Research Assistant Yurdakul AYGÖRMEZ
Dynamics Lecture 3

Introduction

- **Previously, problems dealing with the motion of particles were solved through the fundamental equation of motion,**

$$\Sigma\vec{F} = m\vec{a}.$$

- **The current chapter introduces two additional methods of analysis.**

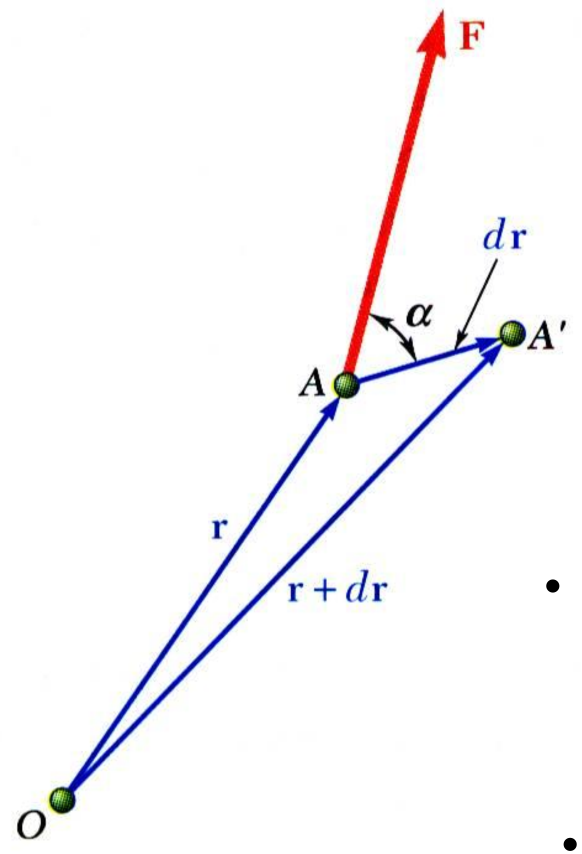
  - ***Method of work and energy*: directly relates force, mass, velocity and displacement.**

  - ***Method of impulse and momentum*: directly relates force, mass, velocity, and time.**

Introduction

Forces and Accelerations	Velocities and Displacements	Velocities and Time
↓	↓	↓
Newton's Second Law	**Work-Energy**	**Impulse-Momentum**
$\sum \vec{F} = m\vec{a}_G$	$T_1 + U_{1\to 2} = T_2$	$m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\, dt = m\vec{v}_2$

Work of a Force



- Differential vector $d\vec{r}$ is the *particle displacement*.

  - *Work of the force* is

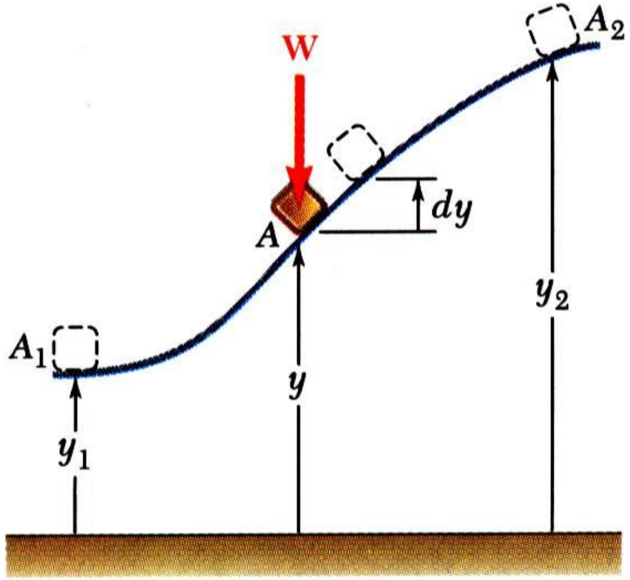
$$
\begin{aligned}
dU &= \vec{F} \bullet d\vec{r} \\
&= F\,ds\,\cos\alpha \\
&= F_x dx + F_y dy + F_z dz
\end{aligned}
$$

- Work is a ***scalar*** quantity, i.e., it has magnitude and sign but not direction.

- Dimensions of work are length $\times$ force. Units are

$$1\,\mathrm{J}\,(\mathit{joule}) = (1\,\mathrm{N})(1\,\mathrm{m}) \qquad 1\,\mathrm{ft}\cdot\mathrm{lb} = 1.356\,\mathrm{J}$$

Work of a Force



- Work of the force of gravity,

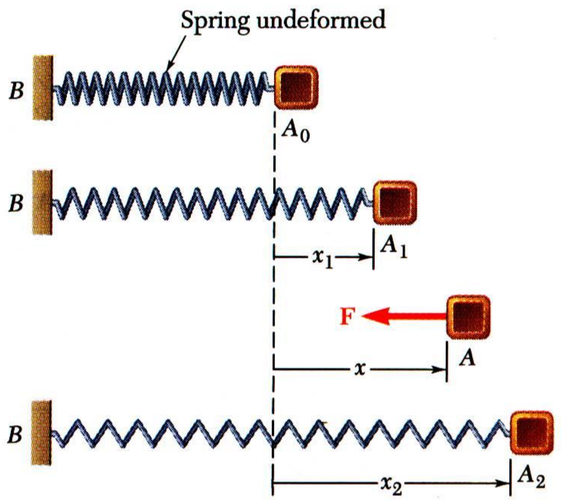
$$dU = F_x dx + F_y dy + F_z dz$$
$$= -W\ dy$$
$$U_{1\to 2} = -\int_{y_1}^{y_2} W\ dy$$
$$= -W(y_2 - y_1) = -W\ \Delta y$$

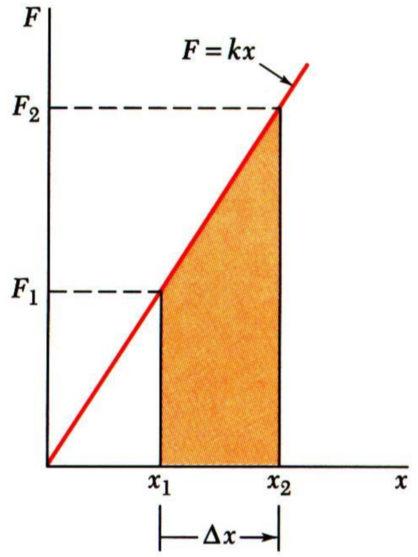
- Work *of the weight* is equal to product of weight *W* and vertical displacement Δy.

- **In the figure above, when is the work done by the weight positive?**

a) Moving from y_1 to y_2 | b) Moving from y_2 to y_1 | c) Never

Work of a Force





- Magnitude of the force exerted by a spring is proportional to deflection,

$$F = kx$$

$$k = \text{spring constant } (\text{N/m or lb/in.})$$

- Work *of the force exerted by spring*,

$$dU = -F\,dx = -kx\,dx$$

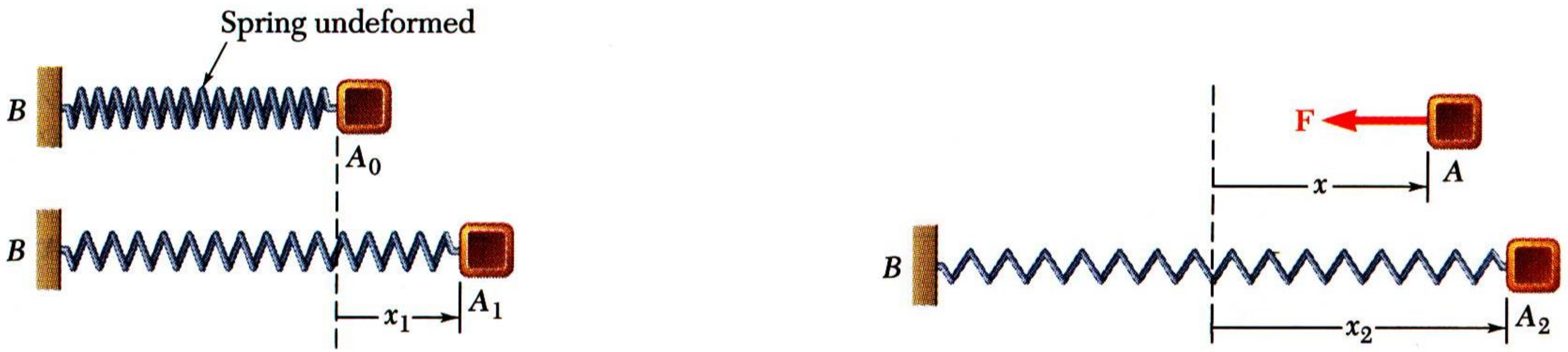
$$U_{1\to 2} = -\int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

- Work *of the force exerted by spring* is positive when $x_2 < x_1$, i.e., when the spring is returning to its undeformed position.

- Work of the force exerted by the spring is equal to negative of area under curve of F plotted against x,

$$U_{1\to 2} = -\tfrac{1}{2}(F_1 + F_2)\,\Delta x$$

Work of a Force



As the block moves from A_0 to A_1, is the work positive or negative?

Positive

Negative

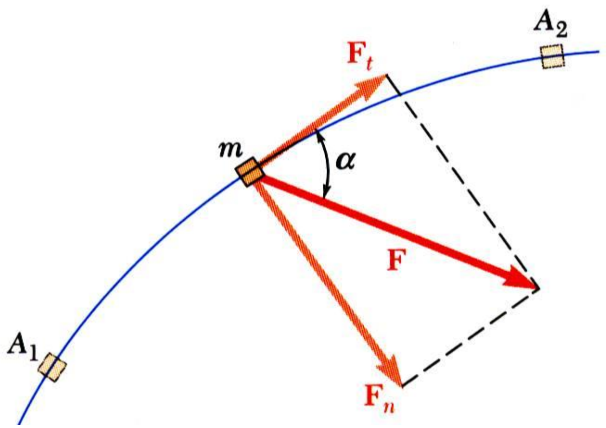
Displacement is in the opposite direction of the force

As the block moves from A_2 to A_o, is the work positive or negative?

Positive

Negative

Particle Kinetic Energy: Principle of Work & Energy



- Consider a particle of mass m acted upon by force $\vec{F}$

$$F_t = ma_t = m\frac{dv}{dt}$$

$$= m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds}$$

$$F_t\,ds = mv\,dv$$

- Integrating from A_1 to A_2 ,

$$\int_{s_1}^{s_2} F_t ds = m\int_{v_1}^{v_2} v\,dv = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

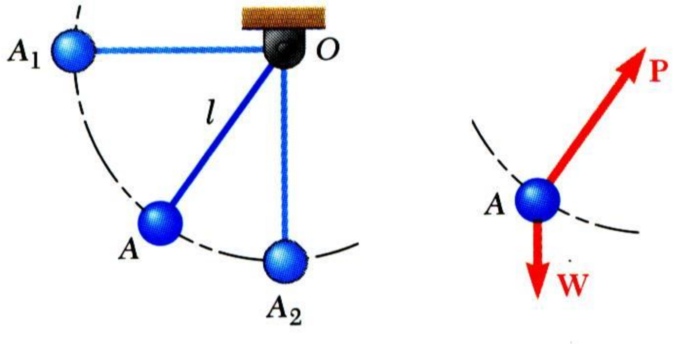
$$U_{1\to 2} = T_2 - T_1 \qquad T = \tfrac{1}{2}mv^2 = kinetic\ energy$$

- *The work of the force $\vec{F}$ is equal to the change in kinetic energy of the particle.*

- Units of work and kinetic energy are the same:

$$T = \tfrac{1}{2}mv^2 = \mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)\mathrm{m} = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

Applications of the Principle of Work and Energy



- The bob is released from rest at position A_1. Determine the velocity of the pendulum bob at A_2 using work & kinetic energy.

- Force $\vec{P}$ acts normal to path and does no work.

$$T_1 + U_{1\to 2} = T_2$$

$$0 + Wl = \frac{1}{2}\frac{W}{g}v_2^2$$

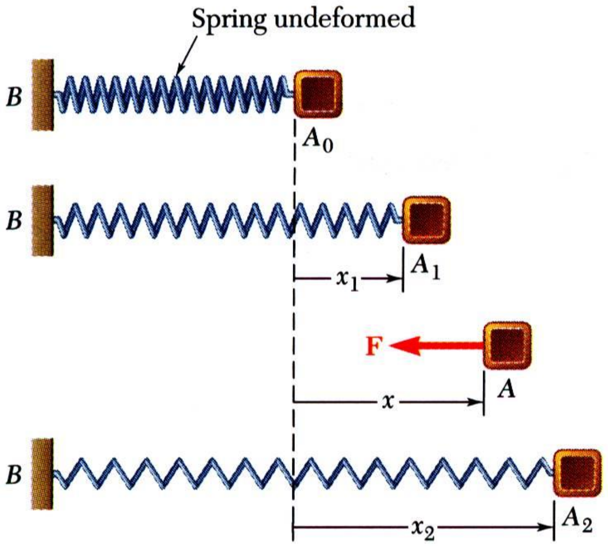
$$v_2 = \sqrt{2gl}$$

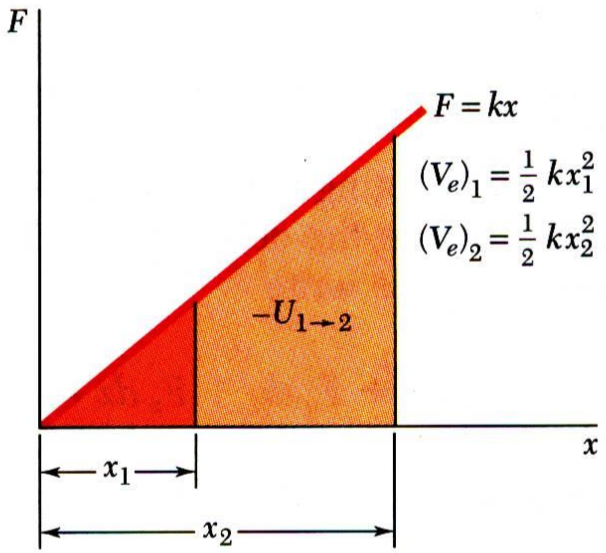
- Velocity is found without determining expression for acceleration and integrating.

- All quantities are scalars and can be added directly.

- Forces which do no work are eliminated from the problem.

