Project Scheduling

What you will learn at the end of this chapter

- What Schedule is.
- What Critical Path Method (CPM) is.
- How to draw and understand CPM diagrams.
- How to determine the critical activities.
- How to extend the duration of activities without affecting the duration of the project.

Project Management

- Project management is the application of various tools and techniques for utilizing diverse resources in order to complete one largely unique task within established budget, limited time and desired quality.
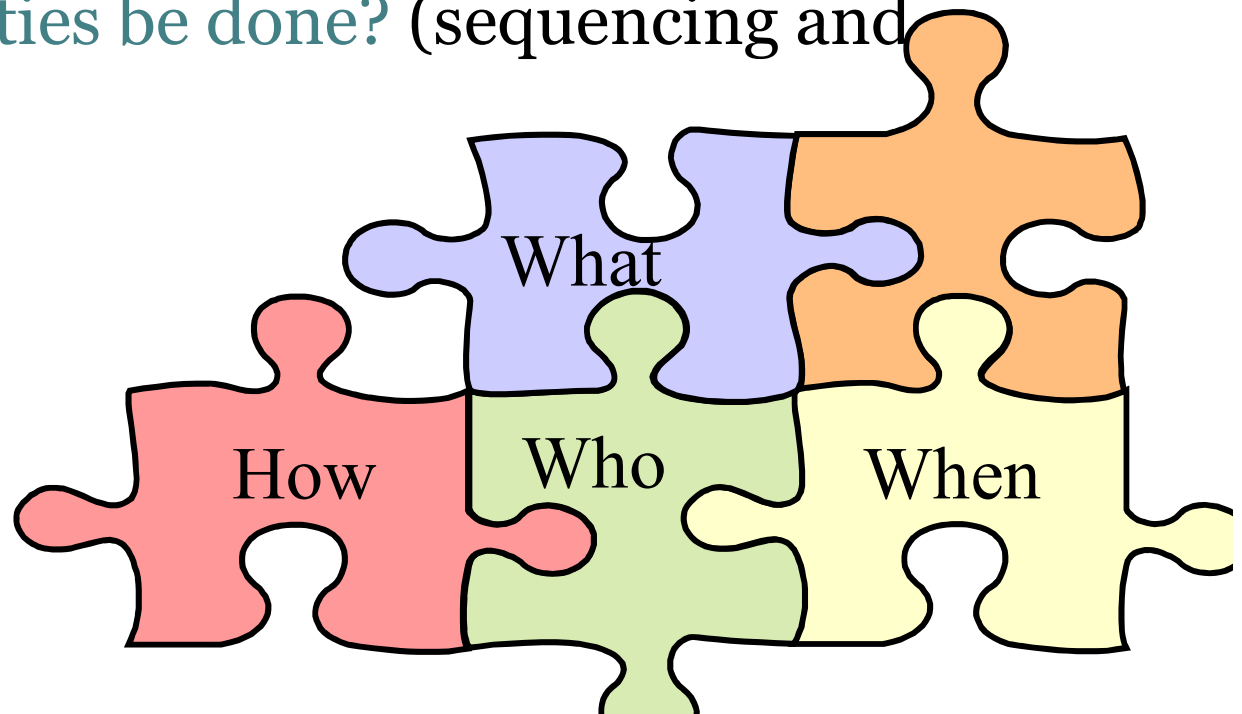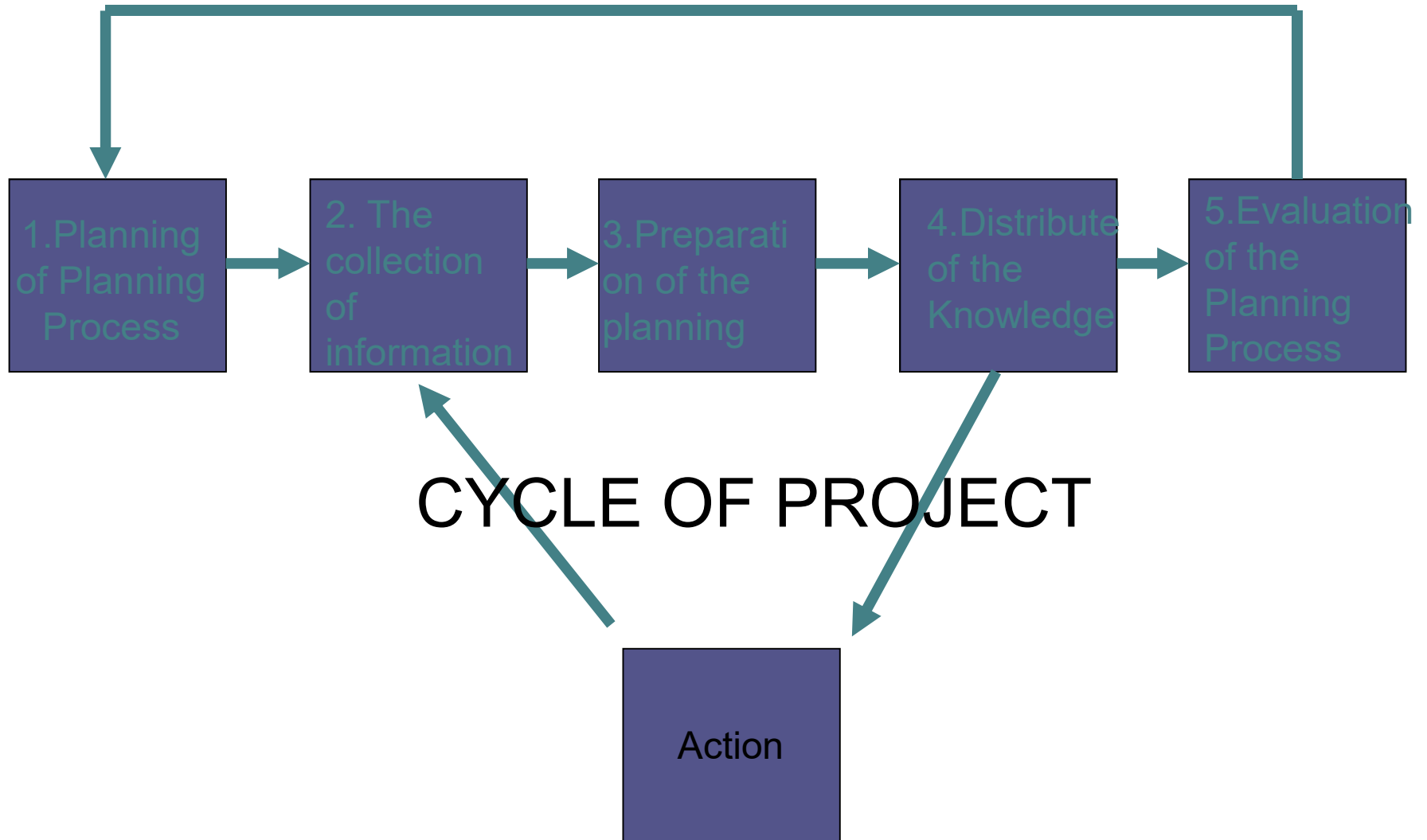
Planning

- Planning is a decision making, process performed in advance of action which endeavors to design a desired future and effective ways of bringing it about.
