Homework Assignment

MAT2092 Algebra-II, 2019-2020 Spring Semester

Yildiz Technical University, Department of Mathematics

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Deadline: 06.05.2020, 11.00

You must show all the details of your work to get full credit. Correct answers with no detail will get zero credit. Copying someone else's is not acceptable. No late homework will be accepted. Homework scores will contribute %30 to the final grade.

1) (i)Let $S = \left\{ \begin{bmatrix} a & a-b \\ a-b & b \end{bmatrix} \mid \quad a, b \in \mathbf{Z} \right\}$. Prove or disprove that S is a subring of $M_2(\mathbf{Z})$.

(ii) Show that the set of nilpotent elements forms an ideal.

2) Let d be a square free integer(that is, d cannot divisible by square of a prime). Show that $\mathbf{Z}[\sqrt{d}]$ is an integral domain.

3) Let $\phi : R \longrightarrow S$ be a ring homomorphism and R be an principal ideal ring. Show that $\phi(R)$ is also a principal ideal ring. Is it true for principal ideal domain?

4) Let n be a square free positive integer. Prove that $\mathbf{Z}_n$ has no nonzero nilpotent elements.

5) Find all subrings and ideals of $\mathbf{Z}$.

6) (i)Prove or disprove that $\mathbf{Z}[\sqrt{-6}]$ is a unique factorization domain.

(ii) Prove or disprove that $\mathbf{Z}[\sqrt{-6}]$ is a principal ideal domain.

(iii) Prove or disprove that $\mathbf{Z}[i]$ is a unique factorization domain.

7) Prove that every zero divisor in $\mathbf{Z}_{p^n}$ is a nilpotent element.

8) (i) Prove or disprove that 2 and 3 (independently) are irreducible element in $\mathbf{Z}[i]$.

(ii) Prove that $1 + 3\sqrt{-5}$ is not a prime element in $\mathbf{Z}[\sqrt{-5}]$.