Potential Energy





- Work of the force exerted by a spring depends only on the initial and final deflections of the spring,

$$U_{1\to 2} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

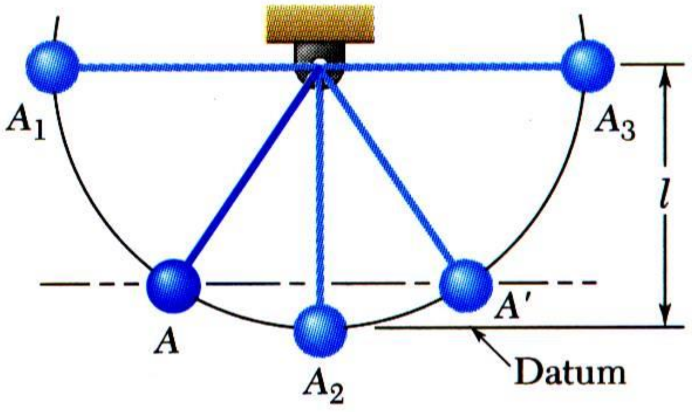
- The potential energy of the body with respect to the elastic force,

$$V_e = \tfrac{1}{2}kx^2$$

$$U_{1\to 2} = (V_e)_1 - (V_e)_2$$

- Note that the preceding expression for V_e is valid only if the deflection of the spring is measured from its undeformed position.

Conservation of Energy



$$T_1 = 0 \quad V_1 = W\ell$$
$$T_1 + V_1 = W\ell$$

$$T_2 = \tfrac{1}{2} m v_2^2 = \frac{1}{2}\frac{W}{g}(2g\ell) = W\ell \quad V_2 = 0$$
$$T_2 + V_2 = W\ell$$

- Work of a conservative force,

$$U_{1\to 2} = V_1 - V_2$$

- Concept of work and energy,

$$U_{1\to 2} = T_2 - T_1$$

- Follows that

$$T_1 + V_1 = T_2 + V_2$$
$$E = T + V = \text{constant}$$

- When a particle moves under the action of conservative forces, the total mechanical energy is constant.
- Friction forces are not conservative. Total mechanical energy of a system involving friction decreases.
- Mechanical energy is dissipated by friction into thermal energy. Total energy is constant.

Prove that a force $F(x, y, z)$ is conservative if, and only if, the following relations are satisfied:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

For a conservative force, Equation must be satisfied.

$$F_x = -\frac{\partial V}{\partial x} \quad F_y = -\frac{\partial V}{\partial y} \quad F_z = -\frac{\partial V}{\partial z}$$

We now write $\frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial x \partial y} \quad \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial y \partial x}$

Since $\frac{\partial^2 V}{\partial x \partial y} = \frac{\partial^2 V}{\partial y \partial x}$:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \blacktriangleleft$$

We obtain in a similar way

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} \blacktriangleleft$$

$$F_x\,dx + F_y\,dy + F_z\,dz = -\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)$$

from which it follows that

$$F_x = -\frac{\partial V}{\partial x} \qquad F_y = -\frac{\partial V}{\partial y} \qquad F_z = -\frac{\partial V}{\partial z} \tag{13.22}$$

It is clear that the components of **F** must be functions of the coordinates x, y, and z. Thus, a *necessary* condition for a conservative force is that it depend only upon the position of its point of application. The relations (13.22) can be expressed more concisely if we write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} = -\left(\frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j} + \frac{\partial V}{\partial z}\mathbf{k}\right)$$

The vector in parentheses is known as the *gradient of the scalar function* V and is denoted by **grad** V. We thus write for any conservative force

$$\mathbf{F} = -\mathbf{grad}\,V \tag{13.23}$$

1) The force $\boldsymbol{F} = (yz\vec{\imath} + zx\vec{\jmath}/xy\vec{k})/xyz$ acts on the particle *P*(*x*, *y*, *z*) which moves in space. (***a***) Show that this force is a conservative force. (***b***) Determine the potential function associated with **F**.

(*a*) $$F_x = \frac{yz}{xyz} \quad F_y = \frac{zx}{xyz} \quad F_z = \frac{xy}{xyz}$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(\frac{1}{x}\right)}{\partial y} = 0 \quad \frac{\partial F_y}{\partial x} = \frac{\partial\left(\frac{1}{y}\right)}{\partial x} = 0$$

Thus, $$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The other two equations derived are checked in a similar way.

(*b*) Recall that

$$F_x = -\frac{\partial V}{\partial x}, \qquad F_y = -\frac{\partial V}{\partial y}, \qquad F_z = -\frac{\partial V}{\partial z}$$

$$F_x = \frac{1}{x} = -\frac{\partial V}{\partial x} \qquad V = -\ln x + f(y, z) \tag{1}$$

$$F_y = \frac{1}{y} = -\frac{\partial V}{\partial y} \qquad V = -\ln y + g(z, x) \tag{2}$$

$$F_z = \frac{1}{z} = -\frac{\partial V}{\partial z} \qquad V = -\ln z + h(x, y) \tag{3}$$

Equating (1) and (2)

$$-\ln x + f(y,z) = -\ln y + g(z,x)$$

Thus,

$$f(y,z) = -\ln y + k(z) \tag{4}$$

$$g(z,x) = -\ln x + k(z) \tag{5}$$

Equating (2) and (3)

$$-\ln z + h(x,y) = -\ln y + g(z,x)$$

$$g(z,x) = -\ln z + l(x)$$

From (5),

$$g(z,x) = -\ln x + k(z)$$

Thus,

$$k(z) = -\ln z$$

$$l(x) = -\ln x$$

From (4),

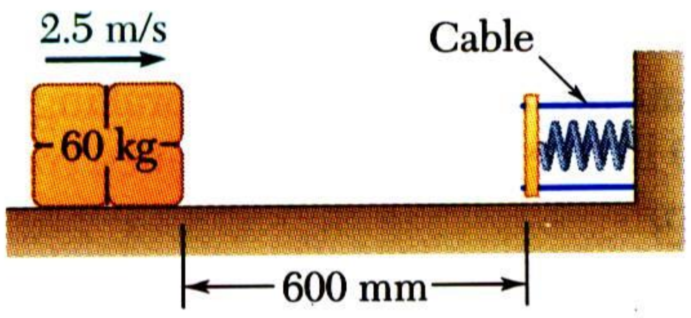
$$f(y,z) = -\ln y - \ln z$$

Substitute for $f(y,z)$ in (1)

$$V = -\ln x - \ln y - \ln z$$

$$V = -\ln xyz \blacktriangleleft$$

2)

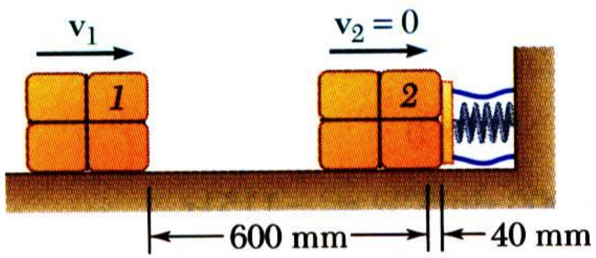


A spring is used to stop a 60 kg package which is sliding on a horizontal surface. The spring has a constant k = 20 kN/m and is held by cables so that it is initially compressed 120 mm. The package has a velocity of 2.5 m/s in the position shown and the maximum deflection of the spring is 40 mm.

Determine *(a)* the coefficient of kinetic friction between the package and surface and *(b)* the velocity of the package as it passes again through the position shown.

SOLUTION:

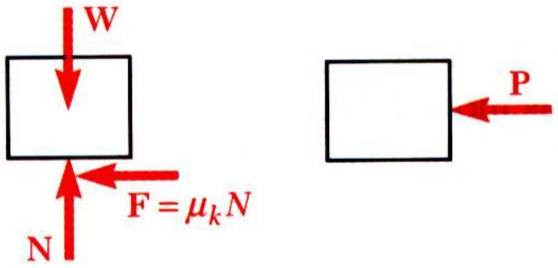
- Apply the principle of work and energy between the initial position and the point at which the spring is fully compressed and the velocity is zero. The only unknown in the relation is the friction coefficient.
- Apply the principle of work and energy for the rebound of the package. The only unknown in the relation is the velocity at the final position.



SOLUTION:

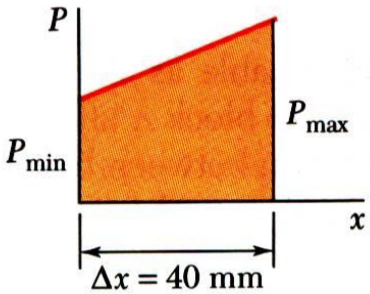
- Apply principle of work and energy between initial position and the point at which spring is fully compressed.

$$T_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2}(60\,\text{kg})(2.5\,\text{m/s})^2 = 187.5\,\text{J} \qquad T_2 = 0$$



$$\left(U_{1\to 2}\right)_f = -\mu_k W x$$
$$= -\mu_k (60\,\text{kg})\left(9.81\,\text{m/s}^2\right)(0.640\,\text{m}) = -(377\,\text{J})\mu_k$$

$$P_{\min} = k x_0 = (20\,\text{kN/m})(0.120\,\text{m}) = 2400\ \text{N}$$
$$P_{\max} = k(x_0 + \Delta x) = (20\,\text{kN/m})(0.160\,\text{m}) = 3200\ \text{N}$$
$$\left(U_{1\to 2}\right)_e = -\tfrac{1}{2}\left(P_{\min} + P_{\max}\right)\Delta x$$
$$= -\tfrac{1}{2}(2400\ \text{N} + 3200\ \text{N})(0.040\,\text{m}) = -112.0\,\text{J}$$



$$U_{1\to 2} = \left(U_{1\to 2}\right)_f + \left(U_{1\to 2}\right)_e = -(377\,\text{J})\mu_k - 112\,\text{J}$$

$$T_1 + U_{1\to 2} = T_2 :$$
$$187.5\,\text{J} - (377\ \text{J})\mu_k - 112\ \text{J} = 0$$

$$\boxed{\mu_k = 0.20}$$

$$P_{\min} = kx_0 = (20\,\text{kN/m})(0.120\,\text{m}) = 2400\text{ N}$$

$$P_{\max} = k(x_0 + \Delta x) = (20\,\text{kN/m})(0.160\,\text{m}) = 3200\text{ N}$$

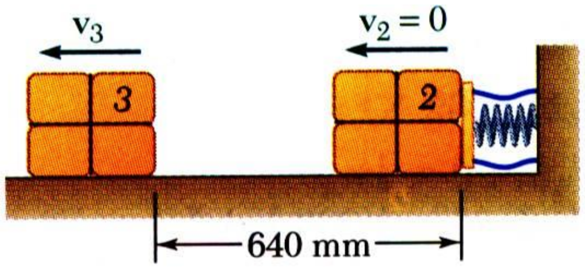
$$(U_{1\to 2})_e = -\tfrac{1}{2}(P_{\min} + P_{\max})\Delta x$$

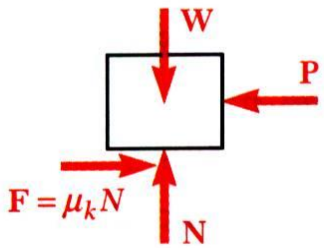
$$= -\tfrac{1}{2}(2400\text{ N} + 3200\text{ N})(0.040\,\text{m}) = -112.0\text{ J}$$

$$U_{1\to 2} = \frac{1}{2}kx_1^{\,2} - \frac{1}{2}kx_2^{\,2}$$

$$= \frac{1}{2}(20000)(0.120)^2 - \frac{1}{2}(20000)(0.160)^2$$

$$= -112\text{ J}$$





- Apply the principle of work and energy for the rebound of the package.

$$T_2 = 0 \qquad T_3 = \tfrac{1}{2} m v_3^2 = \tfrac{1}{2}(60\,\text{kg})v_3^2$$

$$U_{2\to3} = \left(U_{2\to3}\right)_f + \left(U_{2\to3}\right)_e = -(377\ \text{J})\mu_k + 112\ \text{J}$$

$$= +36.5\ \text{J}$$

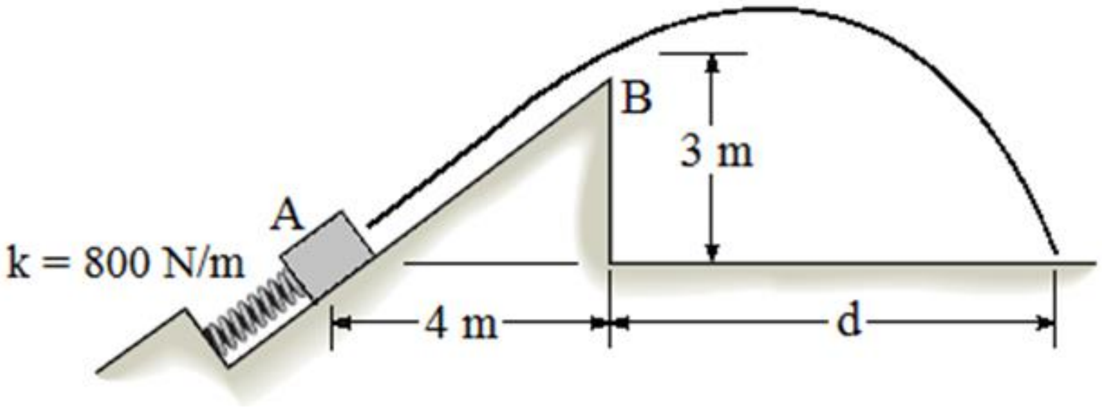
$$T_2 + U_{2\to3} = T_3:$$

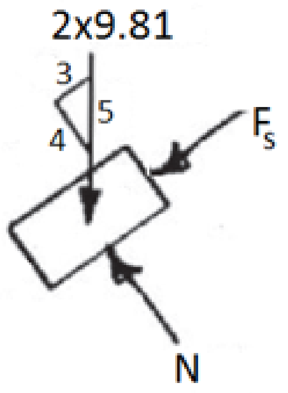
$$0 + 36.5\,\text{J} = \tfrac{1}{2}(60\,\text{kg})v_3^2$$

$$\boxed{v_3 = 1.103\,\text{m/s}}$$

3)

The 2-kg block is pressed against the spring so as to compress it 0.5 m when it is at A. If the coefficient of kinetic friction between the block and the surface AB is $\mu_k = 0.25$, determine the distance d, measured from the wall, to where the block strikes the ground. Neglect the size of the block.





