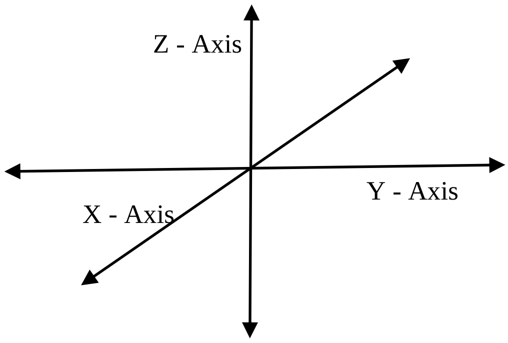
# RECTANGULAR CARTESIAN CO- ORDINATES

Direction cosines of a line – Direction ratios of the join of two points - Projection on a line – Angle between the lines -Equation of a plane in different forms - Intercept form- normal form Angle between two planes - Planes bisecting the angle between two planes, bisector planes.

**Introduction:**

Let X'OX, Y'OY and Z'OZ be three mutually perpendicular lines in space that are concurrent at 0(origin). These three lines, called respectively as x-axis, y-axis and z-axis (and collectively as co-ordinate axes), form the frame of reference, using which the co-ordinates of a point in space are defined.

3D coordinate plane




**Note:**

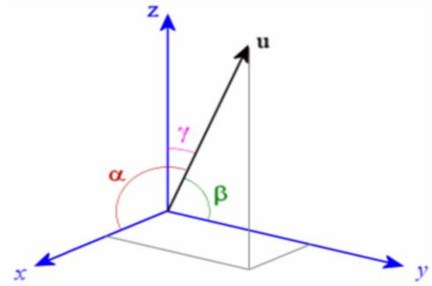
- The positive parts of the co-ordinate axes, namely OX, OY, OZ should form a right-hand system. The plane XOY determined by the x-axis and y-axis is called xoy plane or xy-plane.
- Similarly the yz plane and zx-plane are defined. These three planes called co-ordinate planes, divide the entire space into 8 parts, called the octants. The octant bounded by OX, OY, OZ is called the positive or the first octant.

- In face, the x co-ordinate of any point in the yz-plane will be zero, the y co-ordinate of any point in the zx-plane will be zero and the z co-ordinate of any point in the xy plane will be zero.

- In other words, the equations of the yz, zx and xy-planes are x = 0, y = 0 and z = 0 respectively. The point A lies on the x-axis and hence in the zx and xy-planes. Hence the co-ordinates of A will be (x, 0, 0), similarly the co-ordinates of B and C will be respectively (0, y, 0) and (0, 0, z).

**Definition:** Direction Cosines

The cosine of the angles made by a line with the axes X, Y and Z are called directional cosines of the line. (i.e) The triplet $\cos\alpha$ , $\cos\beta$ , $\cos\gamma$ are called the direction cosines (D.C.'s) of the line and usually denoted as l, m, n. A set of parallel lines will make the same angles with the co-ordinates axes and hence will have the same D. C.' s.



**Note :**

1. If the D.C.'s of line PQ are l, m, n, then the D.C.'s of QP are -l, -m, -n, as the angles made byQP with the co-ordinates axes are $180^0 - \alpha, 180^0 - \beta, 180^0 - \gamma$ when the angles made by PQ with the axes are $\alpha, \beta, \gamma$.

2. The D.C.'s of OX, OY, OZ are respectively 1, 0, 0, 0, 1, 0 and 0, 0, 1.

The **direction cosines of** a line parallel to any coordinate axis are equal to the **direction cosines of** the corresponding axis. The dc's are associated by the **relation** $l^2 + m^2 + n^2 = 1$. If the given line is reversed, then the **direction cosines** will be cos (π − α), cos (π − β), cos (π − γ) or − cos α, − cos β, − cos γ.

**Definition: Direction Ratios**

The direction ratios are simply a set of three real numbers *a*, *b*, *c* proportional to *l*, *m*, *n*, i.e.

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$

From this relation, we can write

$$\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \pm\frac{\sqrt{a^2+b^2+c^2}}{\sqrt{l^2+m^2+n^2}} = \sqrt{a^2+b^2+c^2}$$

$$\Rightarrow \boxed{l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}}$$

These relations tell us how to find the direction cosines from direction ratios.

**Note:**

1. $l^2 + m^2 + n^2 = 1$, where as $a^2 + b^2 + c^2 \neq 1$.
2. To specify the direction of a line in space its direction angles, direction cosines or direction ratios must be known.
3. The D.R.'s of two parallel lines are proportional.

**Formulae:**

1. Direcion Ratios (D.R.'S) of a line joining Two points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ are $\mathbf{x_2 - x_1 , y_2 - y_1 , z_2 - z_1}$ .

**Angle between Two Lines:**

If $l_1, m_1, n_1$ and $l_2, m_2, n_2$ are the direction ratios of the lines $L_1$ and $L_2$, then

$$\boxed{\cos\theta = l_1l_2 + m_1m_2 + n_1n_2}$$

**Corollary 1**

If the two lines are perpendicular, then $\theta = 90^0$ or $\cos\theta = 0$

$$\boxed{\text{i-e., } l_1 l_2 + m_1 m_2 + n_1 n_2 = 0}$$

we recall that, if the two lines are parallel, then $\boxed{l_1 = l_2, m_1 = m_2 \text{ and } n_1 = n_2.}$

**Corollary 2**

If the D.R.'s of the two lines are $a_1$, $b_1$, $c$ and $a_2$, $b_2$, $c_2$ then their D.C.'s are $\left(\frac{a_1}{\sqrt{\Sigma a_1^2}}, \frac{b_1}{\sqrt{\Sigma a_1^2}}, \frac{c_1}{\sqrt{\Sigma a_1^2}}\right)$ and $\left(\frac{a_2}{\sqrt{\Sigma a_2^2}}, \frac{b_2}{\sqrt{\Sigma a_2^2}}, \frac{c_2}{\sqrt{\Sigma a_2^2}}\right)$

