2) Infinite intervals

Let $c \in \mathbb{R}$.

i) $[c, \infty) = \{x \mid x \geq c\}$

ii) $(-\infty, c] = \{x \mid x \leq c\}$

iii) $(c, \infty) = \{x \mid x > c\}$

iv) $(-\infty, c) = \{x \mid x < c\}$

v) $(-\infty, \infty) = \mathbb{R}$.

Line Equation on $\mathbb{R}^2$.

Let $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$.

An equation of the line going through P_1, P_2 is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad \text{if } x_2 \neq x_1.$$

Example. Write an equation of the line going through the points $P_1 = (-2, 3)$ and $P_2 = (4, -1)$.

$$y - 3 = \frac{-1-3}{4-(-2)}(x+2) = -\frac{2}{3}(x+2)$$

or

$$2x + 3y - 5 = 0.$$

Let $L_1: \cancel{A}\, y = m_1 x + n_1$
$L_2: \quad y = m_2 x + n_2.$

L_1 is perpendicular to L_2

$\Longleftrightarrow m_1 \cdot m_2 = -1.$

slope of $L_1 = m_1$
slope of $L_2 = m_2.$

Example. Are the lines
$2x + 5y - 7 = 0$ and $15x - 6y + 4 = 0$
perpendicular?

Solution. $L_1: 2x + 5y - 7 = 0 \Rightarrow$
$y = -\frac{2}{5}x + \frac{7}{5} \Rightarrow m_1 = -2/5.$

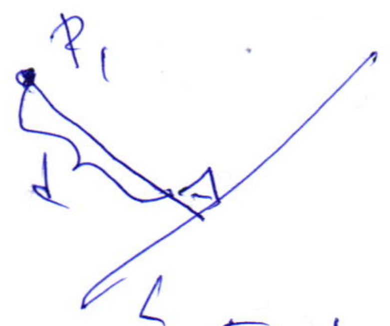
$L_2: 15x - 6y + 4 = 0 \Rightarrow$
$y = \frac{15}{6}x + \frac{4}{6} \Rightarrow m_2 = \frac{15}{6}.$

$m_1 \cdot m_2 = (-2/5) \cdot \left(\frac{\cancel{15}^{3}}{\cancel{6}_{2}}\right) = -1.$

So, $L_1 \perp L_2.$

Distance between the point $P_1=(x_1,y_1)$ and the line L with equation $Ax+By+C=0$ is given by

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}.$$



Example. Find the distance between the point $P=(3,1)$ and the line $3x+4y-3=0$.

Solution $d=\dfrac{|3.3+4.1-3|}{\sqrt{9+16}}=\dfrac{10}{\sqrt{25}}=\dfrac{10}{5}=2$

The distance between two points $P_1=(x_1,y_1)$, $P_2=(x_2,y_2)$ in the plane $\mathbb{R}^2$ is given by

$$|P_1P_2|=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}.$$

Let $L_1 : y = m_1 x + n_1$

$L_2 : y = m_2 x + n_2$.

L_1 is parallel to $L_2 \iff m_1 = m_2$.

Circle

Let the center (a,b) and the radius $r>0$ of a circle. Then, equation of circle is

$$(x-a)^2 + (y-b)^2 = r^2.$$

Functions

Definition: $D, S \subseteq \mathbb{R}$

A function f on a set D into a set S is a rule that assigns a unique element $f(x)$ in S to each element in D.

Here, D is called the domain of f, $D = D(f)$.

The range $R(f)$ of f is the subset of S consisting of all values $f(x)$ of the function.

Example. $f(x) = x^2+1$

f is a function.

$f(0) = 0+1 = 1$

$f(1) = 1+1 = 2$

$f(2) = 4+1 = 5$.

In $y = f(x)$, x is called independent variable, y is called dependent variable.

Example. $f(x) = \sqrt{x}$

Domain of $f = [0, \infty) = \{x \mid 0 \le x\}$

Example. $f(x) = \dfrac{x}{x^2-4}$

Domain of $f = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Example. $f(x) = \sqrt{1-x^2}$.

Domain of $f = \{x \mid 1-x^2 \geq 0\}$
$= [-1, 1]$.

Definition:

1) We say that f is an even function if $f(-x) = f(x)$ for every $x \in D(f)$.

2) We say that f is an odd function if $f(-x) = -f(x)$ for ~~every~~ $x \in D(f)$.

Example.

$f(x) = x^2$ is an even function

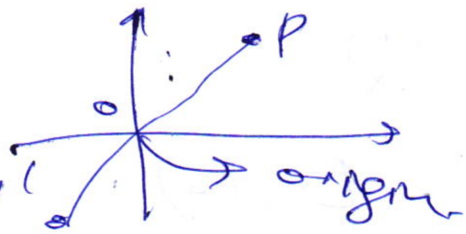
since $f(-x) = f(x) = x^2$.