If the work-energy principle is applied between A and B (with $\delta = 5$ m)

$$T_A + \sum U_{A\to B} = T_B$$

$$\frac{1}{2}mv_A^2 + \left[\left(V_g\right)_A + (V_e)_A\right] - F_s \cdot \delta = \frac{1}{2}mv_B^2 + \left[\left(V_g\right)_B + (V_e)_B\right]$$

$$\frac{1}{2}\cdot m\cdot v_A{}^2 + \left(m\cdot g\cdot h_A + \frac{1}{2}\cdot k\cdot x_A{}^2\right) - \mu\cdot m\cdot g\cdot\delta = \frac{1}{2}\cdot m\cdot v_B{}^2 + \left(m\cdot g\cdot h_B + \frac{1}{2}\cdot k\cdot x_B{}^2\right)$$

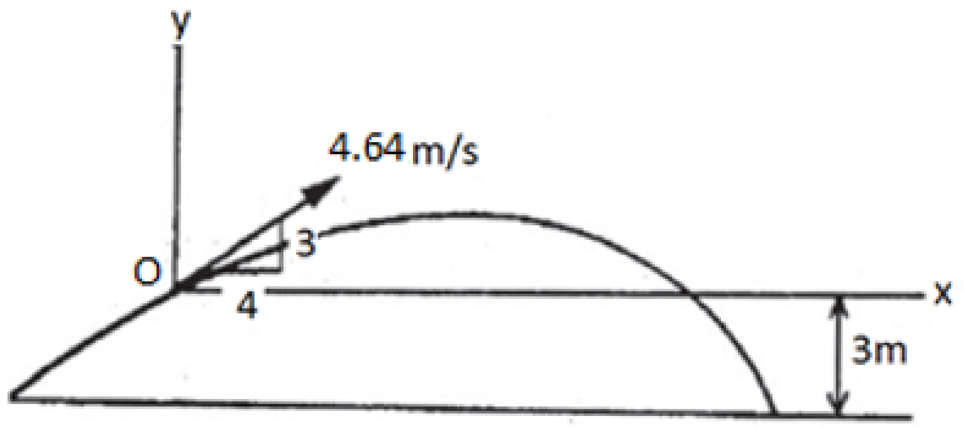
$$\frac{1}{2}\cdot 2\cdot 0^2 + \left(2\cdot 9.81\cdot 0 + \frac{1}{2}\cdot 800\cdot 0.5^2\right) - 0.25\cdot 2\cdot 9.81\cdot\frac{4}{5}\cdot 5 = (2\cdot 9.81\cdot 3 + 0) + \frac{1}{2}\cdot 2\cdot v_B{}^2$$

$$0 + 0 + \frac{1}{2}\cdot 800\cdot 0.5^2 - 0.25\cdot 2\cdot 9.81\cdot\frac{4}{5}\cdot 5 = 2\cdot 9.81\cdot 3 + 0 + \frac{1}{2}\cdot 2\cdot v_B{}^2$$

$$0 + 0 + 100 - 19.62 = 58.86 + 0 + v_B^2$$

$$v_B = 4.64\ \mathrm{m/s}$$

$v_B = 4.64\ \mathrm{m/s}$



If the velocity and motion equations of the block are written

$$y = y_0 + v_0\ \sin\alpha\, t - \frac{1}{2} g\, t^2 \quad (1)$$

$$-3 = 0 + 4.64 \cdot \frac{3}{5} \cdot t - \frac{1}{2} \cdot 9.81 \cdot t^2$$

$$4.905\, t^2 - 2.784t - 3 = 0$$

$t = 1.116\ s$ elde edilir.

$$x = x_0 + v_0\ \cos\alpha\, t \quad (2)$$

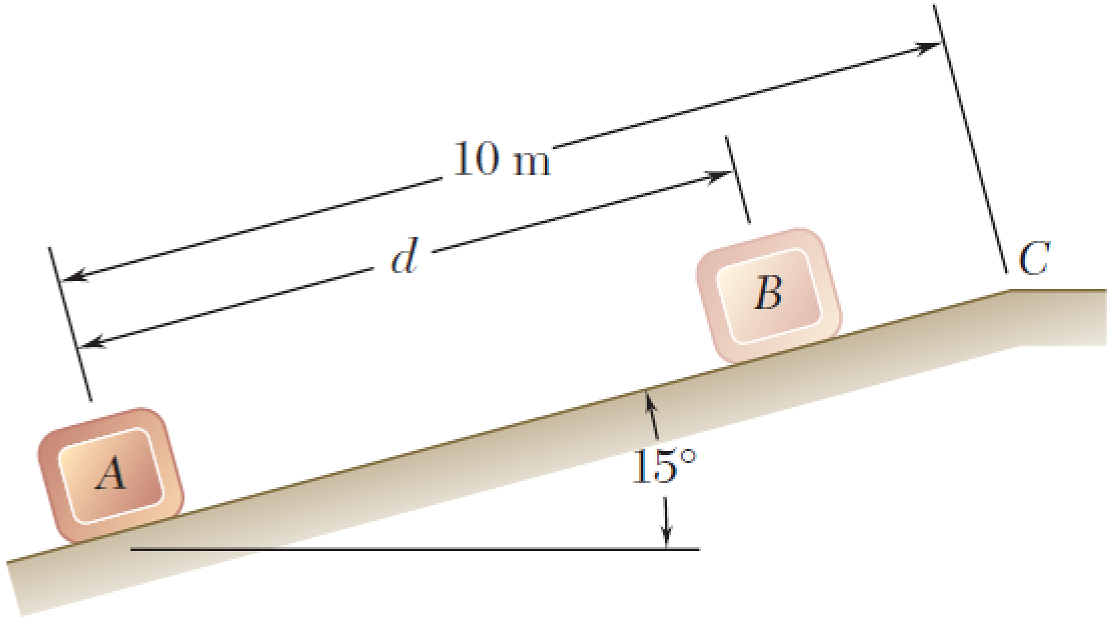
$$d = 0 + 4.64 \cdot \frac{4}{5} \cdot t = 0 + 4.64 \cdot \frac{4}{5} \cdot 1.116 = 4.14\ m$$

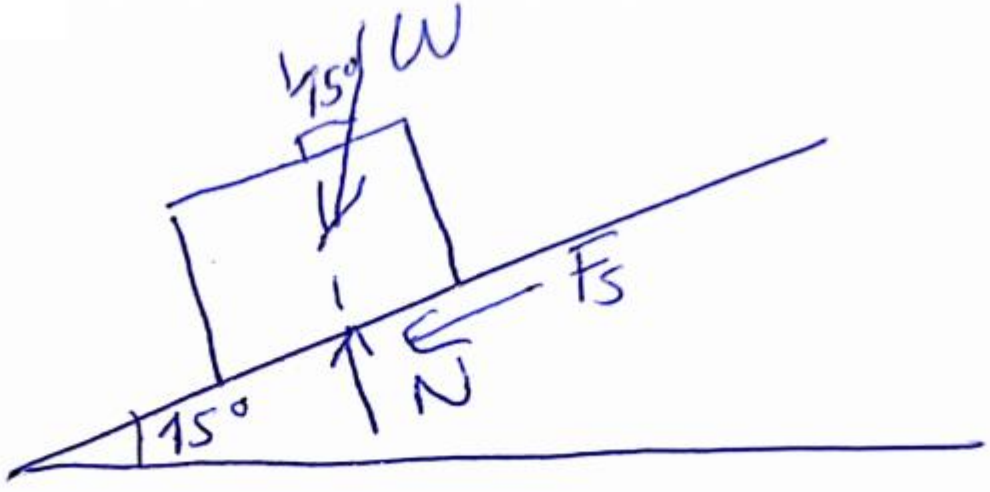
4)

A package is projected up a 15° incline at A with an initial velocity of 8 m/s. Knowing that the coefficient of kinetic friction between the package and the incline is 0.12, determine

(a) the maximum distance d that the package will move up the incline,

(b) the velocity of the package as it returns to its original position.





$$F_s = \mu_k N$$

(*a*) Up the plane from *A* to *B*:

$$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}\frac{W}{g}(8\text{ m/s})^2 = 32\frac{W}{g} \qquad T_B = 0$$

$$U_{A-B} = (-W\ \sin 15° - F)d \qquad F = \mu_k N = 0.12\,\text{N}$$

$$\nwarrow \Sigma F = 0 \quad N - W\cos 15° = 0 \quad N = W\cos 15°$$

$$U_{A-B} = -W(\sin 15° + 0.12\cos 15°)d = -Wd(0.3747)$$

$$T_A + U_{A-B} = T_B: \quad 32\frac{W}{g} - Wd(0.3743) = 0$$

$$d = \frac{32}{(9.81)(0.3747)} \qquad d = 8.71\text{ m} \blacktriangleleft$$

(*b*) Down the plane from *B* to *A*: (*F* reverses direction)

$$T_A = \frac{1}{2}\frac{W}{g}v_A^2 \qquad T_B = 0 \qquad d = 8.71\text{ m}$$

$$U_{B-A} = (W\ \sin 15° - F)d$$

$$= W(\sin 15° - 0.12\cos 15°)(8.71\text{ m}\quad)$$

$$U_{B-A} = 1.245W$$

$$T_B + U_{B-A} = T_A \quad 0 + 1.245W = \frac{1}{2}\frac{W}{g}v_A^2$$

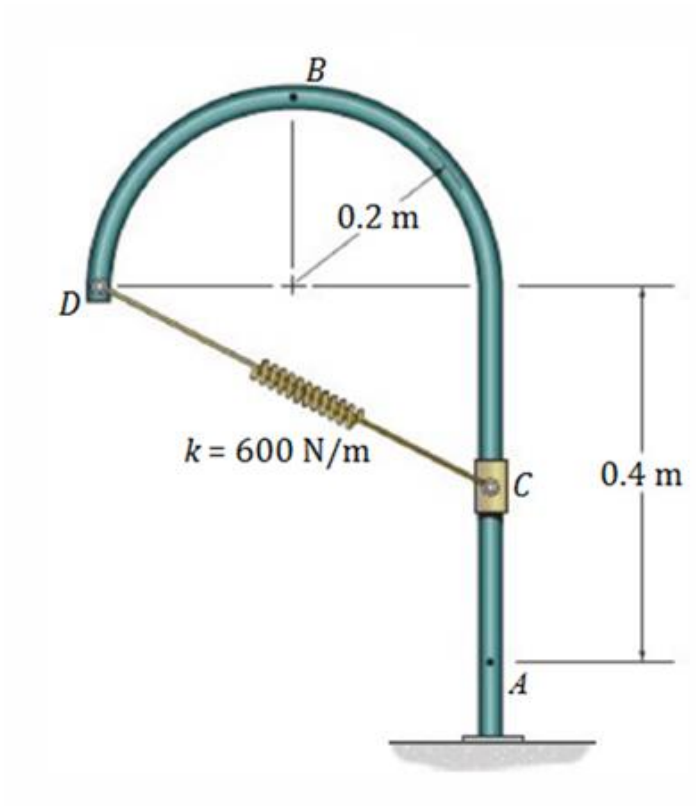
$$v_A^2 = (2)(9.81)(1.245)$$

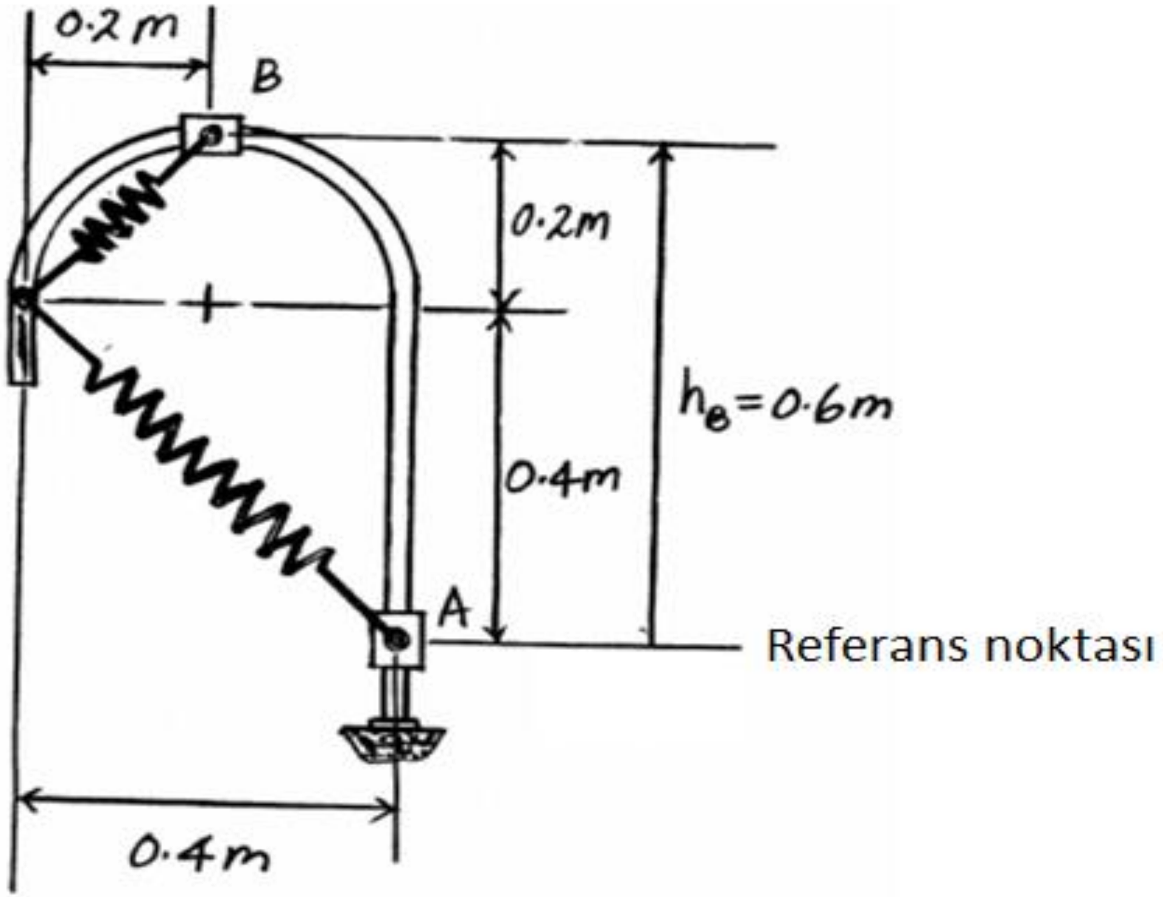
$$= 24.43$$

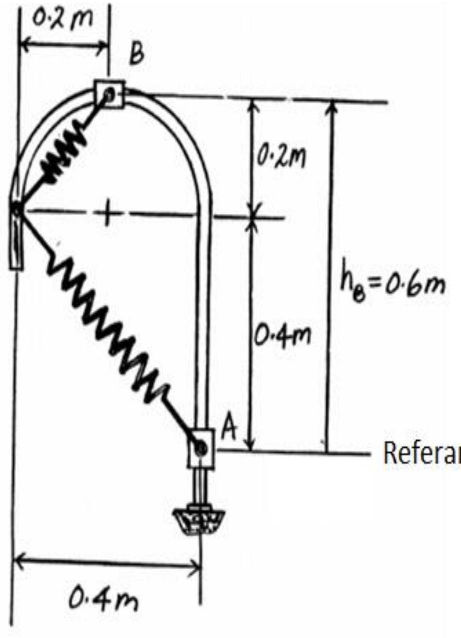
$$v_A = 4.94\text{ m/s} \qquad \mathbf{v}_A = 4.94\text{ m/s} \ \measuredangle\ 15° \ \blacktriangleleft$$

5)

The 2-kg collar is released from rest at A and travels along the smooth vertical guide. Determine the speed of the collar when it reaches position B. Also, find the normal force exerted on the collar at this position. The spring has an unstretched length of 200 mm.