If $\theta$ is the angle between the two lines, then

$$\boxed{\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{\left(a_1^2 + b_1^2 + c_1^2\right)\left(a_2^2 + b_2^2 + c_2^2\right)}}}$$

If $a_1$, $b_1$, $c_1$ and $a_2$, $b_2$, $c_2$ are the direction ratios of the lines $L_1$ and $L_2$, and if they are perpendicular, then $\cos\theta = a_1 a_2 + b_1 b_2 + c_1, c_2 = 0$ or $\theta = 90^o$.

we recall that if the two lines are parallel then

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.}$$

**Projection of a Line segment on a given Line :**

Let AB be a given line and PQ be any line then the the Dr's of the line PQ are **$x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$** and the Dc's of the given line AB are l, m, and n then

The projection of PQ on $AB = l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

**THE PLANE**

A plane is a surface which is such that the straight line joining any two points on it lies completely on it. This characteristic property of a plane is not true for any other surface.

**General Equation of a Plane:**

The first degree equation in x, y, z namely $\boldsymbol{ax + by + cz + d = 0}$ always represents a plane, where a, b, c are not all zero.

**Equation of a plane passing through a point:**

**If** $ax + by + cz + d = 0$ is a plane equation and it passes through a given point $P(x_1, y_1, z_1)$ , then the required plane is **$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$**

**Equation of the plane making intercepts a, b, c on the coordinate axes is**
$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

**Equation of the plane passing through three points A($x_1,y_1,z_1$) , B($x_2,y_2,z_2$)and C($x_3,y_3,z_3$) is**

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

**Equation of a plane in the normal form is** $x\cos\alpha + y\cos\beta + z\cos\gamma = \rho$ , where $\rho$ is the length of the perpendicular from the origin on it and $\cos\alpha$ , $\cos\beta$ , $\cos\gamma$ are the direction cosines of the perpendicular line.

**Length of the perpendicular from the origin 'O' to the given plane** $ax + by + cz + d = 0$ **is given by**

$$\rho = \frac{-d}{\sqrt{a^2+b^2+c^2}}$$

**Length of the perpendicular from the point** P($x_1,y_1,z_1$) **to the plane** $ax + by + cz + d = 0$ **is given by**

$$\rho = \pm\frac{(ax_1+by_1+cz_1+d)}{\sqrt{a^2+b^2+c^2}}$$

**Plane through the Intersection of Two given Planes** $P_1: ax + by + cz + d_1 = 0$ **and** $P_2: ax + by + cz + d_2 = 0$ **is** $ax + by + cz + d_1 + k(ax + by + cz + d_2) = 0$

**Distance between two parallel planes** $P_1: ax + by + cz + d_1 = 0$ **and** $P_2: ax + by + cz + d_2 = 0$ **is**

$$d = \frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$

**Problems**

1. Find the equation of the plane passing through the point (2,-1,1) and parallel to the plane 3x+7y-10z=5

   **Solution**

   Given plane equation is $3x + 7y + 10z - 5 = 0$ ___(1)

   Any plane parallel to (1) is of the form $3x + 7y + 10z - 5 + k = 0$ ___(2)

   Plane (2) passes through (2, -1, 1)

$\therefore\ 4(2)+2(-4)-7(5)+k=0$

$$\boxed{k=35}$$

$\therefore$ The required plane equation is $4x+2y-7z+35=0$

**2.** Find the equation of the plane passing through the points (1, -2, 2) and (-3, 1, -2) and Perpendicular to the plane $2x + y - z + 6 = 0$

**Solution**

Let the required plane equation be $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ ___(1)

Plane (1) passes through (1, -2, 2)

$\therefore\ a(x-1)+b(y+2)+c(z-2)=0$ ___(2)

(2) passes through (-3, 1, -2)

$\therefore\ a(-3-1)+b(1+2)+c(-2-2)=0$

$$-4a+3b-4c=0 \qquad \text{\_\_\_(3)}$$

now plane (2) is perpendicular to $2x+y-z+6=0 \Rightarrow 2a+b-c=0$ ___(4)

from (3) & (4), using rule of cross multiplication,

$$\frac{a}{\begin{vmatrix}3 & -4\\ 1 & -1\end{vmatrix}}=\frac{b}{\begin{vmatrix}-4 & -4\\ -1 & 2\end{vmatrix}}=\frac{c}{\begin{vmatrix}-4 & 3\\ 2 & 1\end{vmatrix}}$$

$$\frac{a}{1}=\frac{b}{-12}=\frac{c}{-10}=k$$

Using these values in (2)

$$1(x-1)-12(y+2)-10(z-2)=0$$

$$x-12y-10z-5=0.$$

**3.** Find the equation of the plane which passes through the points (1, 0, -1) and (2, 1, 1) and parallel to the line joining the points (-2, 1, 3) and (5, 2, 0).

**Solution**

Equation of a plane passing from a point $(x_1, y_1, z_1)$ is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

since is passes through (1, 0, -1)

$$\Rightarrow a(x-1) + b(y-0) + c(z+1) = 0 \qquad \text{\_\_\_\_\_(1)}$$

Plane (1) passes through (2, 1, 1)

$$a(2-1) + b(1-0) + c(1+1) = 0$$

$$a + b + 2c = 0 \qquad \text{\_\_\_\_\_(2)}$$

D.R.'s of the line joining (-2, 1, 3) and (5, 2, 0) are 7, 1, -3

Plane (1) is parallel to this line

$\therefore$ Any normal of plane (1) is $\perp_r$ to this line D.R.'s 7, 1, -3

$$\therefore \; a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$7a + b - 3c = 0 \qquad \text{\_\_\_\_\_(3)}$$

Eliminating a, b, c from (1), (2) and (3)

$$\frac{a}{\begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 2 & 1 \\ -3 & 7 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix}}$$

$$\frac{a}{-5} = \frac{b}{17} = \frac{c}{-6} = k \;\; \text{(say)}$$

substituting a, b, c in (1)

$$-5(x-1) + 17y - 6(z+1) = 0$$

$$-5x + 17y - 6z + 5 - 6 = 0$$

$\boxed{-5x + 17y - 6z - 1 = 0}$ is the required equation of the plane.