Example.

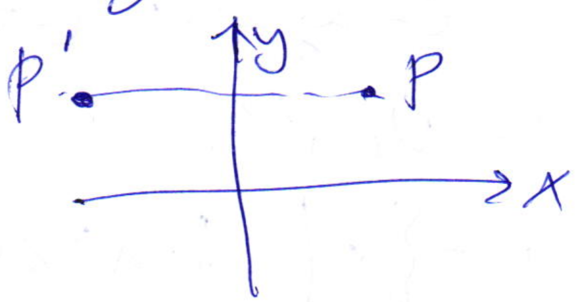
$f(x) = x^3$ is an odd function

$f(x) = \sin x$ " " "

The graph of an odd function is symmetric about the origin.



the graph of an even function is symmetric about the y-axis



Definition (1-1 function).

Let $f: A \longrightarrow B$ be a function.

We say that f is 1-1 (one-one) if $x \neq x'$, then $f(x) \neq f(x')$ or

$f(x) = f(x') \Longrightarrow x = x'$

Definition (onto = surjective) function.
Let $f : A \to B$ be a function.
We say that f is onto if for every $y \in B$ there is an $x \in A$ such that $y = f(x)$.

Graph of $f = \{ (x,y) \mid y = f(x), x \in D(f) \}$

Definition.

Let f and g be functions.
~~fo~~ For every x that belongs to ~~the~~ both $D(f)$ and $D(g)$

then,

1) $(f+g)(x) = f(x) + g(x)$
2) $(f-g)(x) = f(x) - g(x)$
3) $(fg)(x) = f(x)\,g(x)$
4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$.

Composite Function.

If f and g are functions, then the composite function $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x)).$$

$$D(f \circ g) = \{x \in D(g) : g(x) \in D(f)\}$$

Example.

Let $f(x) = \frac{1-x}{1+x}$, $x \neq -1$.

Calculate $(f \circ f)(x) = ?$

Solution:

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{1-x}{1+x}\right)$$

$$= \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = \frac{1+x-1+x}{1+x+1-x} = x.$$

$$D(f \circ f) = (-\infty, -1) \cup (-1, \infty).$$

Piecewise defined functions.

Example. The absolute value function

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Example. The signum function

$$f(x) = \operatorname{sgn}(x) = \begin{cases} 1 & , x > 0 \\ -1 & , x < 0 \\ \text{undefined} & x = 0 \end{cases}$$

Example. The greatest integer function

$$f(x) = [\![x]\!].$$

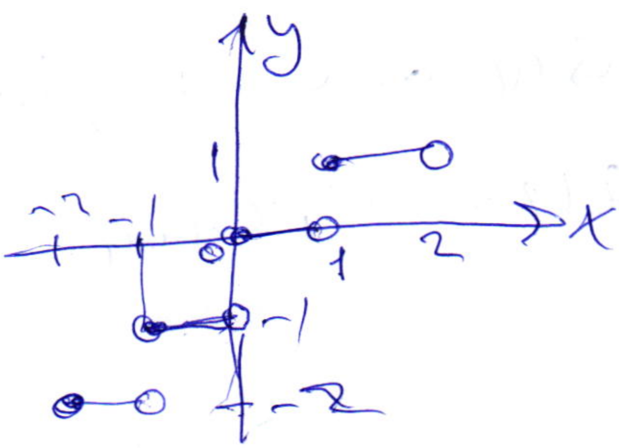
$$n \leq x < n+1 \Rightarrow [\![x]\!] = n \quad , n \in \mathbb{Z}.$$

$$[\![-0.3]\!] = -1$$

$$[\![-2]\!] = -2.$$

$$[\![3.4]\!] = 3.$$

$$[\![0]\!] = 0.$$



$$1 \leq x < 2 \Rightarrow [\![x]\!] = 1$$

$$0 \leq x < 1 \Rightarrow [\![x]\!] = 0$$

$$-1 \leq x < 0 \Rightarrow [\![x]\!] = -1$$

$$-2 \leq x < -1 \Rightarrow [\![x]\!] = -2.$$

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Polynomial functions

$f(x) = P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$,
where $a_0, a_1, \dots, a_n$ called coefficients of the polynomial are constants, $n \in \mathbb{N}$, n is the degree of the polynomial if n is the highest power of x.

Ex. $f(x) = 3$ degree 0.
$f(x) = 2 + x$ degree 1.

Rational Functions

If $P(x)$ and $Q(x)$ are polynomial function and $Q(x) \neq 0$, then

$$R(x) = \frac{P(x)}{Q(x)}$$

is called a rational function.

Example. $R(x) = f(x) = \dfrac{x+1}{x^2-4}$, $x \neq \mp 2$
is a rational funct.