0.2 m
B
0.2m
h_B=0.6m
0.4m
A
Referans noktası
0.4m



a)

Since the unstretched length of the spring is 200 mm, the spring deflection in the first case (A)

$$x_A = \sqrt{0.4^2 + 0.4^2} - 0.2 = 0.3657\text{m}$$

The spring deflection in the second case (B)

$$x_B = \sqrt{0.2^2 + 0.2^2} - 0.2 = 0.08284\text{m}$$

If we choose the reference point A point, if the law of conservation of energy is applied between A and B

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + \left[\left(V_g\right)_A + (V_e)_A\right] = \frac{1}{2}mv_B^2 + \left[\left(V_g\right)_B + (V_e)_B\right]$$

$$\frac{1}{2}.m.v_A{}^2 + \left(m.g.h_A + \frac{1}{2}.k.x_A{}^2\right) = \frac{1}{2}.m.v_B{}^2 + \left(m.g.h_B + \frac{1}{2}.k.x_B{}^2\right)$$

$$\frac{1}{2}.2.0^2 + \left(2.9.81.0 + \frac{1}{2}.600.0.3657^2\right) = \frac{1}{2}.2.v_B{}^2 + \left(2.9.81.0.6 + \frac{1}{2}.600.0.08284^2\right)$$

$$0 + (0 + 40.118) = \frac{1}{2}(2)v_B^2 + (11.772 + 2.0589)$$

$$v_B = 5.127\text{ m/s} = 5.13\text{ m/s}$$

b) While the collar is at point B

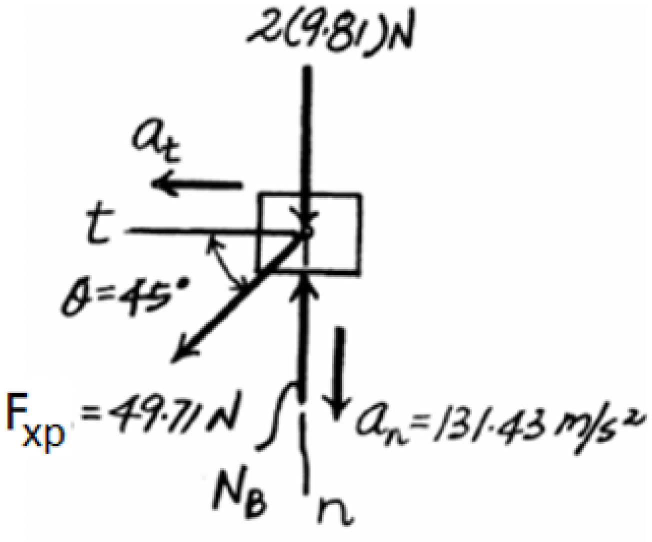
$$\theta = \tan^{-1}\left(\frac{0.2}{0.2}\right) = 45^0$$

Force on the spring

$$F_{xp} = kx_B = 600(0.08284) = 49.71 \text{ N}$$

$$a_n = \frac{v^2}{\rho} = \frac{v_B^2}{0.2} = \frac{5.13^2}{0.2} = 131.43 \text{ m/s}^2$$

If a free body diagram for the collar is drawn at point B



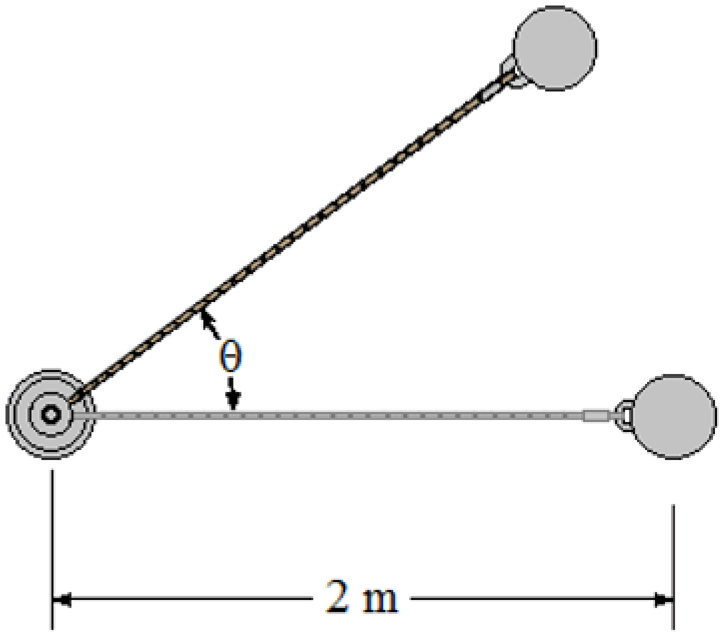
$$\sum F_n = ma_n$$

$$2(9.81) + 49.71 \sin 45^0 - N_B = 2(131.43)$$

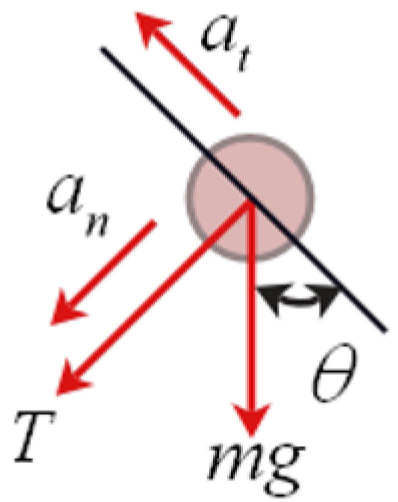
$$N_B = -208.09 \text{ N} = 208 \text{ N} \downarrow$$

6)

The 2-kg pendulum bob moves in the vertical plane with a velocity of 6 m/s when $\theta = 0°$. Determine the angle θ where the tension in the cord becomes zero.



If a free body diagram of a pendulum ball is drawn at any θ position



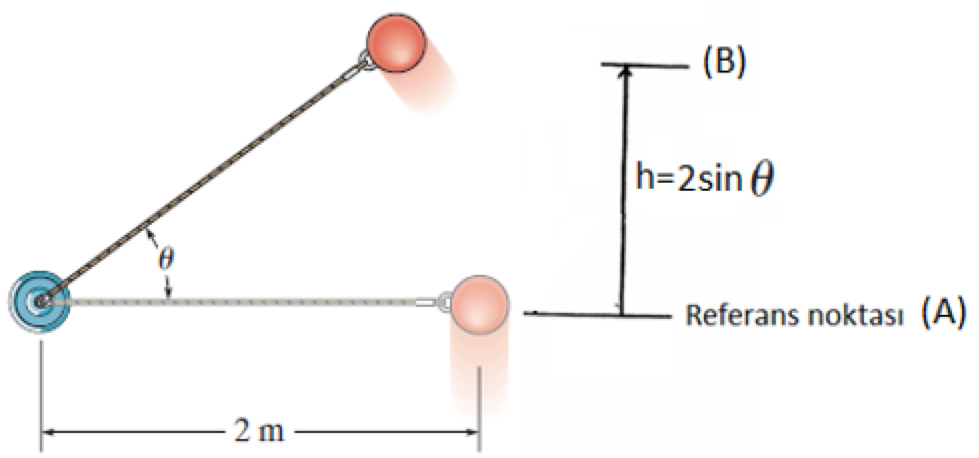
If T = 0

$$+\nwarrow \sum F_t = ma_t\,; \qquad -2(9.81)\cos\theta = 2a_t$$

$$a_t = -9.81\cos\theta$$

$$+\swarrow \sum F_n = ma_n\,; \qquad 2(9.81)\sin\theta = ma_n = m\frac{v^2}{\rho} = 2\frac{v^2}{2}$$

$$v^2 = 19.62\sin\theta$$



If we choose the reference point $\theta = 0^o$ point, if the law of conservation of energy is applied between $\theta = 0^o$ and any position θ

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} \cdot m \cdot {v_A}^2 + m \cdot g \cdot h_A = \frac{1}{2} \cdot m \cdot {v_B}^2 + m \cdot g \cdot h_B$$

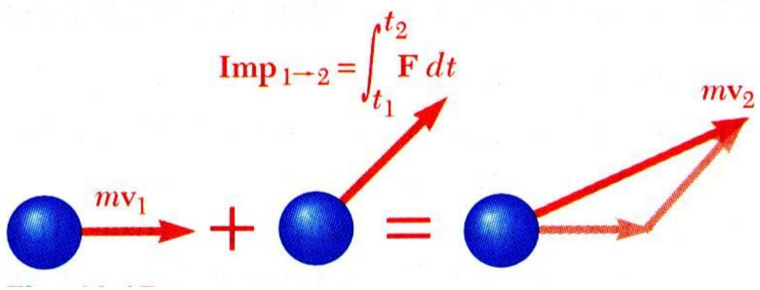
$$\frac{1}{2} \cdot 2 \cdot 6^2 + (2 \cdot 9.81 \cdot 0) = \frac{1}{2} \cdot 2 \cdot (19.62 \sin \theta) + (2 \cdot 9.81 \cdot 2 \sin \theta)$$

$$36 + 0 = (19.62 \sin \theta) + (39.24 \sin \theta)$$

$$58.86 \sin \theta = 36$$

$$\theta = 37.71^o$$

Principle of Impulse and Momentum



- Dimensions of the impulse of a force are

  *force*time.*

- Units for the impulse of a force are

$$\mathrm{N}\cdot\mathrm{s} = \left(\mathrm{kg}\cdot\mathrm{m}/\mathrm{s}^2\right)\cdot\mathrm{s} = \mathrm{kg}\cdot\mathrm{m}/\mathrm{s}$$

- From Newton's second law,

$$\vec{F} = \frac{d}{dt}\left(m\vec{v}\right) \qquad m\vec{v} = \text{linear momentum}$$

$$\vec{F}dt = d\left(m\vec{v}\right)$$

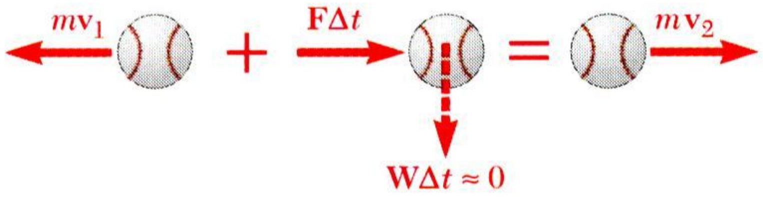
$$\int_{t_1}^{t_2} \vec{F}dt = m\vec{v}_2 - m\vec{v}_1$$

$$\int_{t_1}^{t_2} \vec{F}dt = \mathbf{Imp}_{1\to 2} = \text{impulse of the force } \vec{F}$$

$$m\vec{v}_1 + \mathbf{Imp}_{1\to 2} = m\vec{v}_2$$

- The final momentum of the particle can be obtained by adding vectorially its initial momentum and the impulse of the force during the time interval.

Impulsive Motion



- Force acting on a particle during a very short time interval that is large enough to cause a significant change in momentum is called an *impulsive force*.

- When impulsive forces act on a particle,

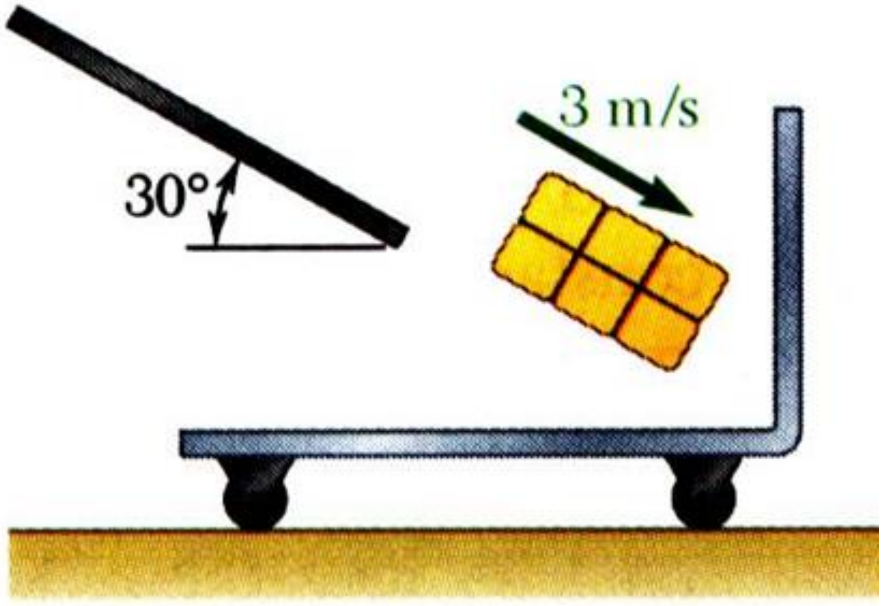
$$m\vec{v}_1 + \sum \vec{F}\,\Delta t = m\vec{v}_2$$

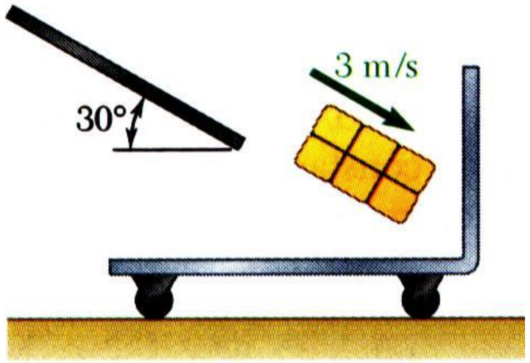
- When a baseball is struck by a bat, contact occurs over a short time interval but force is large enough to change sense of ball motion.

- *Nonimpulsive forces* are forces for which $\vec{F}\Delta t$ is small and therefore, may be neglected – an example of this is the weight of the baseball.

5)

A 10 kg package drops from a chute into a 25-kg cart with a velocity of 3 m/s. Knowing that the cart is initially at rest and can roll freely, determine *(a)* the final velocity of the cart, *(b)* the impulse exerted by the cart on the package, and *(c)* the fraction of the initial energy lost in the impact.





A 10 kg package drops from a chute into a 24 kg cart with a velocity of 3 m/s. Knowing that the cart is initially at rest and can roll freely, determine *(a)* the final velocity of the cart, *(b)* the impulse exerted by the cart on the package, and *(c)* the fraction of the initial energy lost in the impact.

SOLUTION:

- Apply the principle of impulse and momentum to the package-cart system to determine the final velocity.
- Apply the same principle to the package alone to determine the impulse exerted on it from the change in its momentum.