**4.** Find the equation of the plane through (1, -1, 2) and perpendicular to the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$

**Solution**

Equation of the plane passing through (1, -1, 2) is $a(x-1)+b(y+1)+c(z-2)=0$ ___(1)

(1) $\perp r$ to $2x+3y-2z=5 \Rightarrow 2a+3b-2c=0$ ____(2)

(I) $\perp r$ to $x+2y-3z=8 \Rightarrow a+2b-3c=0$ ____(3)

Solving (1), (2) and (3) we get

$$\begin{vmatrix} x-1 & y+1 & z-2 \\ 2 & 3 & -2 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$$(x-1)(-9+4)-(y+1)(-6+2)+(z-2)(4-3)=0$$

$$(x-1)(-5)+4(y+1)+(z-2)=0$$

$\boxed{-5x+4y+z+7=0}$ is the required plane equation.

**5.** Find the equation of the plane passing through the points (2, 5, -3), (-2, -3, 5) and (5, 3, -3).

**Solution**

The equation of the plane passing through three points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

$$(x-2)(16)-(y-5)(-24)+(z+3)(32)=0$$

$\boxed{2x+3y+4y-7=0}$ is the required plane equation.

**6.** Show that the fair points (0, -1, -1), (4, 5, 1), (3, 9, 4) and (-4, 4, 4) lie on a plane.

**Solution:**

The equation of the plane passing through three points (0, -1, -1), (4, 5, 1), (3, 9, 4) is

$$\begin{vmatrix} x-0 & y+1 & z+1 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 5x - 7y + 11z + 4 = 0.$$

To prove (-4, 4, 4) also lies on this plane, we need to prove it satifies the above plane equation $5x - 7y + 11z + 4 = 0$

$$5(-4) - 7(4) + 11(4) + 4 = 0$$

$\therefore$ The given four points lie on $5x - 7y + 11z + 4 = 0$

**7.** Find the angle between the planes $2x + 4y - 6z = 11$ and $3x + 6y + 5z + 4 = 0$ .

Solution

Angle between two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

$$\cos\theta = \pm \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

lie between two planes $2x + 4y - 6z = 11$ and $3x + 6y + 5z + 4 = 0$ is

$$\cos\theta = \pm \frac{3(2) + 4(6) + 5(-6)}{\sqrt{4 + 16 + 36}\sqrt{9 + 36 + 25}}$$

$$\cos\theta = \pm 0$$

$$\boxed{\theta = \pi/2}$$

**8.** Find the equation of the plane which bisects perpendicularly the join of (2, 3, 5) and (5, -2, 7)

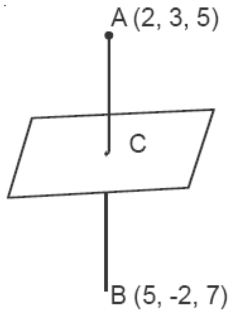
**Solution**

Let C be the midpoint of the line joining two points A(2, 3, 5) and B(5, -2, 7) then C has coordinates

$$C\left(\frac{2+5}{2}, \frac{3-2}{2}, \frac{5+7}{2}\right)$$

$$i.e.\ C\left(\frac{7}{2}, \frac{1}{2}, 6\right)$$

Equation of plane through $C\left(\frac{7}{2}, \frac{1}{2}, 6\right)$ is




$$a\left(x-\frac{7}{2}\right)+b\left(y-\frac{1}{2}\right)+c(z-6)=0 \qquad \text{\_\_\_\_(1)}$$

As AB $\perp r$ to the plane, the DR's of AB are 5 - 2, -2 -3, 7 - 5

i.e. a = 3,b = -5,c = 2)

Substituting in (1)

$$\Rightarrow 3\left(x-\frac{7}{2}\right)-5\left(y-\frac{1}{2}\right)+2(z-6)=0$$

$\boxed{3x-5y+2z-20=0}$ is the required plane equation.

**9.** Find the distance between the planes $x$ - 2$y$+ 2$z$ - 8 = 0 and -3$x$ + 6$y$ - 6$z$ = 57

**Solution**

Distance between two parallel planes

$P_1$: $ax + by + cz + d_1 = 0$ and $P_2$: $ax + by + cz + d_2 = 0$ is

$$d = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

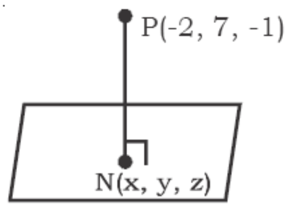
The given planes are $x$ - 2$y$+ 2$z$ - 8 = 0 and $x$-2$y$ + 2$z$ + 57/3 =0

$$d = \frac{\left|-8-\frac{57}{3}\right|}{\sqrt{1^2+(-2)^2+2^2}}$$

$$= \frac{|-27|}{\sqrt{1+4+4}} = 9$$

**10.** Find the foot N of the perpendicular drawn from P(-2, 7, -1) to the plane 2$x$ - $y$ + $z$ = 0

Let N be $(x_1, y_1, z_1)$. N lies on $2x-y+z=0$




$\therefore\ 2x_1 - y_1 + z_1 = 0$ ____(1)

The D.R.'s of PN are

$$x_1+2, y_1-7, z_1+1$$

PN is parallel normal to the plane

$$\frac{x_1+2}{2}=\frac{y_1-7}{-1}=\frac{z_1+1}{1}=k \quad \text{(say)}$$

$$x_1=2k-2, \quad y_1=-k+7, \quad z_1=k-1$$

Substituting in (1), we get

$$2(2k-2)-(-k+7)+(k-1)=0$$

$$\Rightarrow \boxed{k=2}$$

$$\therefore x_1=2, \quad y_1=5, \quad z_1=1$$

Hence the foot of the perpendicular is (2, 5, 1).