- Trying to anticipate what will happen and devising ways of achieving the set of objectives and targets.

Planning of Construction Project aim to answer 4 questions:

1. What will be done? (activity=items of work)
2. How will activities be done? (methods)
3. Who will do all activity and which means? (sources)
4. When will activities be done? (sequencing and scheduling)

Planning Process

```
┌──────────────┐    ┌──────────────┐    ┌──────────────┐    ┌──────────────┐    ┌──────────────┐
│ 1.Planning   │ →  │ 2. The       │ →  │ 3.Preparati  │ →  │ 4.Distribute │ →  │ 5.Evaluation │
│ of Planning  │    │ collection   │    │ on of the    │    │ of the       │    │ of the       │
│ Process      │    │ of           │    │ planning     │    │ Knowledge    │    │ Planning     │
│              │    │ information  │    │              │    │              │    │ Process      │
└──────────────┘    └──────────────┘    └──────────────┘    └──────────────┘    └──────────────┘
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CYCLE OF PROJECT

Action

Outcomes of Planning Processes

- Timely completion of the project
- The realization of the workflow without any delay
- Reduce the risk associated with necessity of rework
- Reduction of misunderstandings and complexities
- Everyone should have more knowledge about the project
- The provision of meaningful and timely reporting to management
- Information about the times to be programmed of the projects key pieces
- Having knowledge about the distribution of the cost of the project
- Defining their responsibilities and authorities

Schedule

- Schedule is a program which is prepared as the job is estimated to determine the duration of the project, so that the time can be controlled and managed by making required modifications throughout the project.

PROJECT PLANNING TECHNIQUES

- Bar Charts / Gantt Charts

- Network Analysis Systems
  - Critical Path Method (CPM)
    - Activity on Arrow
    - Activity on Node
  - Program Evaluation and Review Technique (PERT)

Bar Charts / Gantt Charts
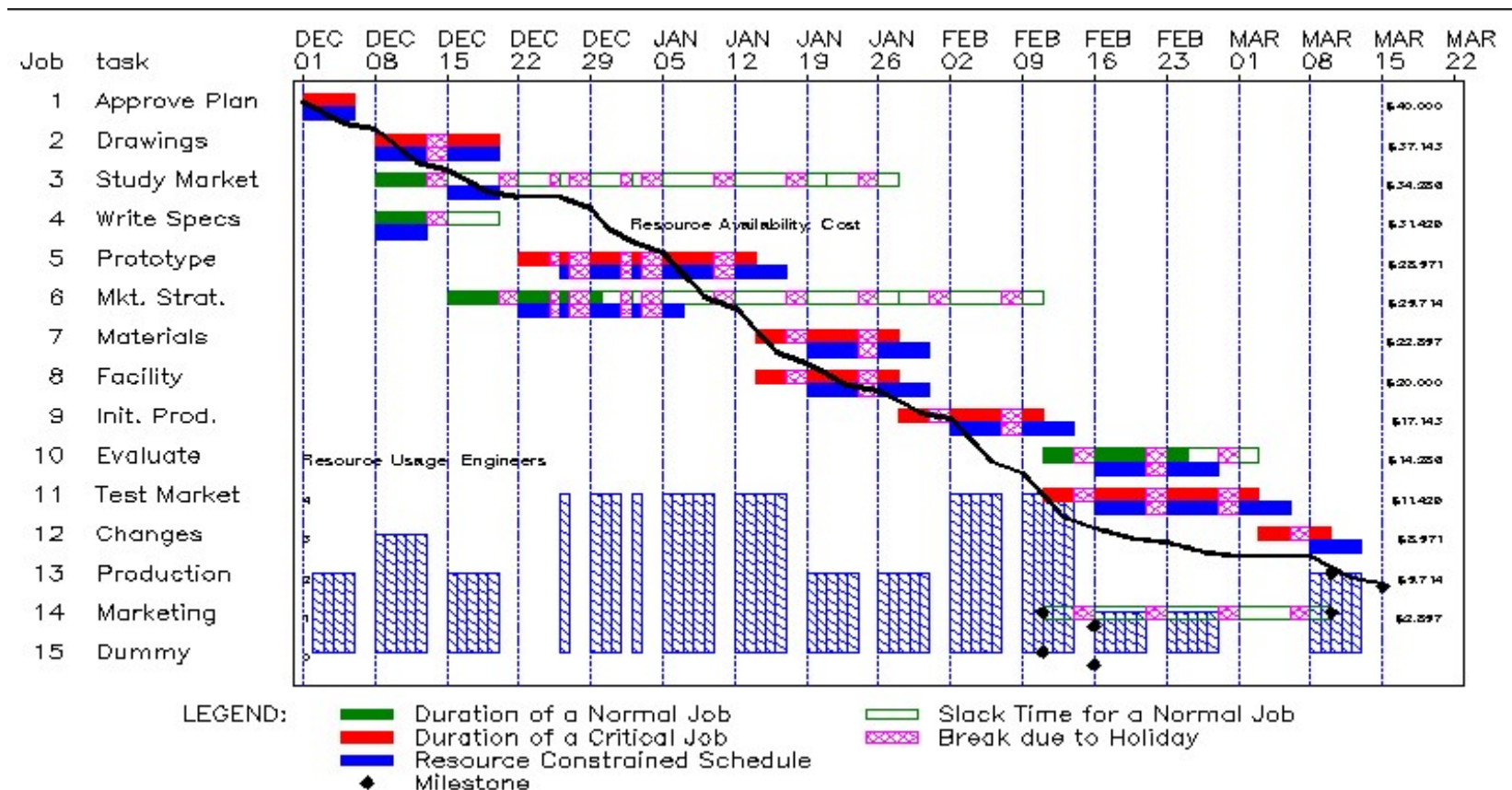
Bar Charts/ Gantt Charts

- A bar chart graphically describes a project consisting of well defined collection of tasks and activities, the completion of which marks its end.