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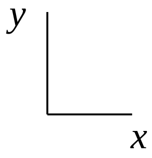
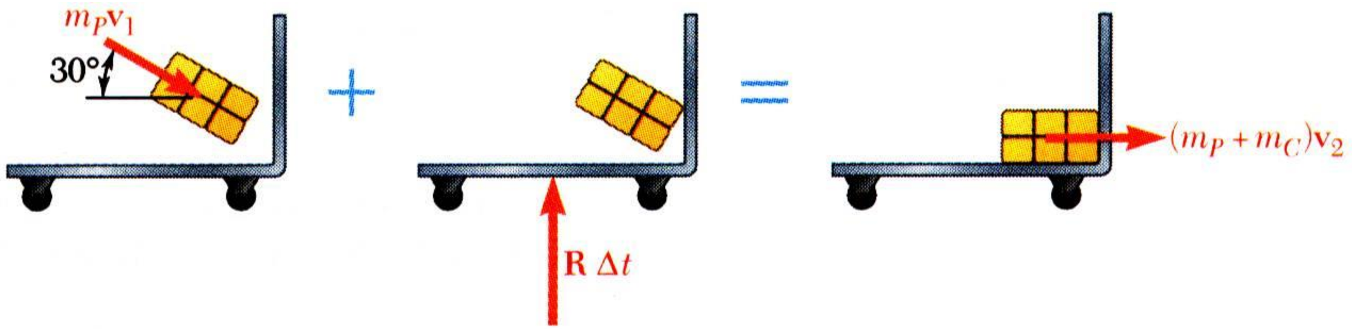




$$m_p \vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = (m_p + m_c)\vec{v}_2$$

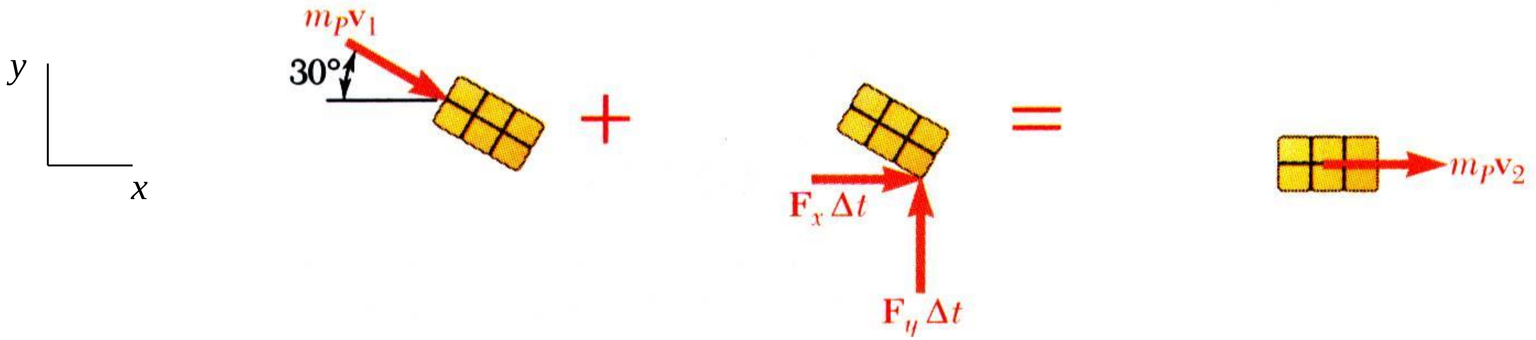
x components:

$$m_p v_1 \cos 30^\circ + 0 = (m_p + m_c) v_2$$

$$(10\ \mathrm{kg})(3\ \mathrm{m/s})\cos 30^\circ = (10\ \mathrm{kg} + 25\ \mathrm{kg}) v_2$$

$$\boxed{v_2 = 0.742\ \mathrm{m/s}}$$

- Apply the same principle to the package alone to determine the impulse exerted on it from the change in its momentum.



$$m_p \vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = m_p \vec{v}_2$$

x components:

$$m_p v_1 \cos 30° + F_x \Delta t = m_p v_2$$

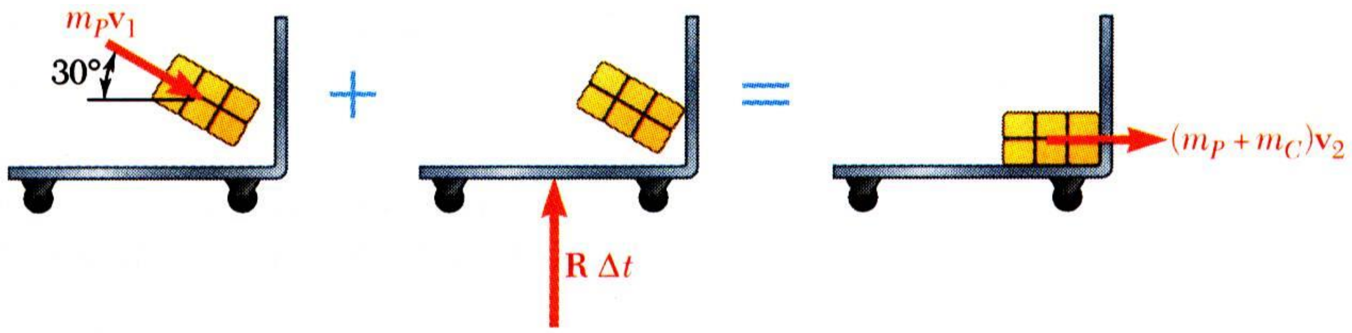
$$(10\ \mathrm{kg})(3\ \mathrm{m/s})\cos 30° + F_x \Delta t = (10\ \mathrm{kg})v_2 \qquad F_x \Delta t = -18.56\ \mathrm{N \cdot s}$$

y components:

$$-m_p v_1 \sin 30° + F_y \Delta t = 0$$

$$-(10\ \mathrm{kg})(3\ \mathrm{m/s})\sin 30° + F_y \Delta t = 0 \qquad F_y \Delta t = 15\ \mathrm{N \cdot s}$$

$$\boxed{\sum \mathbf{Imp}_{1\to 2} = \vec{F}\Delta t = (-18.56\ \mathrm{N \cdot s})\vec{i} + (15\ \mathrm{N \cdot s})\vec{j} \qquad F\Delta t = 23.9\ \mathrm{N \cdot s}}$$



To determine the fraction of energy lost,

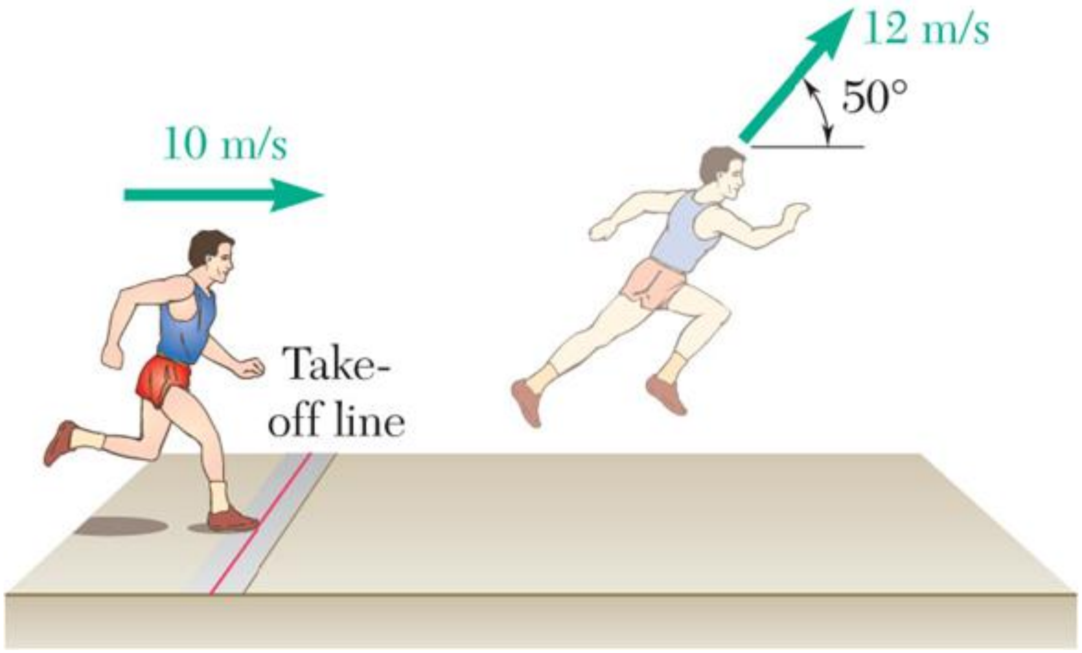
$$T_1 = \tfrac{1}{2} m_p v_1^2 = \tfrac{1}{2}(10 \text{ kg})(3\,\text{m/s})^2 = 45 \text{ J}$$

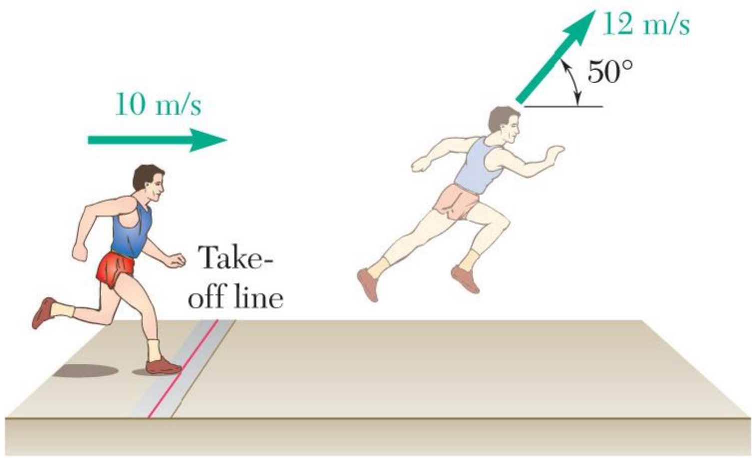
$$T_2 = \tfrac{1}{2}(m_p + m_c)v_2^2 = \tfrac{1}{2}(10 \text{ kg} + 25 \text{ kg})(0.742\,\text{m/s})^2 = 9.63 \text{ J}$$

$$\boxed{\frac{T_1 - T_2}{T_1} = \frac{45 \text{ J} - 9.63 \text{ J}}{45 \text{ J}} = 0.786}$$

6)

The jumper approaches the takeoff line from the left with a horizontal velocity of 10 m/s, remains in contact with the ground for 0.18 s, and takes off at a 50^{o} angle with a velocity of 12 m/s. Determine the average impulsive force exerted by the ground on his foot. Give your answer in terms of the weight *W* of the athlete.





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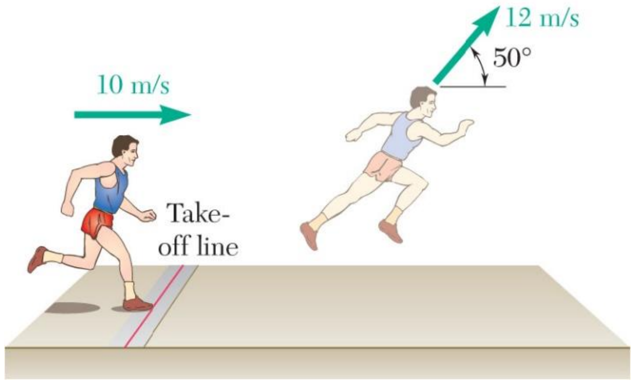
SOLUTION:

- Draw impulse and momentum diagrams of the jumper.
- Apply the principle of impulse and momentum to the jumper to determine the force exerted on the foot.

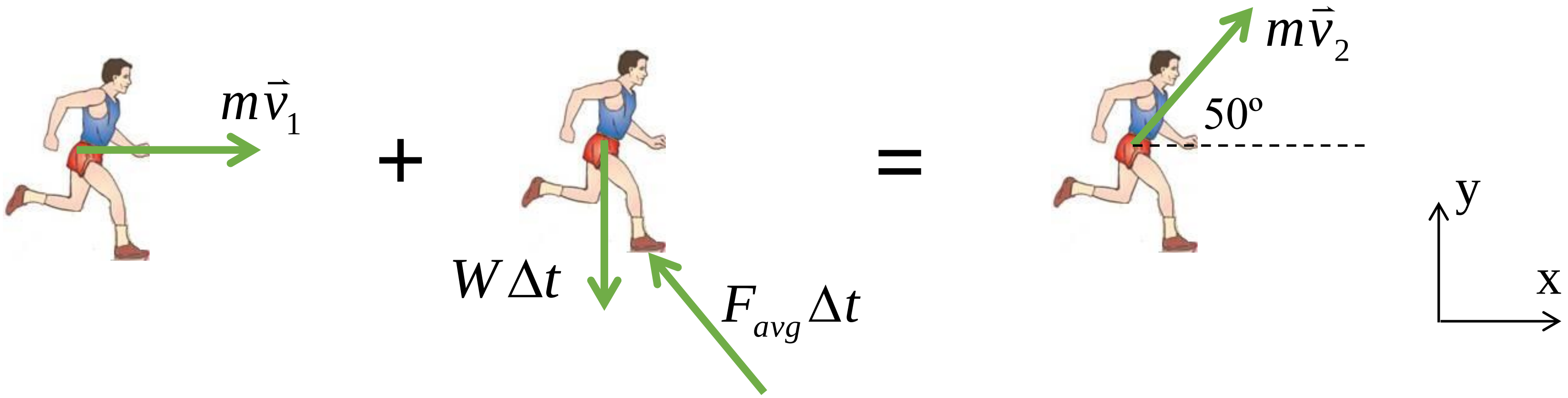
Given: v_1 = 10 m/s, v_2= 12 m/s at 50º,