**11.** The foot of the perpendicular from the given point A(1, 2, 3) on a plane is B(-3, 6, Find the plane equation.

**Solution**

The D.R.'s of AB are $x_2-x_1,\ y_2-y_1,\ z_2-z_1$ i.e., -3-1, 6-2, 1-3 i-e., -4, 4, 2

Since AB is normal to the plane and B (-3, 6, 1) is a point

on the plane, the equation of the plane is $4(x-(-3))-4\,y-6)2(z-1)=0$

$\boxed{2x-2y+z+17=0}$ is the required plane equation.

**12.** Find the image or reflection of the point (5, 3, 2) in the plane $x + y - z = 5$.
Let A be (5, 3, 2)

**Solution**

Let the image of A be $B(x_1, y_1, z_1)$

The mid-point of AB is $L\left(\frac{x_1+5}{2}, \frac{y_1+3}{2}, \frac{z_1+2}{2}\right)$

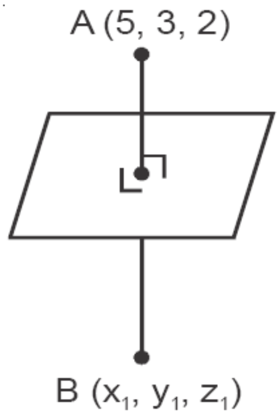
L lies on the plane $x + y - z = 5$ ____ (1)

$$\therefore \frac{x_1+5}{2}+\frac{y_1+3}{2}-\frac{z_1+2}{2}=5$$

$x_1 + y_1 - z_1 = 4$ ____(2)

D.R.'s of AB are $x_1 - 5, y_1 - 3, z_1 - 2$

D.R.'s normal to the plane are 1, 1, -1.
AB is parallel to normal to the plane




$$\therefore \ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{x_1 - 5}{1} = \frac{y_1 - 3}{1} = \frac{z_1 - 2}{-1} = \text{k} \ \ \text{(say)}$$

$x_1 = k + 5, \quad y_1 = k + 3, \quad z_1 = -k + 2.$

Substituting in (2)

$(k+5) + (k+3) - (-k+2) = 4$

$$\boxed{k = \frac{-2}{3}}$$

$\therefore$ The image B is $\left(\frac{-2}{3} + 5, \frac{-2}{3} + 3, \frac{-2}{3} + 2\right)$

i.e., $B\left(\frac{13}{3}, \frac{7}{3}, \frac{8}{3}\right)$

**13.** Find the equation of the plane through the line of intersection of $x + y + z = 1$ and $2x + 3y + 4z = 5$ and

(i) Perpendicular to to $x - y + z = 0$

(ii) passing through (1, 2, 3)

**Solution:**

The equation of the plane passing through the line of intersection of

$x + y + z = 1$ ___(1) and $2x + 3y + 4z - 5 = 0$ ___(2)

(i.e) $(x + y + z - 1) + k(2x + 3y + 4z - 5) = 0$ ___(3)

$(1 + 2k)x + (1 + 3k)y + (1 + 4k)z - (1 + 5k) = 0$ ___(4)

(i) (4) $\perp r$ to $x - y + z = 0$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$(1+2k)\cdot 1 + (1+3k)(-1) + (1+4k)\cdot 1 = 0$$

$$1 + 3k = 0 \Rightarrow \boxed{k = \frac{-1}{3}}$$

substituting in (4)

$$\left(1-\frac{2}{3}\right)x+\left(1-\frac{3}{3}\right)y+\left(1-\frac{4}{3}\right)z-\left(1-\frac{5}{3}\right)=0$$

$$\frac{x}{3}-\frac{z}{3}+\frac{2}{3}=0$$

$\boxed{x-z+2=0}$ is the required plane equation.

(ii) (3) passes through the point (1, 2, 3)

$$(1+2+3-1)+k(2+6+12-5)=0$$

$$5+15k=0$$

$$\boxed{k=\frac{-1}{3}}$$

Substituting in equation (4)

$\boxed{x - z + 2 = 0}$ is the required plane equation

**14.** Find the equation of the plane passing through the line of intersection of the planes $2x + 5y + z = 3$ and $x + y + 4z = 5$ and parallel to the plane $x + 3y + 6z = 1$.

**Solution:**

The given planes are $2x + 5y + z = 3$ ____(1)

$x + y + 4z = 5$ ____(2)

$x + 3y + 6z = 1$____(3)

The required plane equation is is of the form

$(2x + 5y + z - 3) + k(x + y + 4z - 5) = 0$

$(2 + k)x + (k + 5)y + (1+ 4k)z - (3 + 5k) = 0$____(4)

(4) is parallel to (3)

$$\therefore \frac{2+k}{1}=\frac{k-5}{3}=\frac{4k+1}{6}$$

$$\frac{2+k}{1}=\frac{k-5}{3}\Rightarrow k=\frac{-11}{2}$$

Substituting in (4)

$$\left(2-\frac{11}{2}\right)x+\left(\frac{-11}{2}-5\right)y+\left(1+4\left(\frac{-11}{2}\right)\right)z=3+5\left(\frac{-11}{2}\right)$$

$x + 3y + 6z - 7 = 0$ is the required plane equation

**15.** Find the equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ parallel to y-axis.

**Solution:**

The given planes are $x + y + z = 1$ ____(1)

$2x + 3y - z + 4 = 0$ ____(2)

Let the required plane equation be

$(x + y + z - 1) + k(2x + 3y - z + 4) = 0$ ____(3)

$(1 + 2k)x + (1 + 3k)y + (1 - k)z - 1 + 4k = 0$

Nomal to plane (3) is perpendicular *to* y-axis whose D.R.'s are 0, 1, 0

$$\therefore\ (1+2k)(0)+(1+3k)(1)+(1-k)(0)=0$$

$$\Rightarrow \boxed{k=\frac{-1}{3}}$$

Substituting in (3)

$$\therefore\ (x+y+z-1)\frac{-1}{3}(2x+3y-z+4)=0$$

$\boxed{x+4z-7=0}$ is the required plane equation.