- Bar charts are the easiest to understand and most widely used form of planning tools. Even when the network analysis is used, the initial work is presented usually in bar chart form.

Critical Path Method

- The CPM is the most commonly used network analysis technique, Since
  - The concept is simple.
  - The computations require basic arithmetic
  - A large number of computer programs are available to automate the work required of CPM scheduling (Such as **Primavera**, Ms. Project ...)

Critical Path Method

- Predictions about the activity duration

- Logical relations between the activities

Methods of constructing CPM diagrams

- Arrow Diagram (Activity on Arrow (A-o-A))

- Precedence Diagram (Activity on Node (A-o-N))

The elements of Arrow Diagram (Activity on arrow (AoA))

- Activity : the performance of a task required to complete the project (such as design of foundations, review of design, procurement of steel etc...)
- Event: a notation to show starting and ending points of an activity.
- Duration: the estimated time to perform an activity.
- Network: a diagram to represent the relationship of the activities to complete the project.

Arrow Diagram (Activity on arrow (AoA))
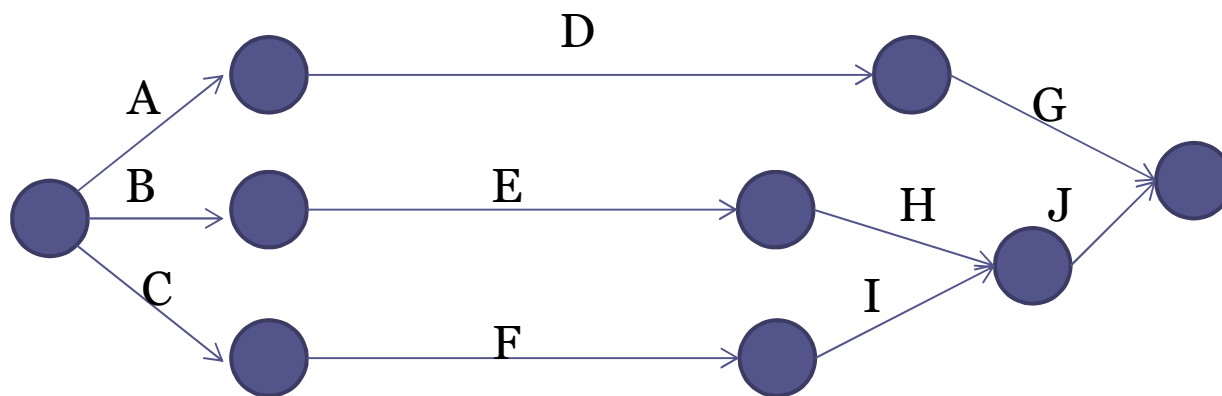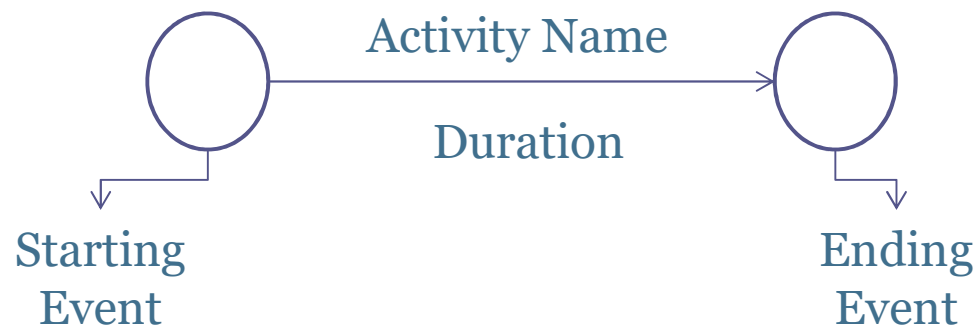
Arrow Diagram (Activity on arrow (AoA))
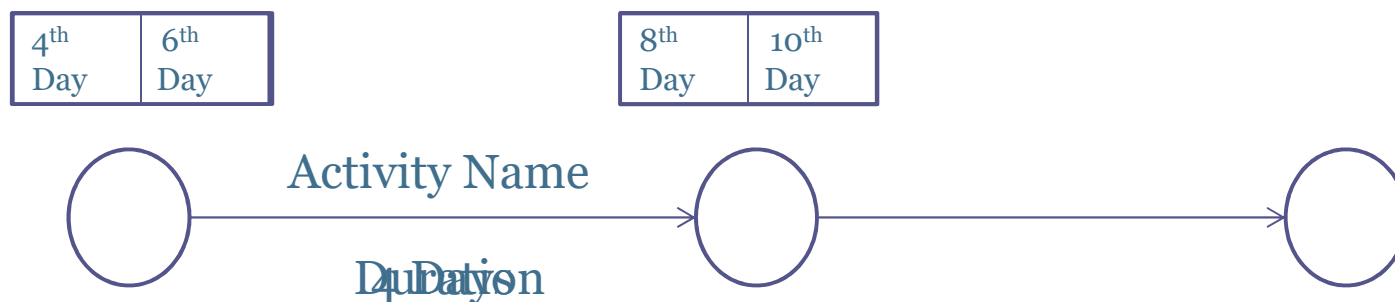
Early Start (ES): The earliest time an activity can be started.