Δt= 0.18 s

Find: F_{avg} in terms of W

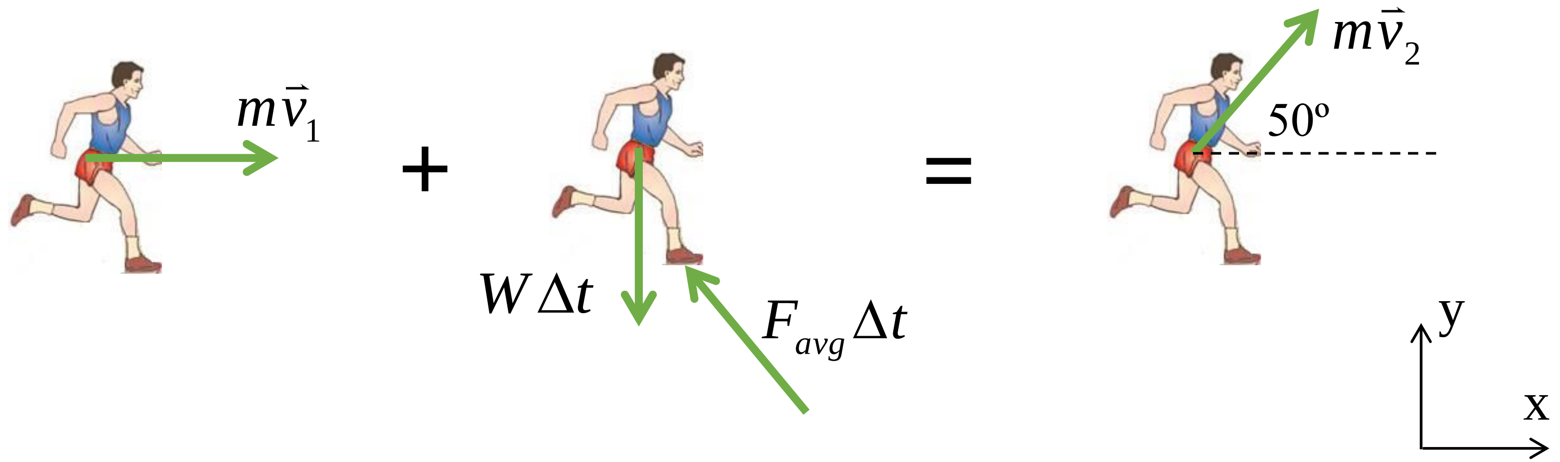


Draw impulse and momentum diagrams of the jumper



Use the impulse momentum equation in y to find F_{avg}

$$m\mathbf{v}_1 + (\mathbf{P} - \mathbf{W})\Delta t = m\mathbf{v}_2 \qquad \Delta t = 0.18 \text{ s}$$



$$m\mathbf{v}_1 + (\mathbf{F}_{avg} - \mathbf{W})\Delta t = m\mathbf{v}_2 \qquad \Delta t = 0.18 \text{ s}$$

Use the impulse momentum equation in x and y to find F_{avg}

$$\frac{W}{g}(10) + (-F_{avg-x})(0.18) = \frac{W}{g}(12)(\cos 50°)$$

$$F_{avg-x} = \frac{10 - (12)(\cos 50°)}{(9.81)(0.18)} W$$

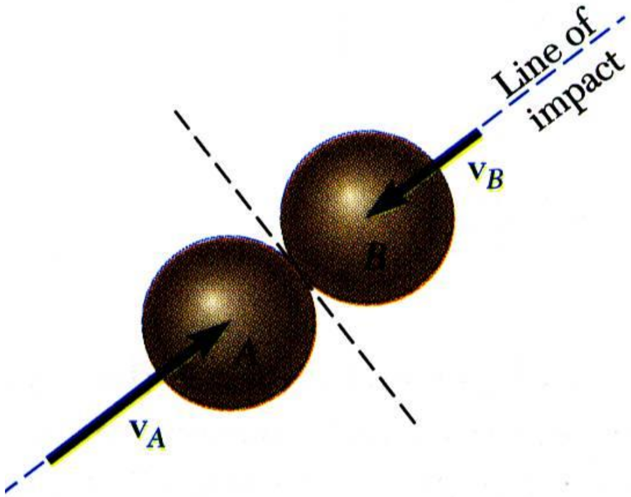
$$0 + (F_{avg-y} - W)(0.18) = \frac{W}{g}(12)(\sin 50°)$$

$$F_{avg-y} = W + \frac{(12)(\sin 50°)}{(9.81)(0.18)} W$$

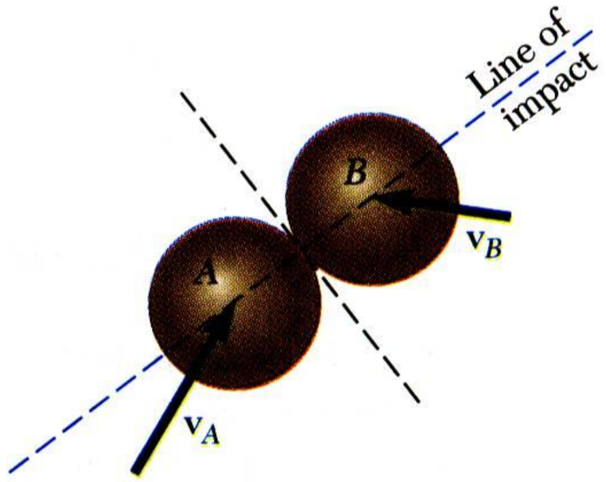
$$\mathbf{F}_{avg} = -1.295W\ \mathbf{i} + 6.21W\ \mathbf{j}$$

F_{avg-x} is positive, which means we guessed correctly (acts to the left)

Impact



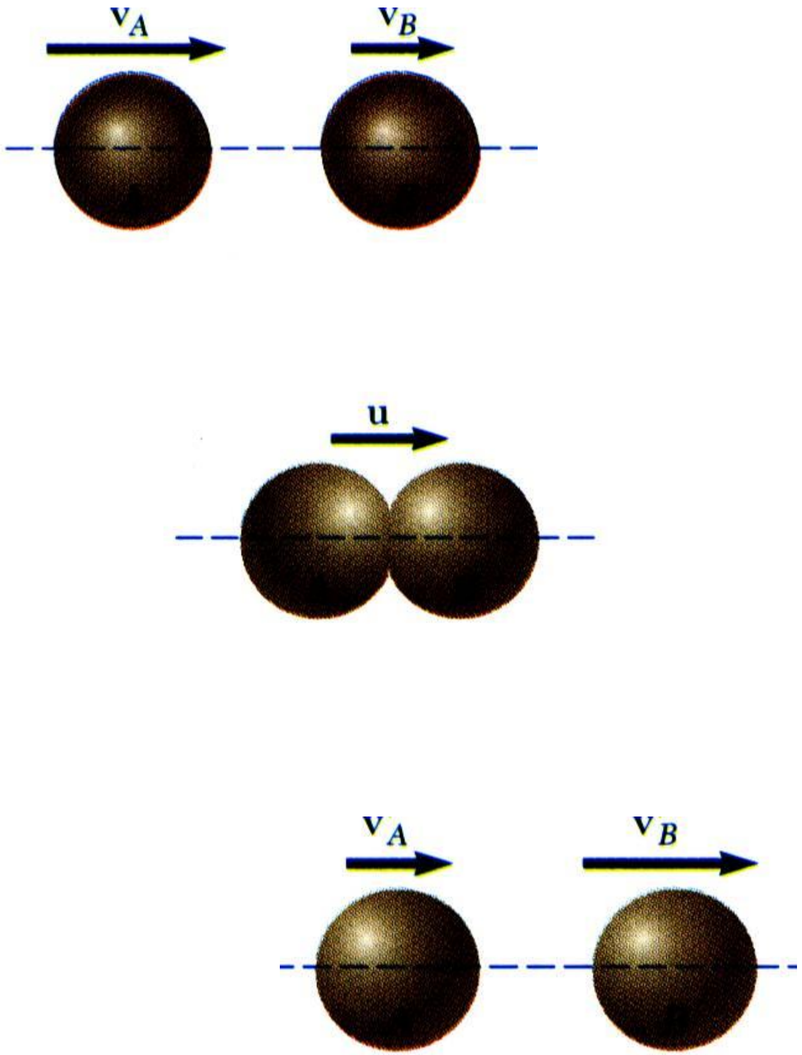
Direct Central Impact



Oblique Central Impact

- *Impact:* Collision between two bodies which occurs during a small time interval and during which the bodies exert large forces on each other.
- *Line of Impact:* Common normal to the surfaces in contact during impact.
- *Central Impact:* Impact for which the mass centers of the two bodies lie on the line of impact; otherwise, it is an *eccentric impact.*.
- *Direct Impact:* Impact for which the velocities of the two bodies are directed along the line of impact.
- *Oblique Impact:* Impact for which one or both of the bodies move along a line other than the line of impact.

Direct Central Impact

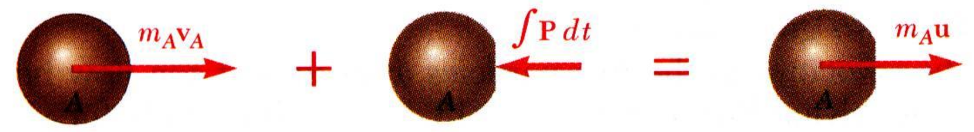


- Bodies moving in the same straight line, $v_A > v_B$.
- Upon impact the bodies undergo a *period of deformation*, at the end of which, they are in contact and moving at a common velocity.
- A *period of restitution* follows during which the bodies either regain their original shape or remain permanently deformed.
- Wish to determine the final velocities of the two bodies. The total momentum of the two body system is preserved,

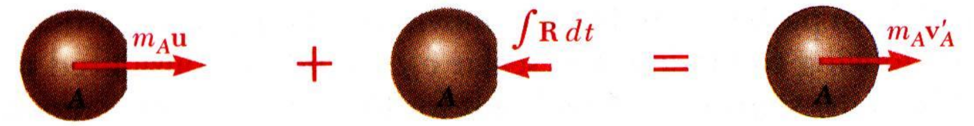
$$m_A v_A + m_B v_B = m_B v_B' + m_B v_B'$$

- A second relation between the final velocities is required.

Direct Central Impact



- Period of deformation: $m_A v_A - \int P dt = m_A u$



- Period of restitution: $m_A u - \int R dt = m_A v'_A$

$$e = \text{coefficient of restitution}$$

$$= \frac{\int R dt}{\int P dt} = \frac{u - v'_A}{v_A - u}$$

$$0 \le e \le 1$$

- A similar analysis of particle *B* yields

$$e = \frac{v'_B - u}{u - v_B}$$

- Combining the relations leads to the desired second relation between the final velocities.

$$v'_B - v'_A = e\left(v_A - v_B\right)$$

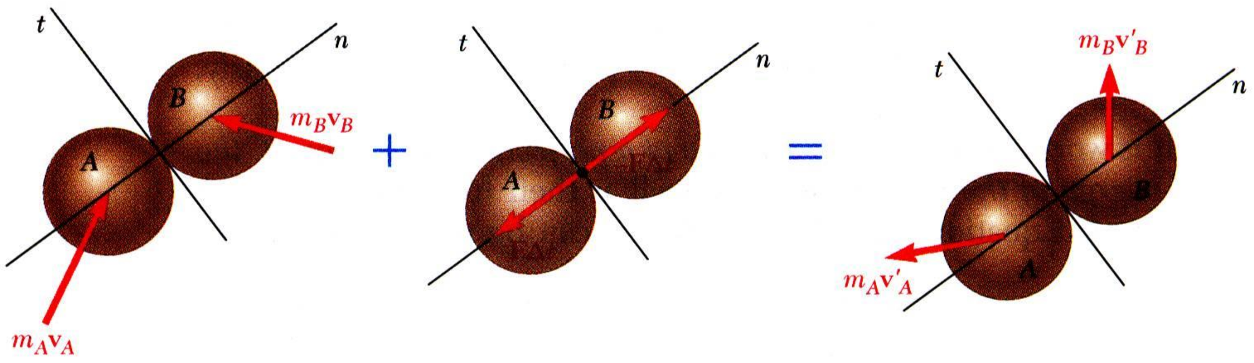
- *Perfectly plastic impact, e* = 0: $v'_B = v'_A = v'$

$$m_A v_A + m_B v_B = \left(m_A + m_B\right) v'$$

- *Perfectly elastic impact, e* = 1:
  Total energy and total momentum conserved.

$$v'_B - v'_A = v_A - v_B$$

Oblique Central Impact



- Final velocities are unknown in magnitude and direction. Four equations are required.

- No tangential impulse component; tangential component of momentum for each particle is conserved.

$$(v_A)_t = (v'_A)_t \qquad (v_B)_t = (v'_B)_t$$

- Normal component of total momentum of the two particles is conserved.

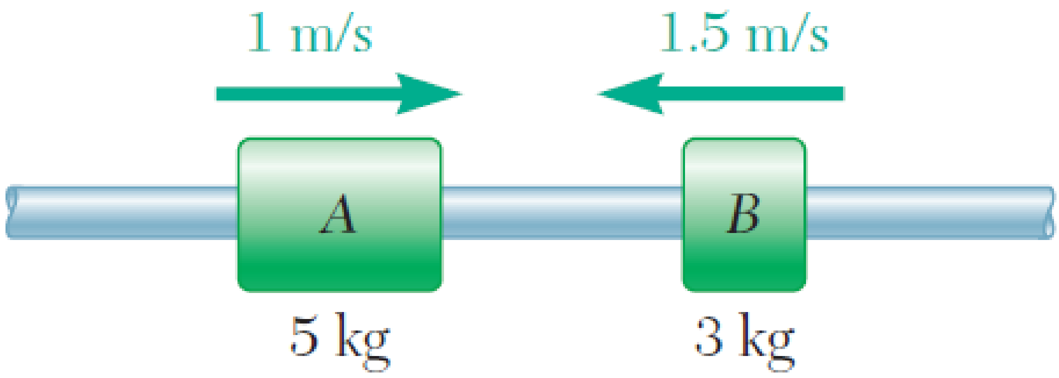
$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

- Normal components of relative velocities before and after impact are related by the coefficient of restitution.

$$(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]$$

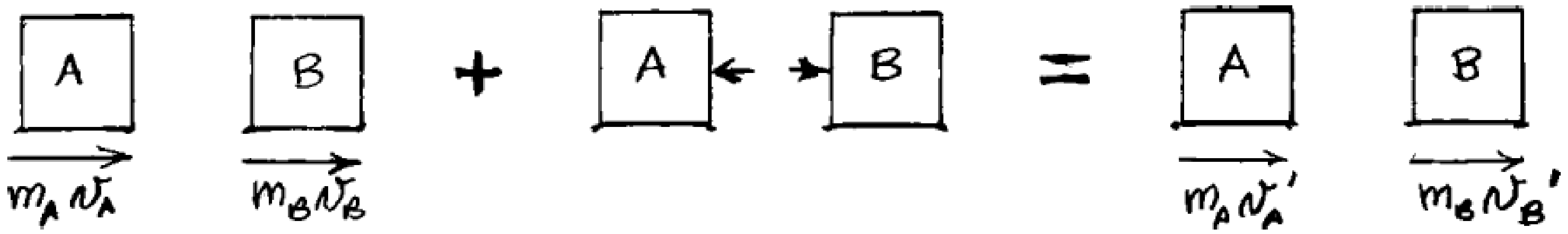
7)

The coefficient of restitution between the two collars is known to be 0.70. Determine (a) their velocities after impact, (b) the energy loss during impact.