2. Equation of a straight line passing through ($x_1$, $y_1$, $z_1$) with direction ratios of the line as a, b, c

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

3. Equation of a straight line passing through two given points ($x_1$, $y_1$, $z_1$) and ($x_2$, $y_2$, $z_2$) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

**Problems:**

1. Find the equation of the straight line which passes through the point (2, 3,4) and making angles 60°, 60°, 45° with positive direction of axes.

**Solution:** Equation of a straight line is $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} \longrightarrow$ (1)

Here $x_1 = 2, y_1 = 3, z_1 = 4$.

$$l = \cos 60 = \frac{1}{2}$$

$$m = \cos 60 = \frac{1}{2}$$

$$n = \cos 45 = \frac{1}{\sqrt{2}}$$

Substituting the values of l, m, n in the straight line equation, we get

$$\frac{x-2}{\frac{1}{2}} = \frac{y-3}{\frac{1}{2}} = \frac{z-4}{\frac{1}{\sqrt{2}}}$$

2. Find the equation of the straight line passing through (2, -1, 1) and parallel to the line joining the points (1, 2, 3) and (-1, 1, 2).

**Solution:**

The direction ratios of the line joining the points (1, 2, 3) and (-1, 1, 2) are -1-1, 1-2, 2-3
i-e., -2, -1, -1

Equation of a straight line passing through the point ($x_1$, $y_1$, $z_1$) with direction ratios a,b,c is

$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ --------(1)

Here $x_1 = 2, y_1 = -1, z_1 = 1$

$a = -2, b = -1, c = -1$

Substituting these values in (1) we get

$$\frac{x-2}{-2} = \frac{y-(-1)}{-1} = \frac{z-1}{-1}$$

(i.e) $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z-1}{1}$ which is the required equation of the line.

**3.** Find the equation of the line joining the points (1, -1, 2) and (4, 2, 3).

**Solution:**

The equation of a straight line is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Here $x_1$=1, $x_2$=-1,$x_3$=2

$x_2 = 4, y_2 = 2, z_2 = 3$.

Hence the equation of the required line is

$$\frac{x-1}{4-1} = \frac{y-(-1)}{2-(-1)} = \frac{z-2}{3-2}$$

i.e., $\frac{x-1}{3} = \frac{y+1}{3} = \frac{z-2}{1}$

**4.** Prove that the points (3, 2, 4) (4, 5, 2) and (5, 8, 0) are collinear.

**Solution:**

Equation of a straight line passing through two given points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Equation of the line passing through (3, 2, 4) and (4, 5, 2) is

$$\frac{x-3}{4-3} = \frac{y-2}{5-2} = \frac{z-4}{2-4}$$

i.e., $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z-4}{-2}$ ____(1)

If the above two points are collinear with (5, 8, 0) then the point (5, 8, 0) must satisfy equation (1)

Substituting $x = 5$, $y = 8$, $z = 0$ in (1), we get

$$\frac{5-3}{1}=\frac{8-2}{3}=\frac{0-4}{-2}$$

$$\Rightarrow \frac{2}{1}=\frac{6}{3}=\frac{-4}{-2}\Rightarrow \frac{2}{1}=\frac{2}{1}=\frac{2}{1}$$

Hence the point (5, 8, 0) satisfies equation (1)

$\therefore$ The three given points are collinear.

**5.** Find the angle between the lines

$$\frac{x+1}{2}=\frac{y+3}{2}=\frac{z-4}{-1} \text{ and } \frac{x-4}{1}=\frac{y+4}{2}=\frac{z+1}{2}$$

**Solution:**

Direction ratios of the first line are 2, 2, -1

Direction cosines of first line are $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$

Direction ratios of the second line are 1, 2, 2.

Direction cosines of second line are $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

Let be the angle between the lines (1) and (2), then

$$\cos\theta = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)+\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)+\left(\frac{-1}{3}\right)\left(\frac{2}{3}\right)=\frac{4}{9} \qquad [\because \cos\theta = l_1l_2+m_1m_2+n_1n_2]$$

$$\theta = \cos^{-1}\left(\frac{4}{9}\right)$$

**Problem for practice**

**6.** Find the equations of the straight line through (a, b, c) which are (i) perpendicular to z-axis (ii) Parallel to z-axis.

**Transform of a general form of a straight line into symmetrical form**

To express the equation of a line in symmetrical form, we need

(i) The coordinates of a point on the line.

(ii) The direction ratios of the straight line

**Method of find a point on the given line**

The general form of a straight line is $a_1x+b_1y+c_1z+d_1=0 = a_2x+b_2y+c_2z+d_2=0$

Let us find the coordinates of the point, where this line meets XOY plane. Then z = 0.

Equations of planes are $a_1x+b_1y+ d_1=0$; $a_2x+b_2y +d_2=0$

Solving these equations, we get

$$\frac{x}{\begin{vmatrix} b_1 & d_1 \\ b_2 & d_2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$\therefore$ Co-ordinates of a point on the line is $\left(\dfrac{b_1d_2 - b_2d_1}{a_1b_2 - a_2b_1}, \dfrac{a_2d_1 - a_1d_2}{a_1b_2 - a_2b_1}, 0\right) = (x_1, y_1, 0)$ (say).