Early Finish (EF): The earliest time an activity can be finished and it is equal to the early start plus duration. EF= ES + Duration

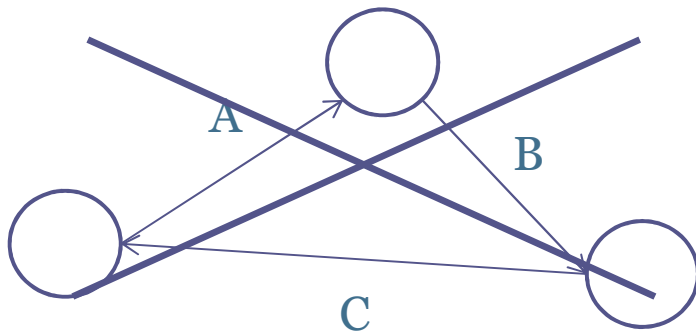Late Finish (LF): The latest time an activity can be finished.

Late Start (LS): The latest time an activity can be started without delaying the completion of the project.

Arrow Diagram (Activity on arrow (AoA))

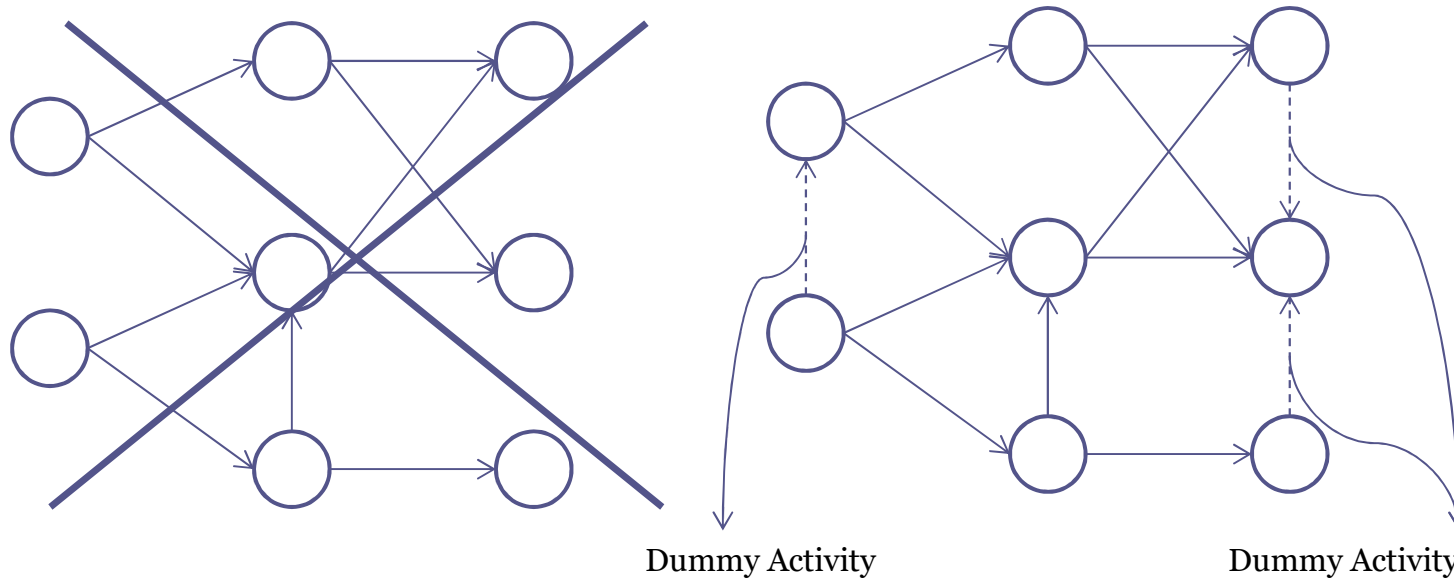- There should be one activity between two events.
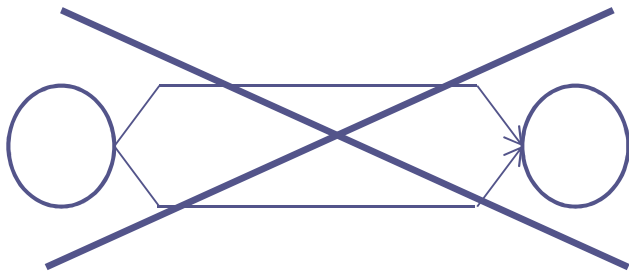
- The activities should not form a loop.

Arrow Diagram (Activity on arrow (AoA))

- There should be only one start event and one end event in a network.

