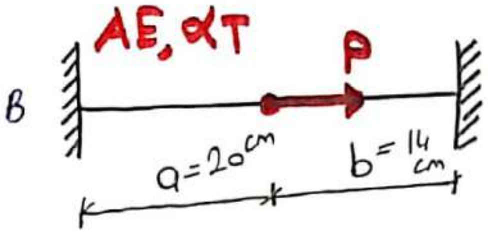
$$0 = \frac{(R_B - P) \cdot 200}{A_{al} E_{al}} + \frac{(R_B + P)\, 200}{A_{al} E_{al}} + \frac{(R_B + P) \cdot 300}{A_{st} E_{st}}$$

$$+ \frac{(R_B) \cdot 300}{A_{st} E_{st}}$$

$N_{max} \Rightarrow al \Rightarrow 5P/4 \quad (\sigma_{allow})_{al}$

$\Rightarrow st \Rightarrow 3P/4 \quad (\sigma_{allow})_{st}$

Example:



The copper bar shown in Figure is fixed connected at its ends. If the change in temperature is $\Delta T = 20°C$, determine the normal stress developed in the bar.

$\alpha_{copper} = 17{,}5 \times 10^{-6} /°C$

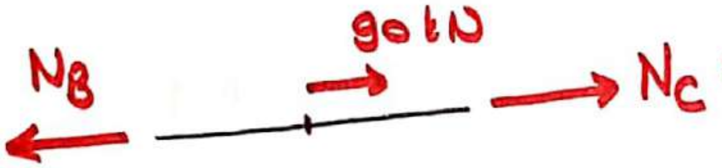
$P = 90\ kN$

$A = 12\ cm^2$

$E = 115\ GPa$

$$\boxed{\Delta L_{Total} = \Delta L_E + \Delta L_T = 0}$$ ... compatibility relation.

$$\Delta L_E = \frac{P.L}{EA} \quad ; \quad \Delta L_T = \alpha . \Delta T . L$$



N_C : support reactions (Normal forces)

$\overset{+}{\rightarrow} \sum F_x = 0$

$N_C + 90 = N_B$ ... (A)

Extensional rigidity

$AE = 1200 \times 115 \times 10^3 = 138 \times 10^6\ N$

$$\underbrace{\frac{(N_C + 90 \times 10^3)(20)}{138 \times 10^6} + \frac{(N_C)(14)}{138 \times 10^6}}_{\Delta L_E} + \underbrace{(17{,}5 \times 10^{-6})(20)(34)}_{\Delta L_T} = 0$$

$N_C = -101\ kN$ ...

$N_B = -11{,}2\ kN$ ...

} support reactions. $\sigma = N/A$