**Note:**
To find a point on the line, we can also take x = 0 or y = 0.

**Method of find the direction ratios.**
Let (l, m, n) be the direction ratios of the required line.

The required line is the intersection of the planes $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2 = 0$

It is perpendicular to these planes whose direction ratios of the normal are $a_1, b_1, c_1$ and $a_2, b_2, c_2$.

By condition of perpendicularity of two lines we get

$$a_1l + b_1m + c_1n = 0$$
$$a_2l + b_2m + c_2n = 0$$

Using the rule of cross multiplication, we get

$$\frac{l}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{-m}{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} = \frac{n}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Therefore, the equation of a straight line passing through a point ($x_1$, $y_1$, $z_1$)with direction ratios a,b,c is

$$\frac{x - x_1}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{y - y_1}{-\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} = \frac{z - 0}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

**7.** Find the symmetrical form the equations of the line $3x + 2y - z - 4 = 0$ and $4x + y - 2z + 3 = 0$ and Find its direction cosines.

**Solution:**
Equation of the given line is

$$\left.\begin{array}{l} 3x + 2y - z - 4 = 0 \\ 4x + y - 2z + 3 = 0 \end{array}\right\} \longrightarrow (1)$$

Let l, m, n be the D.R.'s of line (1).

Since the line is common to both the planes, it is perpendicular to the normals to both the planes. Hence we have

$$3l + 2m - n = 0,$$
$$4l + m - 2n = 0$$

Solving these, we get

$$\frac{l}{-4+1} = \frac{m}{-4+6} = \frac{n}{3-8}$$
$$\Rightarrow \frac{l}{-3} = \frac{m}{2} = \frac{n}{-5}$$

Therefore The D.R.'s of the line (1) ae -3, 2, -5.

$$l = \frac{-3}{\sqrt{38}}, m = \frac{2}{\sqrt{38}}, n = \frac{-5}{\sqrt{38}}$$

Now, to find the co-ordinates of a point on the line given by (1),

Let us find the point where it meets the plane z = 0.

Put z = 0 in the equations given by (1)

we have $\left.\begin{array}{l} 3x + 2y = 4 \\ 4x + y = -3 \end{array}\right\} \longrightarrow (2)$

Solving these two equations, we get

$$\frac{x}{6+4} = \frac{y}{-16-9} = \frac{1}{3-8}$$
$$\Rightarrow \frac{x}{10} = \frac{y}{-25} = \frac{1}{-5}$$
$$\Rightarrow x = -2, y = 5$$

The line meets the plane z = 0 at the point (-2, 5, 0) and has direction ratios -3, 2, -5.

Therefore the equations of the given line in symmetrical form are

$$\frac{x+2}{-3} = \frac{y-5}{2} = \frac{z-0}{-5}.$$

**Problem for Practice**

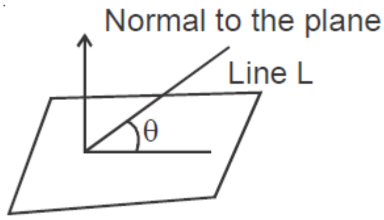
**8.** Find the symmetrical form of the equation of the straight line$2x - 3y + 3z = 4$, $x + 2y - z = -3$

**9.** Find the symmetrical form, the equations of the line formed by planes $x + y + z + 1 = 0$, $4x + y - 2z + 2 = 0$ and find its direction-cosines.

**The Plane and the Straight Line**

**Angle between a Line and Plane**

Angle between a line $L: \frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and a plane $U: ax+by+cz+d=0$




If $\theta$ is the angle between the line L and the plane U, then angle between line L & normal to the plane is $90-\theta$.

Direction ratios of lline L are l, m, n

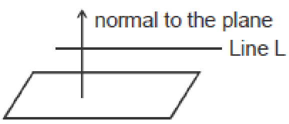
Direction ratios of the normal to the plane U are a, b, c

$$\therefore \cos(90-\theta) = \frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$$

(i.e) $$\theta = \sin^{-1}\left[\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}\right]$$

Consider the line L: $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the plane P: ax+ by + cz + d = 0

**(i)** Perpendicular Condition




Line L is perpendicular to the plane U.

$\Rightarrow$ Line L and normal to the plane are parallel. So their direction ratios are proportional.

Direction ratios of line L : l, m, n

Direction ratios of normal to the plane : a, b, c

Hence

$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

**(ii)** Line L is perpendicular to the plane U.

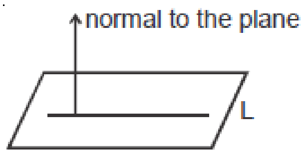
Line L and normal to the plane are parallel. So their direction ratios are proportional.

Hence al + bm + cn = 0

**(iii)** Line L lies on the plane U.

$\Rightarrow$ Every point of line L lies on the plane $ax+by+cz+d=0 \quad \rightarrow (1)$

$\therefore$ the obvious point $(x_1, y_1, z_1)$ lies on the plane (1)




$\therefore ax_1+by_1+cz_1+d=0$ ________ (2)

(1) - (2) gives $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ ________ (2)

Also line L and normal to the plane are perpendicular.

$\therefore al+bm+cn=0 \quad \rightarrow (4)$

Hence if a line L lies on a plane U, then the condition is given by (2) and (4).

And equation of any plane which passes through the given line L is given by (3) and (4).