Dummy Activity   Dummy Activity

Arrow Diagram (Activity on arrow (AoA))

- Dummy Activity: an activity which indicates that any activity following dummy cannot be started until the activity or activities preceding the dummy are completed. The dummy does not require any time and resource.
- There should be one activity between two events.
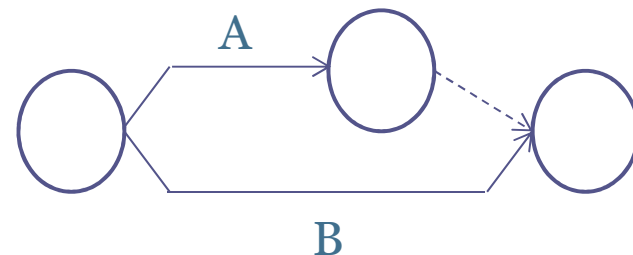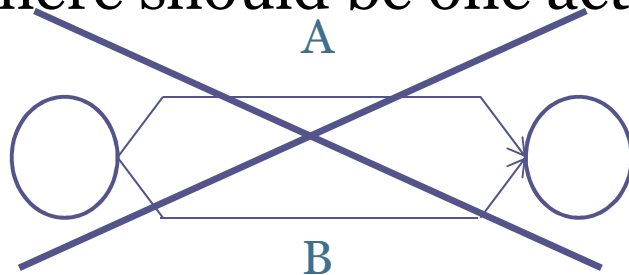
Identity dummy: used to identify activities when two or more parallel independent activities have the same start and finish events.

Arrow Diagram (Activity on arrow (AoA))

- D activity is started after completion of B activity. In addition, C activity is started as A and B activities are completed.
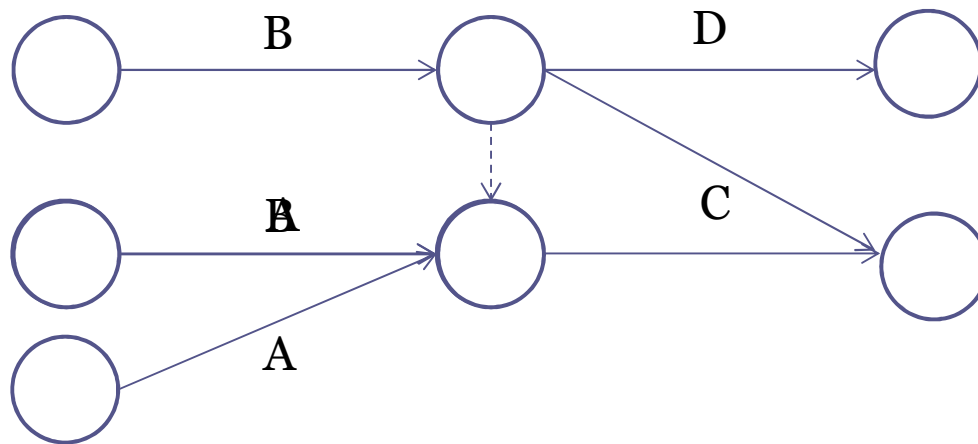
▫ Logic dummy: used to prevent the error in logic arising from the chains of wholly independent activities having a common event.

Arrow Diagram (Activity on arrow (AoA))

Explanations	A-o-A

- C depends on A and B/ A and

  B starts at the same time

- C depends on A and B/D

  depends on A and B
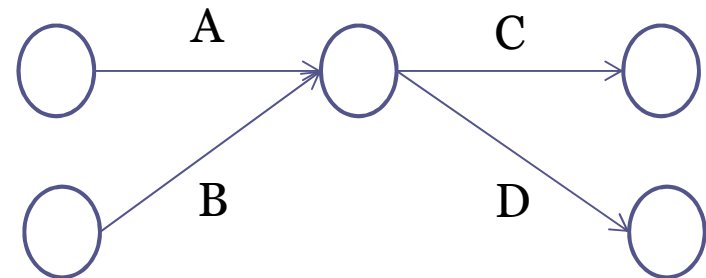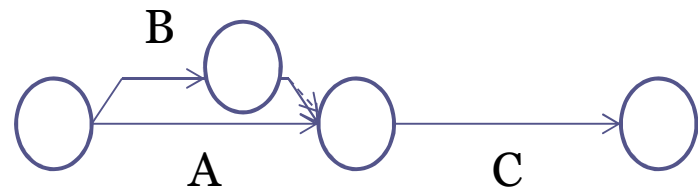
Arrow Diagram (Activity on arrow (AoA))

Explanations	A-o-A
• K depends on A and B/ L depends on B and C.	
• K depends on A / L depends on A and B/ M depends on A, B and C.	

Arrow Diagram (Activity on arrow (AoA))
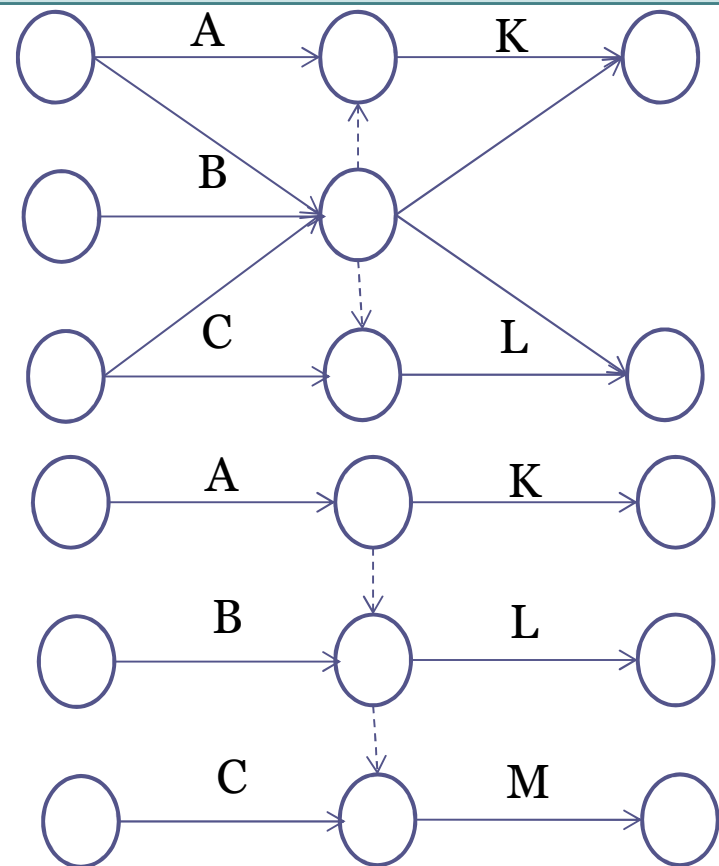
Explanations	A-o-A

- K depends on A / L depends

  on A and B/ M depends on B

  and C.
