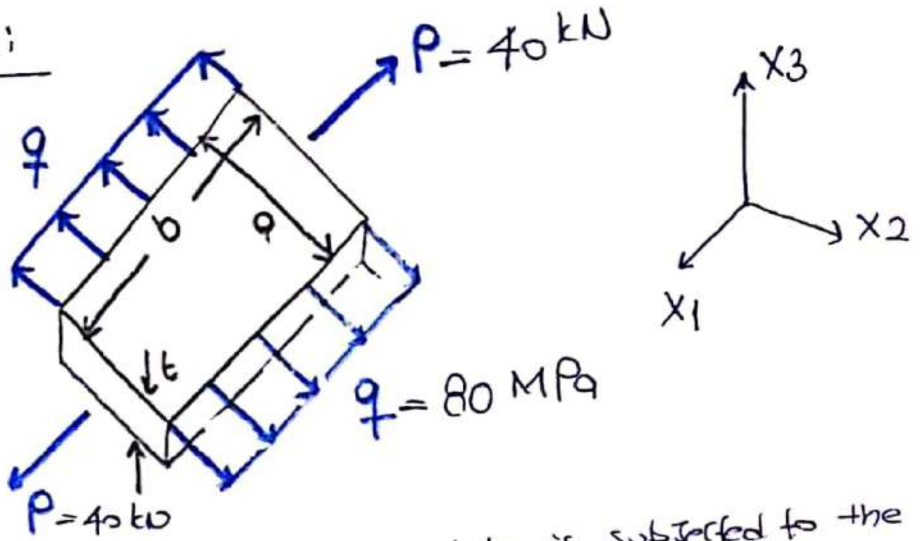
Example:



For the state of plane stress, the plate is subjected to the tensile force P and the tensile stress q in the direction X1 and X2, respectively.

$\nu = 0,3$

$E = 210$ GPa

$a = 40$ mm, $b = 400$ mm, $t = 4$ mm

Determine the strain (elastic strain) energy.

$$\sigma_1 = \frac{P}{at} = \frac{40 \times 10^3 \, N}{40 \times 4 \, mm^2}$$

$$\sigma_1 = 250 \text{ MPa}$$

$$\sigma_2 = q = 80 \text{ MPa}$$

Calculate the volume of the plate:

$$V = a.b.t = 40 \cdot 400 \cdot 4$$
$$= 64 \times 10^3 \, mm^3$$

poisson's ratio

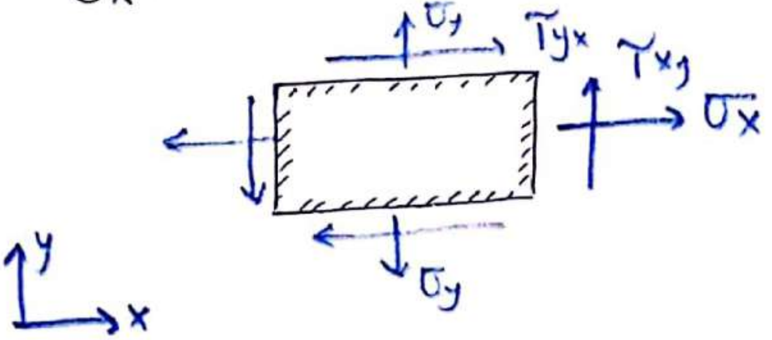
volume

$$U_i = \left[\frac{1}{2E} \left(\sigma_1^2 + \sigma_2^2 \right) - \frac{\nu}{E} \left(\sigma_1 \sigma_2 \right) \right] V$$

$$U_i = 64 \times 10^3 \, mm^3 \left[\frac{1}{2(210 \times 10^3)} \left(250^2 + 80^2 \right) - \frac{0,3}{210 \times 10^3} (250 \cdot 80) \right]$$

$$U_i = 8670 \text{ Nmm}$$

Example: For the state of plane stress shown in Figure, determine the strain energy density ($E = 200$ GPa , $\nu = 0,15$, $\sigma_x = 160$ MPa , $\sigma_y = 80$ MPa , $\tau_{xy} = 30$ MPa)



$$u_i = \frac{U_i}{V} = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2\right) - \frac{\nu}{E}\left(\sigma_x \cdot \sigma_y\right) + \frac{1}{2G}\left(\tau_{xy}^2\right)$$

$$G = \frac{E}{2(1+\nu)}$$

shear modulus

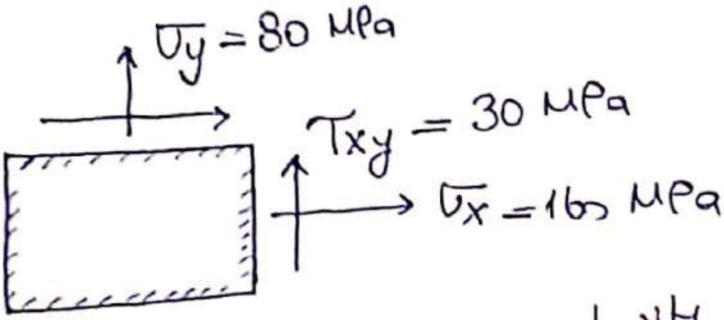
$$u_i = \frac{1}{2(200 \times 10^3)}\left(160^2 + 80^2\right) - \frac{0,15}{200 \times 10^3}\left(160 \cdot 80\right) + \frac{1}{\cancel{2}\left(\underbrace{\frac{E}{\cancel{2}(1+\nu)}}_{G = \text{shear modulus.}}\right)}\left(30^2\right)$$

$$u_i \cong 0,075 \text{ Nmm/mm}^3$$

Homework:

determine the strain energy density in terms of principal stresses.

Example:



Determine the hystrostatic strain energy density and strain energy density due to distortion by considering the state of plane stress in the figure. ($E = 200$ GPa, $\nu = 0{,}15$)

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{\left[\frac{1}{2}(\sigma_x - \sigma_y)\right]^2 + \tau_{xy}^2}$$

principal stresses in 2D

$$\sigma_1 = 170 \text{ MPa} \; ; \; \sigma_2 = 70 \text{ MPa}, \; \cancel{\sigma_3 = 0}$$

$$u_{hyd} = \frac{1-2\nu}{6E}\left(\sigma_1 + \sigma_2 + \cancel{\sigma_3}^{\,0}\right)^2$$

$$u_{hyd} = \frac{1-2(0{,}15)}{6(200\times10^3)}(170+70)^2 = 0{,}0336 \text{ Nmm/mm}^3$$

$$u_{distortion} = \frac{1+v}{3E}\left[\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\cancel{\sigma_3})^2+(\cancel{\sigma_3}-\sigma_1)^2\right\}/2\right]$$

$$= \frac{1+0{,}15}{3(200\times10^3)}\left\{\left[(170-70)^2+(70^2)+(170)^2\right]/2\right\}$$

$$u_{distortion} = 0{,}0419\ \text{Nmm/mm}^3$$