**Problems**

**10.** Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $3x+y+z=7$

**Solution:**

The angle between a line and a plane is

$$\sin\theta = \frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$$

Here $l=2, \quad m=3, \quad n=6$

$a=3, \quad b=1, \quad c=1$

$$\sin\theta = \frac{2(3)+1(3)+1(6)}{\sqrt{3^2+1^2+1^2}\sqrt{2^2+3^2+6^2}}$$

$$=\frac{6+3+6}{\sqrt{11}\sqrt{49}}$$

$$\sin\theta = \frac{15}{7\sqrt{11}}$$

$$\theta = \sin^{-1}\left(\frac{15}{7\sqrt{11}}\right)$$

**11.** Find the equation of the plane which contains the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ and is perpendicular to the plane $x+2y+z=12$ .

**Solution:**

Let a, b, c be the direction ratios of the normal to the required plane

Equation of a plane which contains line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ is given by

$a(x-1)+b(y+1)+c(z-3)=0$----------(1)

Since line L and normal to the plane are perpendicular $2a-b+4c=0$--------(2)

Also given the required plane is perpendicular to the plane $x + 2y + z = 12$

Hence their normals are perpendicular

Therefore $a + 2b + c = 0$---------(3)

Eliminating a, b, c from (1), (2) & (3), we get the required plane equation.

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

(i.e) $9x - 2y - 5z + 4 = 0$ is the required equation of the plane.

**12.** Find the image of the line $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2}$ in the plane $2x - y + z + 3 = 0$.

**Solution:**

The image of the line is the line joining the images of any two points on the line.

It is an advantage to select one of the points as the point of intersection of the line.

$$L : \frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2} \text{ and the plane } 2x - y + z + 3 = 0$$

Any point on the line L is $(3r + 1, 5r + 3, 2r + 4)$

If this point is taken as A, then it lies on the plane, then$2(3r + 1) - (5r + 3) + (2k + 4) + 3 = 0$

(i.e) r = -2.

Substituting $k = -2$, we get the co-ordinate of the point of intersection of the line and the plane A(-5,-7,0).

Let us consider another point on the line L.

Let us choose the obvious point on the line i.e., P(1, 3, 4).

Let the image of the point P (1, 3, 4) on the plane $2x – y + z + 3 = 0$ be P'.

By definition of the image, the midpoint M of PP' lies on the plane and line PP' is normal to the plane.

Let the direction ratios of the line PP' be (, m, n D.R.'s of the normal to the plane are 2, -1, 1

As line PP 'and normal to the plane are parallel their direction ratios are proportional.

Then the image of P is P' (-3, 5, 2).

$$\therefore \frac{l}{2} = \frac{m}{-1} = \frac{n}{1} = k \text{ (say)}$$

Equation of line PP', which passes through (1, 3, 4) with direction ratios (2, -1, 1) is given by $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{-1}$

Any point on this line is given by $(2k + 1, -k + 3, -k + 4)$

Suppose this point is M which meets the plane, then it has to satisfy the equation of the plane.

$$\therefore 2(2k+1)-(-k+3)+(k+4)+3=0$$

i.e., $\boxed{k=-1}$

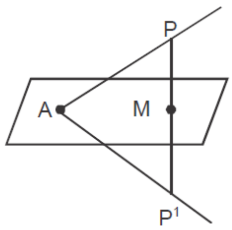
Substituting for k, we get the co-ordinates of M(-1, 4,3)

$\because$ M is the mid point of PP[1]

$$\frac{1+x_1}{2}=-1,\ \frac{1+y_1}{2}-4,\ \frac{1+z_1}{2}=3$$

$$x_1=-3, y_1=5, z_1=2$$

$\because$ the image of is P[1] (-3, 5, 2).




Equation of the image line is given by

$$\frac{x+5}{-3+5}=\frac{y+7}{5+7}=\frac{z-0}{2-0}$$

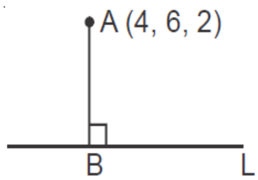
i.e., $$\frac{x+5}{2}=\frac{y+7}{12}=\frac{z}{2}$$

i.e., $$\frac{x+5}{1}=\frac{y+7}{6}=\frac{z}{1}$$

**13.** Find the foot of the perpendicular from a point (4, 6, 2) to the line $\frac{x-2}{3}=\frac{y-2}{2}=\frac{z-2}{1}$. Also find the length and the equation of the perpendicular.

**Solution:**

Let B be the foot of the perpendicular drawn from a point A(4, 6, 2) to the line L: $\frac{x-2}{3}=\frac{y-2}{2}=\frac{z-2}{1}=$ k




Then B has coordinates of the form (3$k$ + 2, 2$k$ + 2, $k$ + 2)

Direction ratios of line AB: (3$k$ - 2, 2$k$ - 4, $k$)

Direction ratios of the line L : (3, 2,1)

Line AB is perpendicular to line L.

Hence 3(3$k$ - 2) + 2(2$k$ - 4) = $k$ = 0

Therefore $k$ =1

Hence the foot of the perpendicular is N (5, 4, 3)

Equation of the perpendicular is the equation of line joining the points (4, 6, 2) and (5, 4, 3) is

$$\frac{x-4}{1}=\frac{y-6}{-2}=\frac{z-2}{1}$$

Length of the perpendicular $= AB = \sqrt{(5-4)^2+(4-6)^2+(3-2)^2}$

$$= \sqrt{6} \text{ units.}$$

**Condition for Co planarity of the lines**

Condition for lines $L_1: \frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $L_2: \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_1}{n_2}$ to be coplanar is

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

Equation of the plane containing the coplanar lines $L_1$ and $L_2$ is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

**Problems**

**14.** Show that the lines L1: $\frac{x-7}{2}=\frac{y-10}{3}=\frac{z-13}{4}$ and L2: $\frac{x-3}{1}=\frac{y-5}{2}=\frac{z-7}{3}$ are coplanar. Find the equation of the plane of co planarity and the coordinates of the point of intersection of the lines.