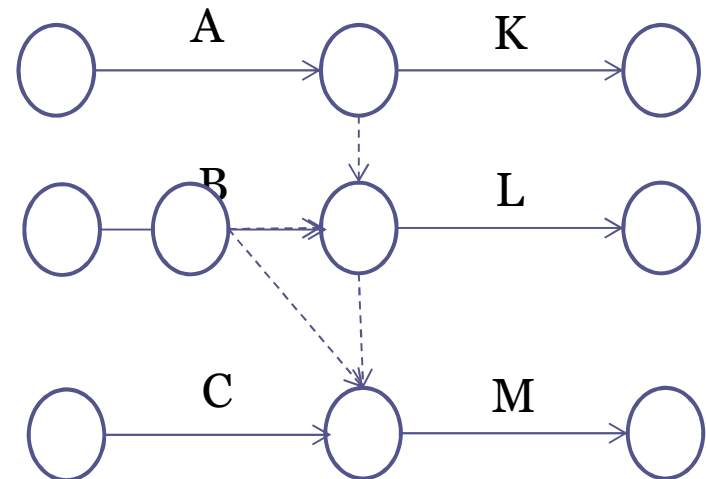
Exercise: Laddering (Merdiven tipi)

A	B	C
10 km	10 km	10 km
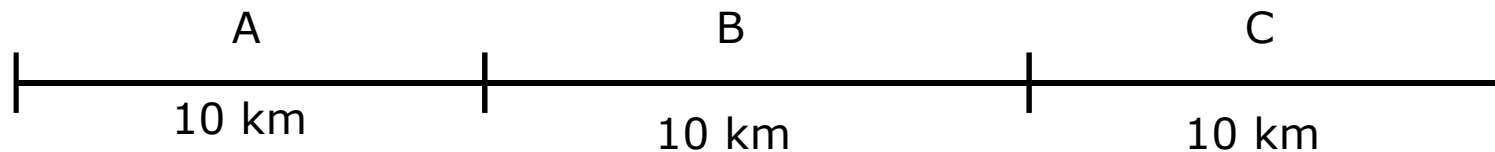
- Laying pipeline whose length is 30 km.
- **ASSUMPTION : There are three activities required to perform the job, namely excavation, laying pipes, and refilling. There is one team for each activity.**
  (For example, One excavator and one driver for excavation)

Abbreviation:
- EXCAVATION: EXC
- LAYING PIPES: PIP
- FILLING: FIL

Exercise: Laddering
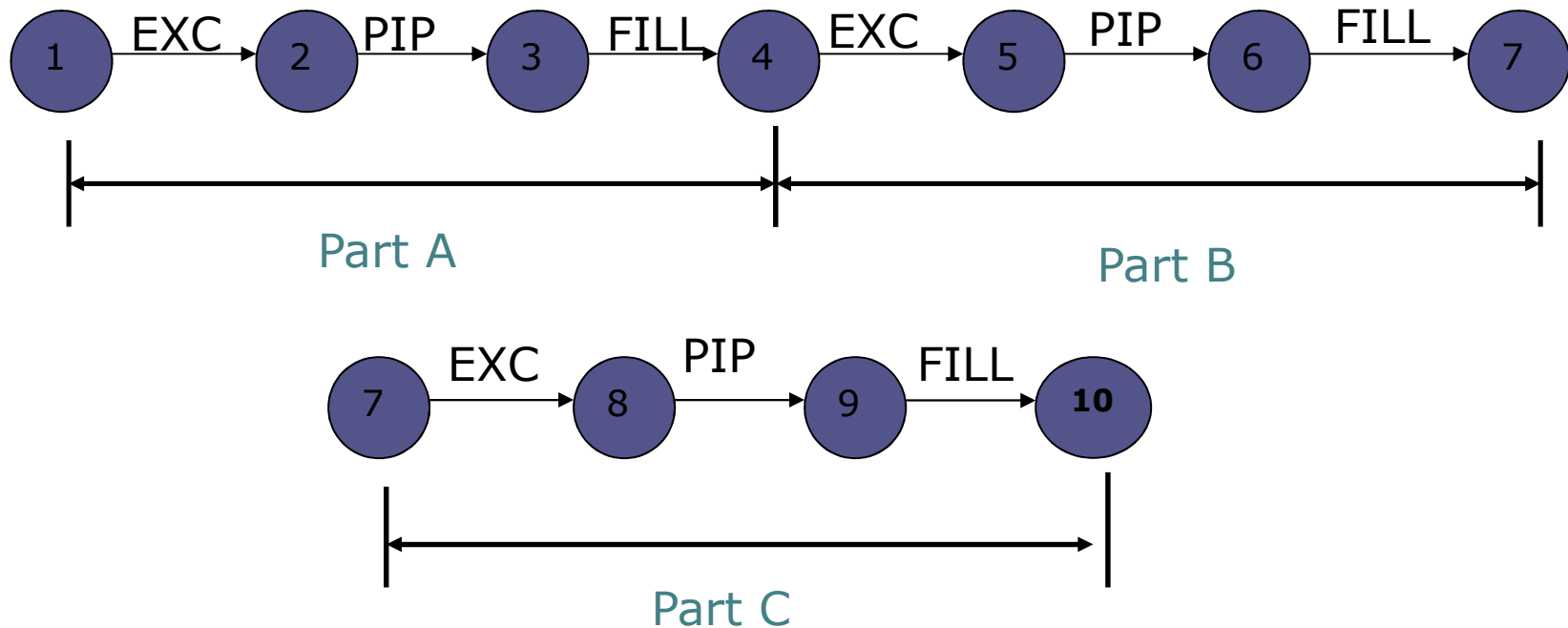
