$y''(x) = p(x)y' + q(x)y + r(x)$

$y(a) = \alpha, y(b) = \beta$

Atış metodu:

1. Başlangıç değer problemi

$y''(x) = p(x)y' + q(x)y + r(x)$

$y(a) = \alpha, y'(a) = 0$

Çözüm: $y_1(x)$

2.Başlangıç değer problemi

$y''(x) = p(x)y' + q(x)y$

$y(a) = 0, y'(a) = 1$

Çözüm: $y_2(x)$

$$y(x) = y_1(x) + \frac{\beta - y_1(b)}{y_2(b)} y_2(x)$$

$x^2 y'' + (2x+1)y' + 3y = 6, \quad y(1) = 2, \; y'(1) = 0.5$

$u_1 = y$

$u_2 = y'$

$u_1' = y' = u_2, \quad u_1(1) = y(1) = 2$

$$u_2' = y'' = -\frac{2x+1}{x^2} u_2 - \frac{3}{x^2} u_1 + \frac{6}{x^2}, \quad u_2(1) = y'(1) = 0.5$$

$$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} u_2 \\ -\frac{2x+1}{x^2} u_2 - \frac{3}{x^2} u_1 + \frac{6}{x^2} \end{bmatrix}, \quad \begin{bmatrix} u_1(1) \\ u_2(1) \end{bmatrix} = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$$

$y' = x^2 - y,\ y(0) = 1$

$x_1 = 0, y_1 = 1$

$x_2 = 0.1, y_2 = ?$

$x_3 = 0.2, y_3 = ?$

$f(x, y) = x^2 - y$

1.iterasyon

$k_1 = hf(x_1, y_1) = 0.1 f(0,1) = -0.1$

$k_2 = hf(x_1 + h/2, y_1 + k_1/2) = 0.1 f(0.05, 0.95) = \ldots$

$k_3 = hf(x_1 + h/2, y_1 + k_2/2) = 0.1 f(0.05, \ldots) = \ldots$

$k_4 = hf(x_1 + h, y_1 + k_3) = 0.1 f(0.1, \ldots) = \ldots$

$y_2 = y_1 + (1/6) * (k_1 + 2k_2 + 2k_3 + k_4) = \ldots$

2.iterasyon

$k_1 = hf(x_2, y_2) = \ldots$

$k_2 = hf(x_2 + h/2, y_2 + k_1/2) = \ldots$

$k_3 = hf(x_2 + h/2, y_2 + k_2/2) = \ldots$

$k_4 = hf(x_2 + h, y_2 + k_3) = \ldots$

$y_3 = y_2 + (1/6) * (k_1 + 2k_2 + 2k_3 + k_4) = \ldots$

1. Başlangıç değer problemi

$y''(x) = p(x)y' + q(x)y + r(x)$

$y(a) = \alpha, y'(a) = 0$

Çözüm: $y_1(x)$

$u_1 = y_1$

$u_2 = y_1'$

$u_1' = u_2, \quad u_1(a) = \alpha$

$u_2' = p(x)u_2 + q(x)u_1 + r(x), \quad u_2(a) = 0$

$$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} u_2 \\ p(x)u_2 + q(x)u_1 + r(x) \end{bmatrix}, \quad \begin{bmatrix} u_1(a) \\ u_2(a) \end{bmatrix} = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$

$h = (b-a)/N$

$x_1 = a, u_{1,1} = \alpha, u_{2,1} = 0$

$x_2 = a+h, u_{1,2} = ?, u_{2,2} = ?$

$\vdots$

$x_{N+1} = b, u_{1,N+1} = ?, u_{2,N+1} = ?$

$$u_1' = u_2 \qquad\qquad u_2' = p(x)u_2 + q(x)u_1 + r(x)$$

$$k_1 = hu_{2,i} \qquad\qquad m_1 = h(p(x_i)u_{2,i} + q(x_i)u_{1,i} + r(x_i))$$

$$k_2 = h(u_{2,i} + m_1/2) \qquad\qquad m_2 = h(p(x_i + h/2)(u_{2,i} + m_1/2) + q(x_i + h/2)(u_{1,i} + k_1/2) + r(x_i + h/2))$$

$$k_3 = h(u_{2,i} + m_2/2) \qquad\qquad m_3 = h(p(x_i + h/2)(u_{2,i} + m_2/2) + q(x_i + h/2)(u_{1,i} + k_2/2) + r(x_i + h/2))$$

$$k_4 = h(u_{2,i} + m_3) \qquad\qquad m_4 = h(p(x_i + h)(u_{2,i} + m_3) + q(x_i + h)(u_{1,i} + k_3) + r(x_i + h))$$

$$u_{1,i+1} = u_{1,i} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad\qquad u_{2,i+1} = u_{2,i} + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)$$

$$i = 1, \ldots, N \qquad\qquad i = 1, \ldots, N$$

$$u_1 = y_1 \qquad\qquad u_2 = y_1'$$

2.Başlangıç değer problemi

$y''(x) = p(x)y' + q(x)y$

$y(a) = 0, y'(a) = 1$

Çözüm: $y_2(x)$

$v_1 = y_2(x)$

$v_2 = y_2'(x)$

$v_1' = v_2, \quad v_1(a) = 0$

$v_2' = p(x)v_2 + q(x)v_1, \quad v_2(a) = 1$

$$\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} v_2 \\ p(x)v_2 + q(x)v_1 \end{bmatrix}, \quad \begin{bmatrix} v_1(a) \\ v_2(a) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$h = (b-a)/N$

$x_1 = a, v_{1,1} = 0, v_{2,1} = 1$

$x_2 = a + h, v_{1,2} = ?, v_{2,2} = ?$

$\vdots$

$x_{N+1} = b, v_{1,N+1} = ?, v_{2,N+1} = ?$

$$v_1' = v_2 \qquad\qquad v_2' = p(x)v_2 + q(x)v_1$$

$$t_1 = hv_{2,i} \qquad\qquad s_1 = h(p(x_i)v_{2,i} + q(x_i)v_{1,i})$$

$$t_2 = h(v_{2,i} + s_1/2) \qquad\qquad s_2 = h(p(x_i + h/2)(v_{2,i} + s_1/2) + q(x_i + h/2)(v_{1,i} + t_1/2))$$

$$t_3 = h(v_{2,i} + s_2/2) \qquad\qquad s_3 = h(p(x_i + h/2)(v_{2,i} + s_2/2) + q(x_i + h/2)(v_{1,i} + t_2/2))$$

$$t_4 = h(v_{2,i} + s_3) \qquad\qquad s_4 = h(p(x_i + h)(v_{2,i} + s_3) + q(x_i + h)(v_{1,i} + t_3))$$

$$v_{1,i+1} = v_{1,i} + \frac{1}{6}(t_1 + 2t_2 + 2t_3 + t_4) \qquad\qquad v_{2,i+1} = v_{2,i} + \frac{1}{6}(s_1 + 2s_2 + 2s_3 + s_4)$$

$$i = 1, \ldots, N \qquad\qquad i = 1, \ldots, N$$

$$v_1 = y_2 \qquad\qquad v_2 = y_2'$$

SINIR DEĞER PROBLEMİNİN ÇÖZÜMÜ:

$$w_i = u_{1,i} + \frac{\beta - u_{1,N+1}}{v_{1,N+1}} v_{1,i}, \quad i = 1, \ldots, N+1$$

$w_i \sim y(x)$