**Solution:**

Consider the lines

$L_1: \frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $L_2: \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_1}{n_2}$

Condition for co planarity of two lines is

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

L1: $\frac{x-7}{2}=\frac{y-10}{3}=\frac{z-13}{4}$

Here $(x_1,y_1,z_1)=(7,10,13)$

$(x_2,y_2,z_2)=(3,5,7)$

$l_1,m_1,n_1=2,3,4$

$l_2,m_2,n_2=1,2,3$

$$\begin{vmatrix} 3-7 & 5-10 & 7-13 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} -4 & -5 & -6 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= -4[9-8]+5[6-4]-6[4-3]=0$$

Therefore the lines are coplanar.

Equation of the plane containing the coplanar lines $L1$ and $L2$ is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

i.e., $$\begin{vmatrix} x-7 & y-10 & z-13 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

i.e., $(x-7)(9-8)-(y-10)(6-4)+(z-13)(4-3)=0$

i.e., $x-2y+z=0$

To find the point of intersection of lines L1 & L2 :

Any point on the line $L_1$: $\frac{x-7}{2} = \frac{y-10}{3} = \frac{z-13}{4} = k$ is $A(2k + 7,+10, 4k +13)$

Any point on the line $L_2$: $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-7}{3} = r$ is $B(r + 3, 2r + 5,3r + 7)$

If $L_1$ and $L_2$ intersect, then for some value of r and k, the coordinates A and B are the same.

$$\begin{aligned} \therefore \quad & 2k+7=r+3 && \text{i.e., } 2k-r=-4 \\ & 3k+10=2r+5 && \text{i.e., } 3k-2r=-5 \\ & 4k+13=3r+7 && \text{i.e., } 4k-3r=-5 \end{aligned}$$

Solving any two equations, we get k = -3 and r = -2.

Hence the common point of intersection of the lines $L_1$ and $L_2$ is (1, 1, 1).

**Problem for practice**

**15.** Show that the lines joining the points (0, 2, -4) & (-1, 1, -2) and (-2, 3, 3) & (-3, -2, 1) are coplanar. Find their point of intersection. Also find the equation of the plane containing them.

## SHORTEST DISTANCE BETWEEN TWO SKEW LINES

Two straight lines which do not lie in the same plane are called non-planar or skew lines. Skew lines are neither parallel nor intersecting. Such lines have a common perpendicular. The length of the segment of this common perpendicular line intercepted between the skew lines is called the shortest distance between them. The common perpendicular line itself is called the shortest distance line.

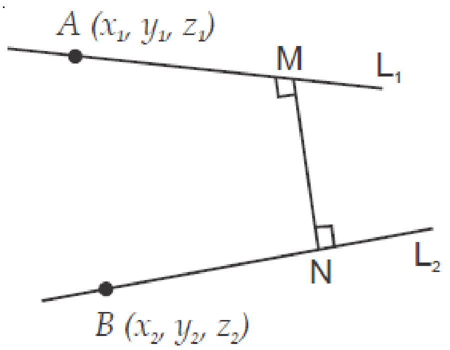
Let us now find the shortest distance and the equations of the shortest distance line between the skew lines.

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1} \qquad \text{\_\_\_\_(1)}$$

and $$\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2} \qquad \text{\_\_\_\_(2)}$$

Let $L_1$, $L_2$ be the skew lines passing through A $(x_1, y_1, z_1)$ and B $(x_2, y_2, z_2)$ respectively.

Let MN be the S.D.between $L_1$ & $L_2$ and let l, m, n be the D.R.'s of the S.D.Line.




The point M may be taken as $(x_1+l_1r_1, y_1+m_1r_1, z_1+n_1r_1)$ and N may be taken as $(x_2+l_2r_2, y_2+m_2r_2, z_2+n_2r_2)$ Then the D.R.'s of MN are found.

Using the fact that MN is perpendicular to both $L_1$ and $L_2$ obtain two equations in $r_1$ and $r_2$, solving which we obtain the values of $r_1$ and $r_2$.

Substituting there values, we know the coordinates of M and N. Then the length and equations of MN can be found.

**Problems**

**16.** Find the length and equations of the shortest distance between the lines L1: $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$

L2: $\frac{x+3}{-3}=\frac{y\mp 7}{2}=\frac{z-6}{4}$

**Solution :**

Let the S.D. line cut the first line at P and the second line at Q.

$$L_1:\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}=r \quad \text{(say)}$$

$$L_2:\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}=s \quad \text{(say)}$$

The point P on $L_1$ has coordinates $(3r+3, -r+8, r+3)$

The point Q on $L_2$ has coordinates $(-3s-3, 2s-7, 4s+6)$

Hence DR's of PQ are $-3x-3s-6,\ 2s+r-15,\ 4s-r+3$

PQ is perpendicular to L1

$$\therefore\ 3(-3r-3s-6)-(2s+r-15)+(4s-r+3)=0$$

i.e., $7s+11r=0$ ____(1)

PQ is perpendicular to $L_2$

$$\therefore -3(-3r-3s-6)+2(2s+r-15)+4(4s-r+3)=0$$

$29s+7r=0$ ____(2)

Solving (1) and (2), we get r = s = 0

Using these values of r and s in the co-ordinates of P and Q, we get

P ( 3, 8, 3) and Q (-3, -7, 6)

Length of S.D is $=\sqrt{270}=3\sqrt{30}$ units

Equation of the S.D. line is

$$\frac{x-3}{-6}=\frac{y-8}{-15}=\frac{z-3}{3}\Rightarrow\frac{x-3}{-2}=\frac{y-8}{-5}=\frac{z-3}{1}$$

**Problem for practice**

**17.** Find the length of the shortest distance between the lines

$$\frac{x-2}{2}=\frac{y+1}{3}=\frac{z}{4};2x+3y-5z-6=0=3x-2y-z+3$$