Impulse-momentum principle (collars A and B):

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



Horizontal components $\xrightarrow{+}$: $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

Using data, $$(5)(1) + (3)(-1.5) = 5v_A' + 3v_B'$$

or $$5v_A' + 3v_B' = 0.5 \qquad (1)$$

Apply coefficient of restitution.

$$v_B' - v_A' = e(v_A - v_B)$$
$$v_B' - v_A' = 0.70[1 - (-0.5)]$$

$$v_B' - v_A' = 1.75 \qquad (2)$$

(a) Solving Eqs. (1) and (2) simultaneously for the velocities,

$v_A' = -0.59375$ m/s $\qquad \mathbf{v}_A = 0.594$ m/s $\longleftarrow$ ◀

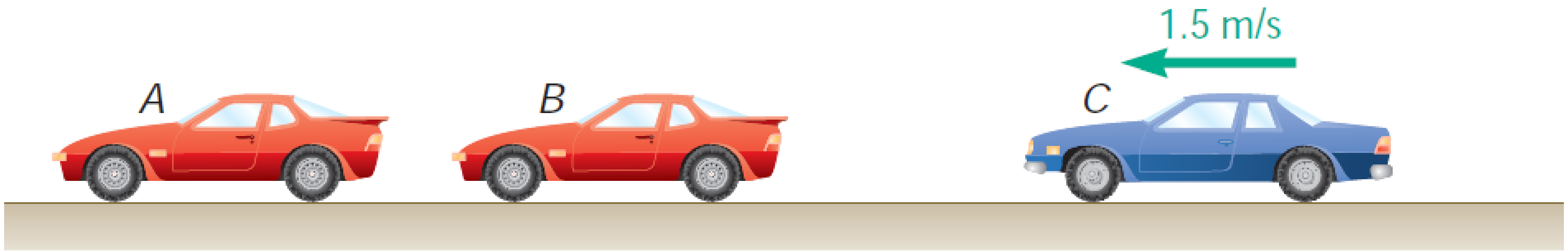
$v_B' = 1.15625$ m/s $\qquad \mathbf{v}_B = 1.156$ m/s $\longrightarrow$ ◀

Kinetic energies: $T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(5)(1)^2 + \frac{1}{2}(3)(-1.5)^2 = 5.875 \text{ J}$

$$T_2 = \frac{1}{2}m_A (v_A')^2 + \frac{1}{2}m_B (v_B')^2 = \frac{1}{2}(5)(-0.59375)^2 + \frac{1}{2}(3)(1.15625)^2 = 2.8867 \text{ J}$$

(*b*) Energy loss: $T_1 - T_2 = 2.99 \text{ J}$ ◀

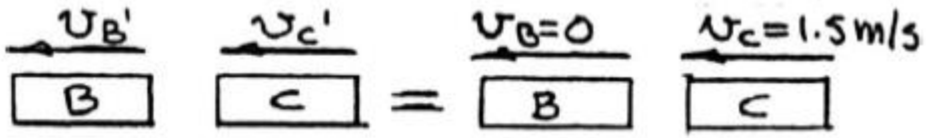
8) Two identical cars A and B are at rest on a loading dock with brakes released. Car C, of a slightly different style but of the same weight, has been pushed by dockworkers and hits car B with a velocity of 1.5 m/s. Knowing that the coefficient of restitution is 0.8 between B and C and 0.5 between A and B, determine the velocity of each car after all collisions have taken place.



$$m_A = m_B = m_C = m$$

Collision between B and C:

The total momentum is conserved:



$$\overset{+}{\leftarrow} \quad mv_B' + mv_C' = mv_B + mv_C$$

$$v_B' + v_C' = 0 + 1.5 \tag{1}$$

Relative velocities:

$$(v_B - v_C)(e_{BC}) = (v_C' - v_B')$$
$$(-1.5)(0.8) = (v_C' - v_B')$$
$$-1.2 = v_C' - v_B' \tag{2}$$

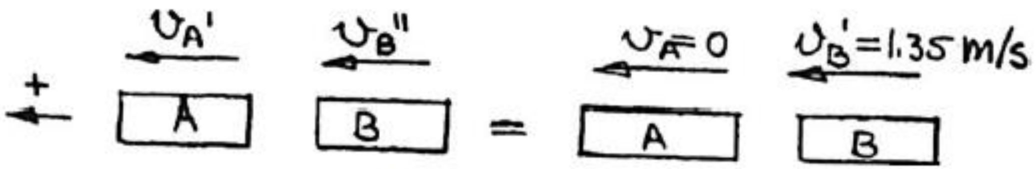
Solving (1) and (2) simultaneously,

$$v_B' = 1.35 \text{ m/s}$$
$$v_C' = 0.15 \text{ m/s}$$

$v_C' = 0.150$ m/s $\longleftarrow$ ◀

Since $v_B' > v_C'$, car B collides with car A.

Collision between A and B:



$$mv_A' + mv_B'' = mv_A + mv_B'$$
$$v_A' + v_B'' = 0 + 1.35 \tag{3}$$

Relative velocities:

$$(v_A - v_B')e_{AB} = (v_B'' - v_A')$$
$$(0 - 1.35)(0.5) = v_B'' - v_A'$$
$$v_A' - v_B'' = 0.675 \tag{4}$$

Solving (3) and (4) simultaneously,

$$2v_A' = 1.35 + 0.675$$

$$\mathbf{v}_A' = 1.013 \text{ m/s} \longleftarrow \blacktriangleleft$$

$$\mathbf{v}_B'' = 0.338 \text{ m/s} \longleftarrow \blacktriangleleft$$

Since $v_C' < v_B'' < v_A'$, there are no further collisions.

9) At an amusement park there are 200-kg bumper cars A, B, and C that have riders with masses of 40 kg, 60 kg, and 35 kg, respectively. Car A is moving to the right with a velocity of 2 m/s and car C has a velocity of 1.5 m/s to the left, but car B is initially at rest. The coefficient of restitution between each car is 0.8. Determine the final velocity of each car, after all impacts, assuming cars A and C hit car B at the same time.

Assume that each car with its rider may be treated as a particle. The masses are:

$$m_A = 200 + 40 = 240 \text{ kg},$$
$$m_B = 200 + 60 = 260 \text{ kg},$$
$$m_C = 200 + 35 = 235 \text{ kg}.$$

Assume velocities are positive to the right. The initial velocities are:

$$v_A = 2 \text{ m/s} \quad v_B = 0 \quad v_C = -1.5 \text{ m/s}$$

Let v'_A, v'_B, and v'_C be the final velocities.

(*a*) Cars *A* and *C* hit *B* at the same time. Conservation of momentum for all three cars.

$$m_A v_A + m_B v_B + m_C v_C = m_A v'_A + m_B v'_B + m_C v'_C$$
$$(240)(2) + 0 + (235)(-1.5) = 240v'_A + 260v'_B + 235v'_C \qquad (1)$$

Coefficient of restition for cars *A* and *B*.

$$v'_B - v'_A = e(v_A - v_B) = (0.8)(2 - 0) = 1.6 \qquad (2)$$

Coefficient of restitution for cars *B* and *C*.

$$v'_C - v'_B = e(v_B - v_C) = (0.8)[0 - (-1.5)] = 1.2 \qquad (3)$$

Solving Eqs. (1), (2), and (3) simultaneously,

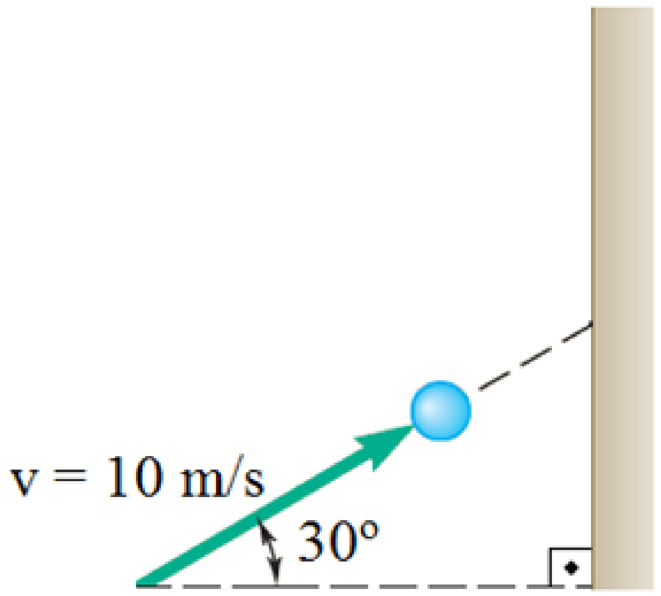
$$v'_A = -1.288 \text{ m/s} \quad v'_B = 0.312 \text{ m/s} \quad v'_C = 1.512 \text{ m/s}$$

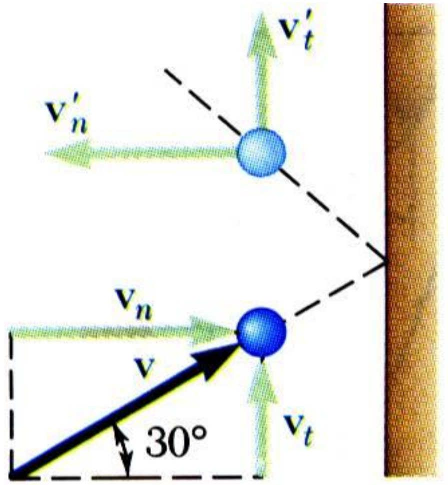
$$\mathbf{v}'_A = 1.288 \text{ m/s} \longleftarrow \blacktriangleleft$$

$$\mathbf{v}'_B = 0.312 \text{ m/s} \longrightarrow \blacktriangleleft$$

$$\mathbf{v}'_C = 1.512 \text{ m/s} \longrightarrow \blacktriangleleft$$

10) A ball is thrown against a frictionless, vertical wall. Immediately before the ball strikes the wall, its velocity has a magnitude v and forms an angle of 30° with the horizontal. Knowing that e=0.90 and v=10 m/s, determine the magnitude and direction of the velocity of the ball as it rebounds from the wall. Note that the direction of the velocity of the ball as it rebounds from the wall should be shown in a figure.

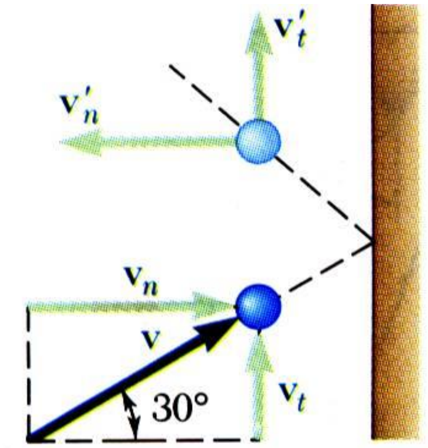


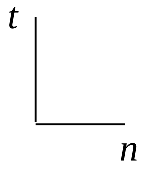


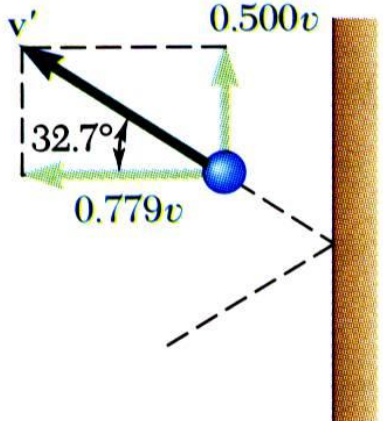
A ball is thrown against a frictionless, vertical wall. Immediately before the ball strikes the wall, its velocity has a magnitude *v* and forms angle of 30^o with the horizontal. Knowing that *e* = 0.90, determine the magnitude and direction of the velocity of the ball as it rebounds from the wall.

<u>SOLUTION</u>:

- Resolve ball velocity into components normal and tangential to wall.
- Impulse exerted by the wall is normal to the wall. Component of ball momentum tangential to wall is conserved.
- Assume that the wall has infinite mass so that wall velocity before and after impact is zero. Apply coefficient of restitution relation to find change in normal relative velocity between wall and ball, i.e., the normal ball velocity.







SOLUTION:

- Resolve ball velocity into components parallel and perpendicular to wall.

$$v_n = v\cos 30° = 0.866v \qquad v_t = v\sin 30° = 0.500v$$

- Component of ball momentum tangential to wall is conserved.

$$v_t' = v_t = 0.500v$$

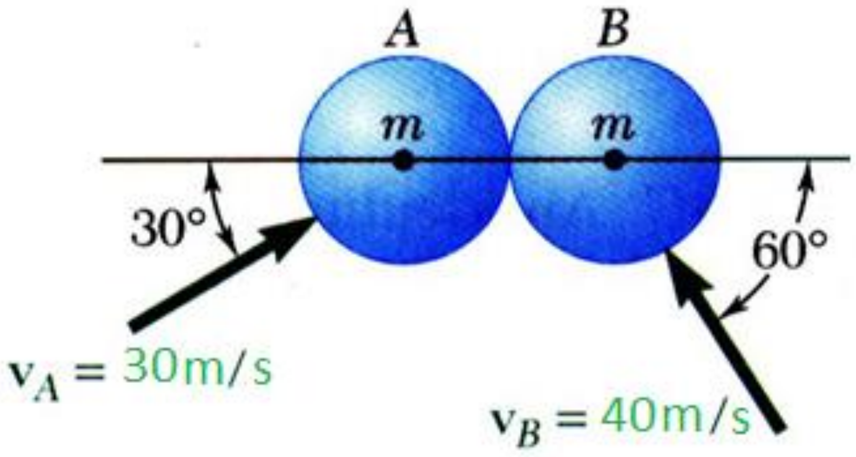
- Apply coefficient of restitution relation with zero wall velocity.

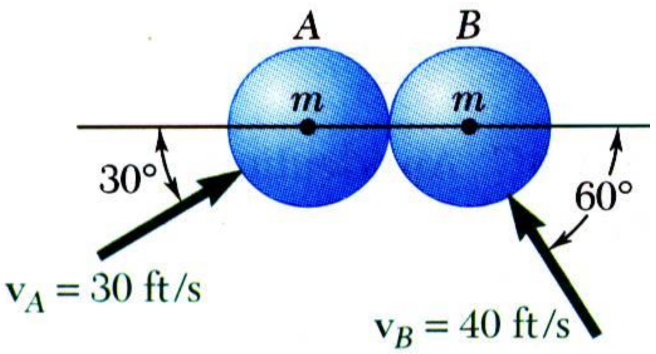
$$0 - v_n' = e(v_n - 0)$$
$$v_n' = -0.9(0.866v) = -0.779v$$

$$\boxed{\begin{aligned}&\vec{v}' = -0.779v\,\vec{\lambda}_n + 0.500v\,\vec{\lambda}_t\\ &v' = 0.926v \qquad \tan^{-1}\left(\frac{0.779}{0.500}\right) = 32.7°\end{aligned}}$$

11)

The magnitude and direction of the velocities of two identical frictionless balls before they strike each other are as shown. Assuming e = 0.9, determine the magnitude and direction of the velocity of each ball after the impact.

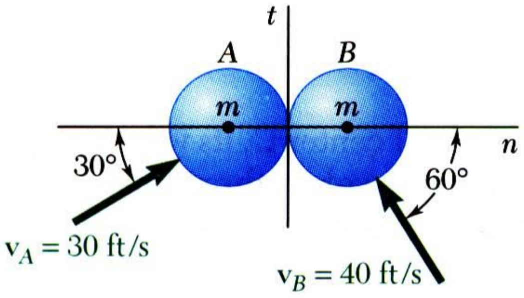




The magnitude and direction of the velocities of two identical frictionless balls before they strike each other are as shown. Assuming $e = 0.9$, determine the magnitude and direction of the velocity of each ball after the impact.

<u>SOLUTION</u>:

- Resolve the ball velocities into components normal and tangential to the contact plane.
- Tangential component of momentum for each ball is conserved.
- Total normal component of the momentum of the two ball system is conserved.
- The normal relative velocities of the balls are related by the coefficient of restitution.
- Solve the last two equations simultaneously for the normal velocities of the balls after the impact.

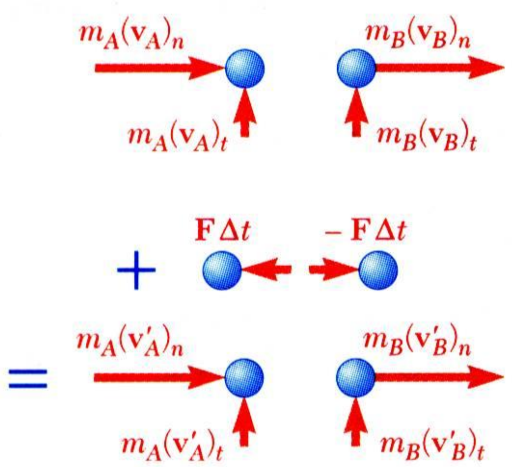


SOLUTION:

- Resolve the ball velocities into components normal and tangential to the contact plane.

$$(v_A)_n = v_A \cos 30° = 26.0\,\text{ft/s} \qquad (v_A)_t = v_A \sin 30° = 15.0\,\text{ft/s}$$

$$(v_B)_n = -v_B \cos 60° = -20.0\,\text{ft/s} \qquad (v_B)_t = v_B \sin 60° = 34.6\,\text{ft/s}$$



- Tangential component of momentum for each ball is conserved.

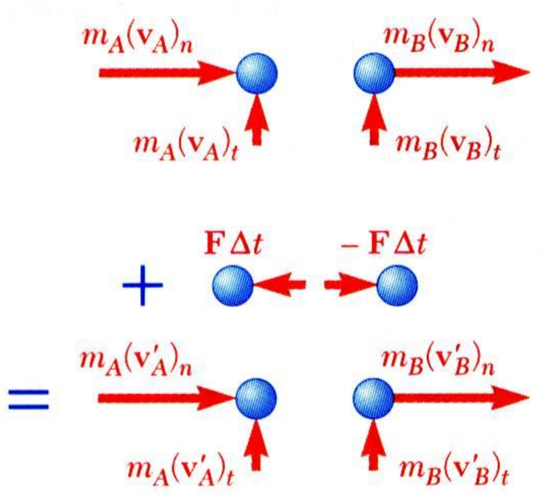
$$(v'_A)_t = (v_A)_t = 15.0\,\text{ft/s} \qquad (v'_B)_t = (v_B)_t = 34.6\,\text{ft/s}$$

- Total normal component of the momentum of the two ball system is conserved.

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$m(26.0) + m(-20.0) = m(v'_A)_n + m(v'_B)_n$$

$$(v'_A)_n + (v'_B)_n = 6.0$$

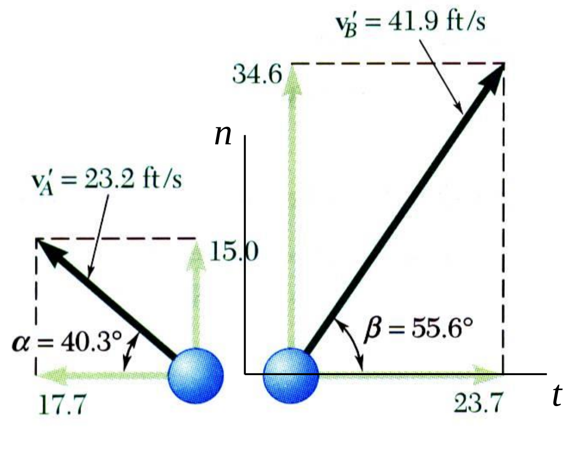


- The normal relative velocities of the balls are related by the coefficient of restitution.

$$(v'_A)_n - (v'_B)_n = e\left[(v_A)_n - (v_B)_n\right]$$
$$= 0.90\left[26.0 - (-20.0)\right] = 41.4$$

- Solve the last two equations simultaneously for the normal velocities of the balls after the impact.

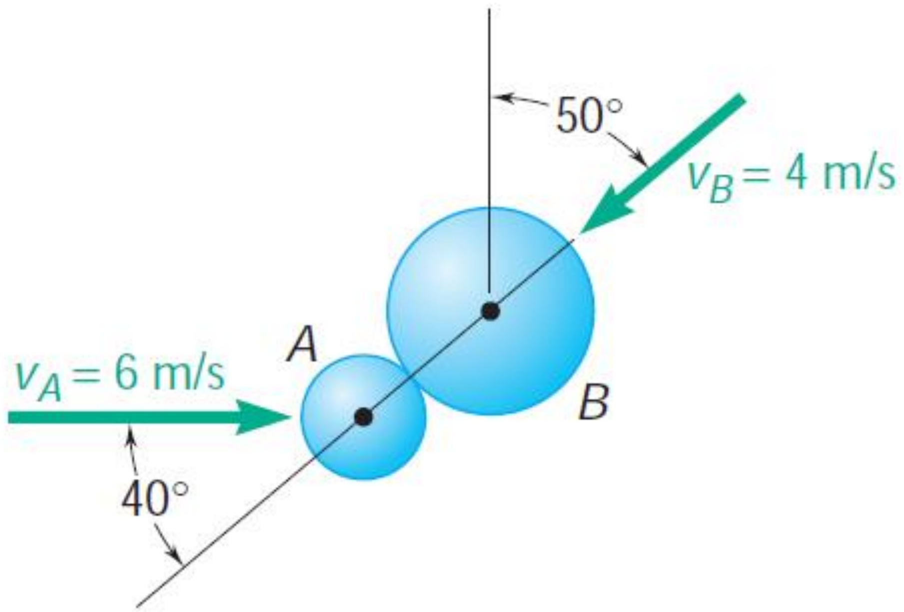
$$(v'_A)_n = -17.7\,\mathrm{ft/s} \qquad (v'_B)_n = 23.7\,\mathrm{ft/s}$$

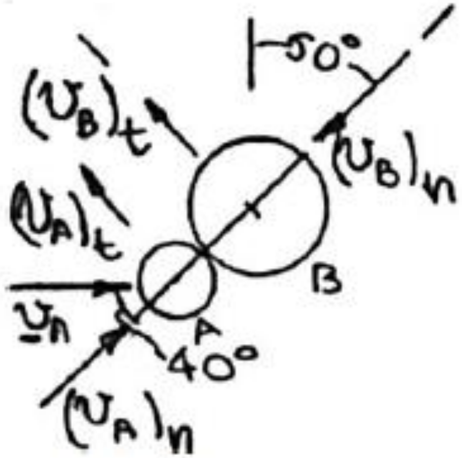


$$\vec{v}'_A = -17.7\vec{\lambda}_t + 15.0\vec{\lambda}_n$$
$$v'_A = 23.2\,\mathrm{ft/s} \quad \tan^{-1}\left(\frac{15.0}{17.7}\right) = 40.3°$$
$$\vec{v}'_B = 23.7\vec{\lambda}_t + 34.6\vec{\lambda}_n$$
$$v'_B = 41.9\,\mathrm{ft/s} \quad \tan^{-1}\left(\frac{34.6}{23.7}\right) = 55.6°$$

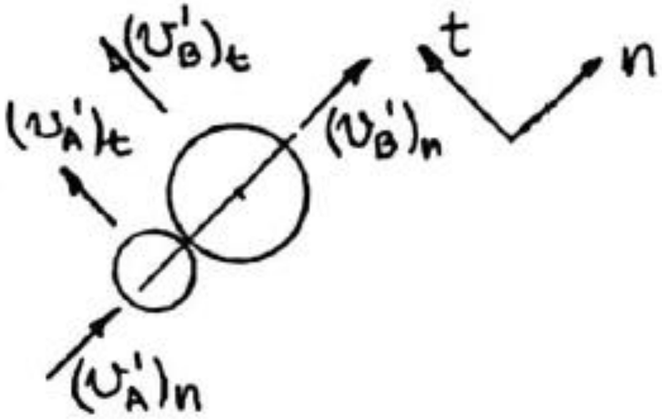
12)

A 600-g ball *A* is moving with a velocity of magnitude 6 m/s when it is hit as shown by a 1-kg ball *B* which has a velocity of magnitude 4 m/s. Knowing that the coefficient of restitution is 0.8 and assuming no friction, determine the velocity of each ball after impact.





Before



After

$$v_A = 6 \text{ m/s}$$

$$(v_A)_n = (6)(\cos 40°) = 4.596 \text{ m/s}$$

$$(v_A)_t = -6(\sin 40°) = -3.857 \text{ m/s}$$

$$v_B = (v_B)_n = -4 \text{ m/s}$$

$$(v_B)_t = 0$$

t-direction:

Ball A alone:

Momentum conserved:

$$m_A(v_A)_t = m_A(v'_A)_t \quad -3.857 = (v'_A)_t$$

$$(v'_A)_t = -3.857 \text{ m/s} \tag{2}$$

Ball B alone:

Momentum conserved:

$$m_B \ (v_B)_t = m_B \ (v'_B)_t$$

$$(v'_B)_t = 0$$

n-direction:

Relative velocities:

$$[(v_A)_n - (v_B)_n]e = (v'_B)_n - (v'_A)_n$$
$$[(4.596) - (-4)](0.8) = (v'_B)_n - (v'_A)_n$$
$$6.877 = (v'_B)_n - (v'_A)_n \tag{3}$$

Total momentum conserved:

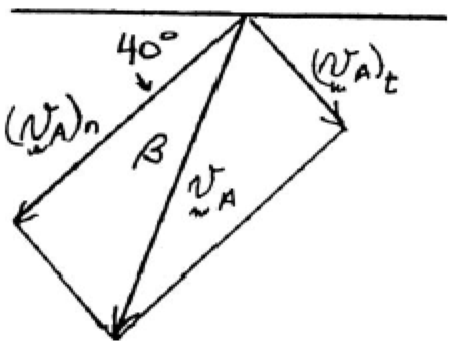
$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$
$$(0.6 \text{ kg})(4.596 \text{ m/s}) + (1 \text{ kg})(-4 \text{ m/s}) = (1 \text{ kg})(v'_B)_n + (0.6 \text{ kg})(v'_A)_n$$
$$-1.2424 = (v'_B)_n + 0.6(v'_A)_n \tag{4}$$

Solving Eqs. (4) and (3) simultaneously,

$$(v'_A)_n = 5.075 \text{ m/s}$$
$$(v'_B)_n = 1.802 \text{ m/s}$$

Velocity of A:

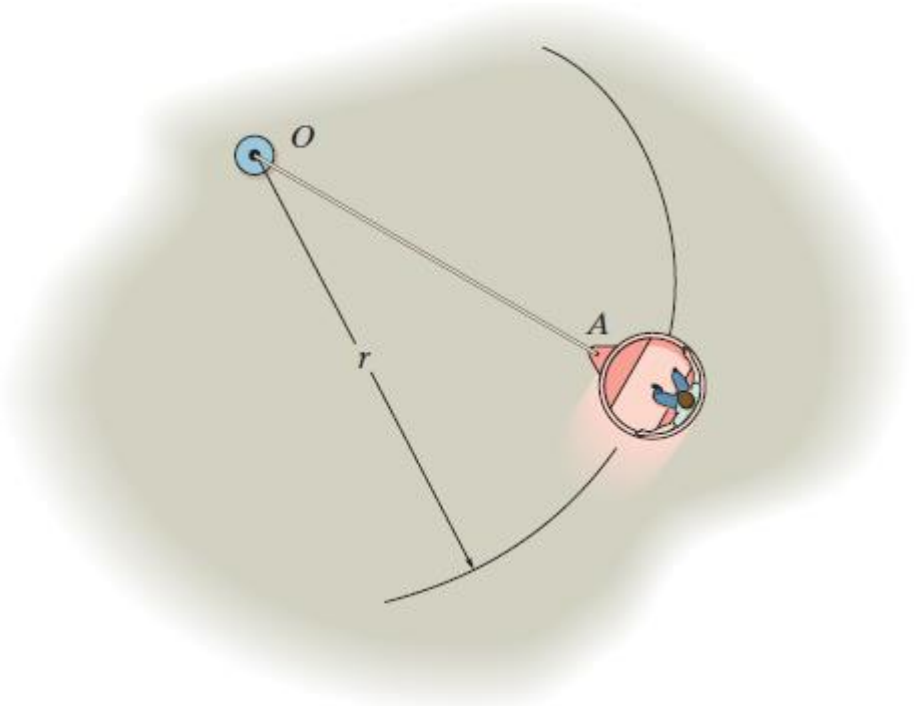
$$\tan\beta = \frac{|(v_A)_t|}{|(v_A)_n|} = \frac{3.857}{5.075}$$
$$\beta = 37.2^\circ \qquad \beta + 40^\circ = 77.2^\circ$$
$$v'_A = \sqrt{(3.857)^2 + (5.075)^2} = 6.37 \text{ m/s}$$



$$\mathbf{v}'_A = 6.37 \text{ m/s} \measuredangle 77.2^\circ \blacktriangleleft$$

13)

The amusement park ride consists of a 200-kg car and passenger that are traveling at 3 m/s along a circular path having a radius of 8 m. If at t = 0, the cable OA is pulled in toward O at 0.5 m/s, determine the speed of the car when t = 4 s. Also, determine the work done to pull in the cable.



Conservation of Angular Momentum. At $t = 4$ s, $r_2 = 8 - 0.5(4) = 6$ m.

$$(H_0)_1 = (H_0)_2$$

$$r_1 m v_1 = r_2 m (v_2)_t$$

$$8[200(3)] = 6[200(v_2)_t]$$

$$(v_2)_t = 4.00 \text{ m/s}$$

Here, $(v_2)_r = 0.5$ m/s. Thus

$$v_2 = \sqrt{(v_2)_t^2 + (v_2)_r^2} = \sqrt{4.00^2 + 0.5^2} = 4.031 \text{ m/s} = 4.03 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(200)(3^2) + \Sigma U_{1-2} = \frac{1}{2}(200)(4.031)^2$$

$$\Sigma U_{1-2} = 725 \text{ J} \qquad \textbf{Ans.}$$