*** We can finish the Project by following this sequence, however this is not an effective method, since two teams have to wait for the completion of the activity of one team. ***

Exercise: Laddering

Part A — 1 →EXC→ 2 →LAY→ 3 →FILL→ 10

Part B — 4 →EXC→ 5 →LAY→ 6 →FILL→ 10

Part C — 7 →EXC→ 8 →LAY→ 9 →FILL→ 10
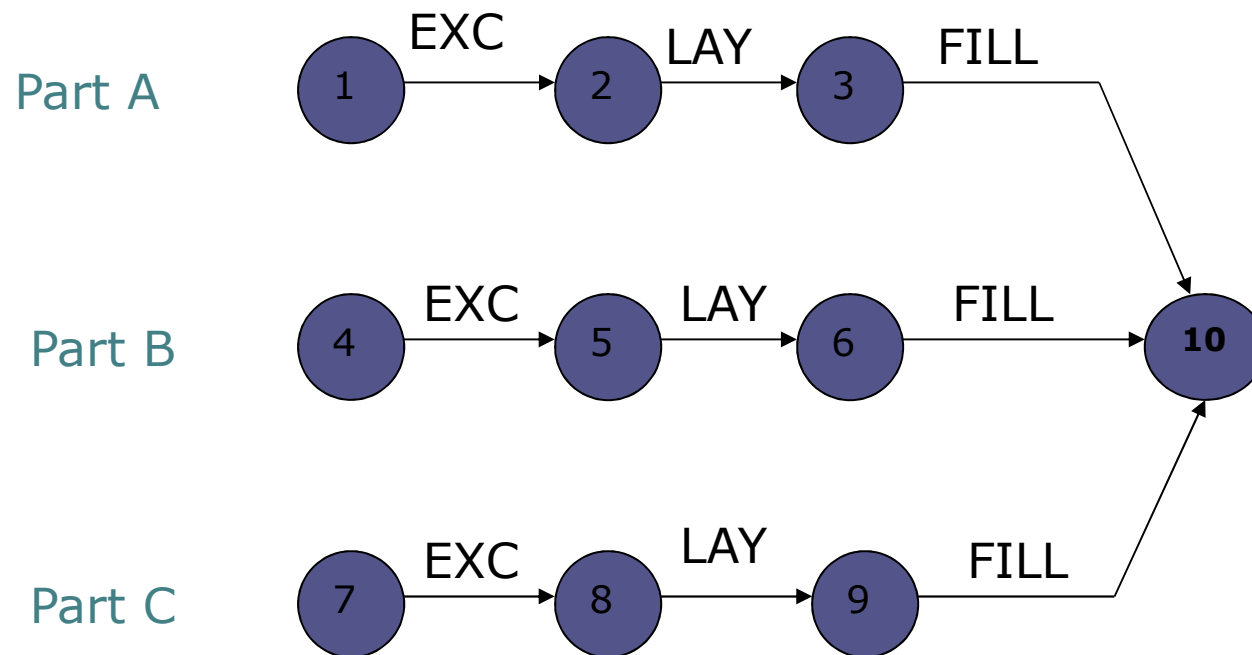
*** This procedure is very effective, however We do not have three teams performing for each activity. ***

Exercise: Laddering
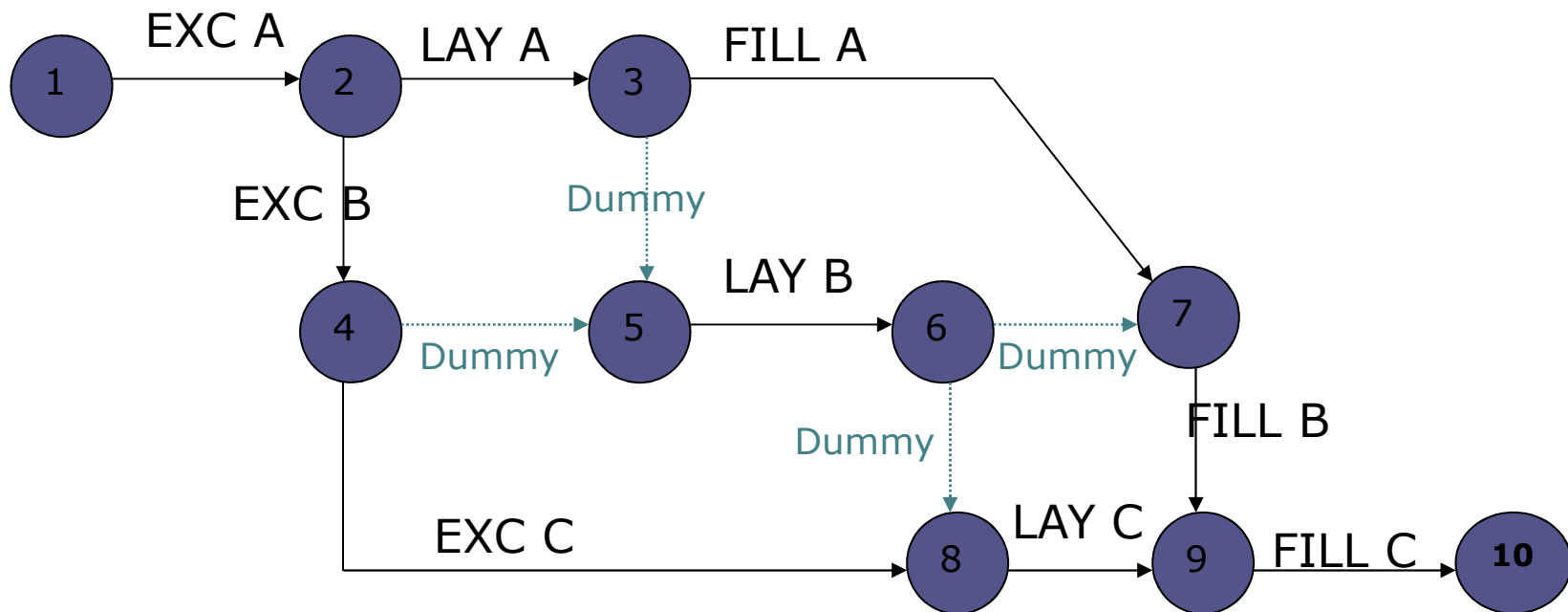
*** With using Dummy activities, the Project can be completed by using resources effectively in the   minimum time duration. ***
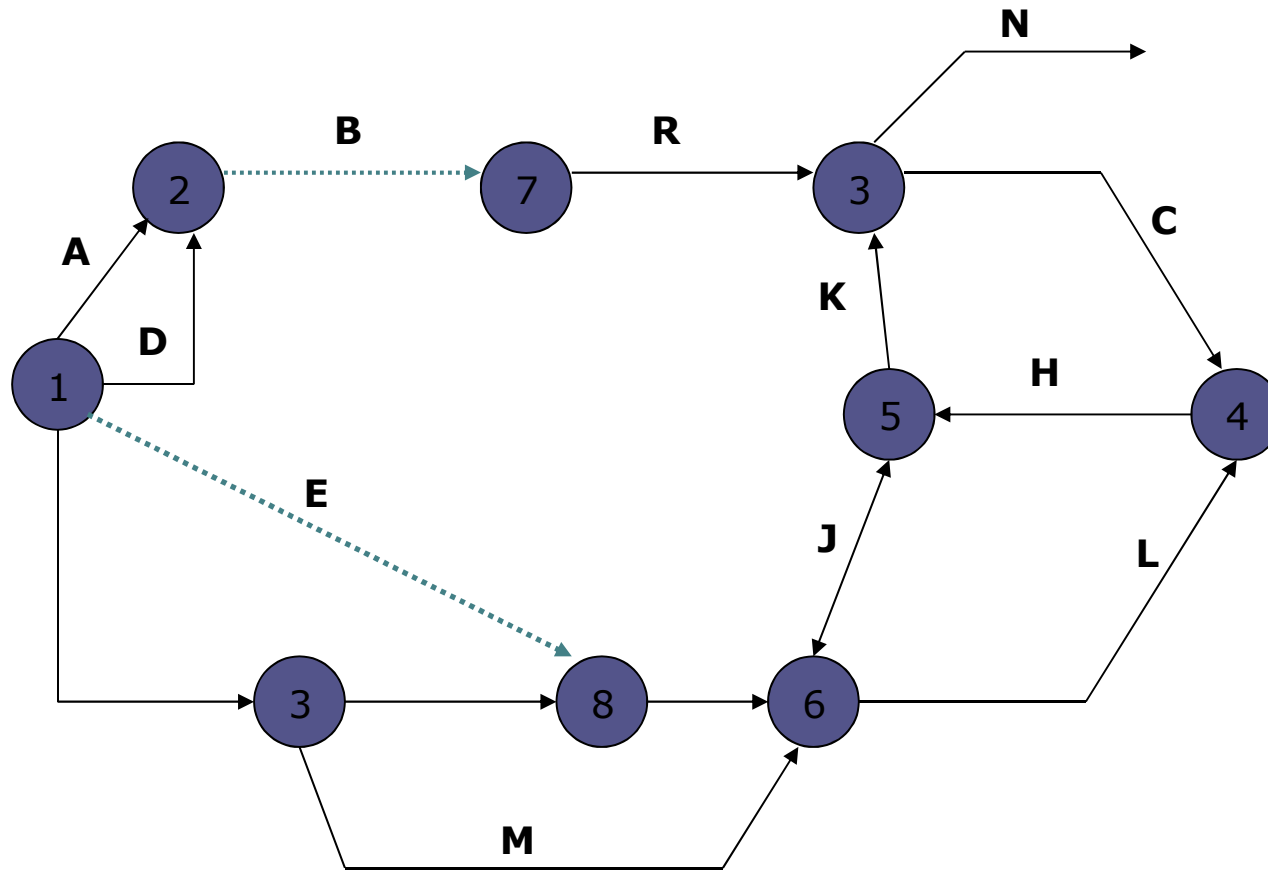
Exercise: Find the errors made in preparation of the network diagram.

Exercise: Find the errors made in preparation of the network diagram.

The starting node is bigger than ending node.

This activity has no ending node.

B

R

N

It should not be dummy activity.

There are two arrows between two nodes.

A

D

K

C

There is a loop.

H

E **It should not be dummy activity.**

J

L

Two headed arrow

M

• **The order of numeration is not correct.**

• **These activities are not assigned.**

• **Two nodes are numerated with 3.**
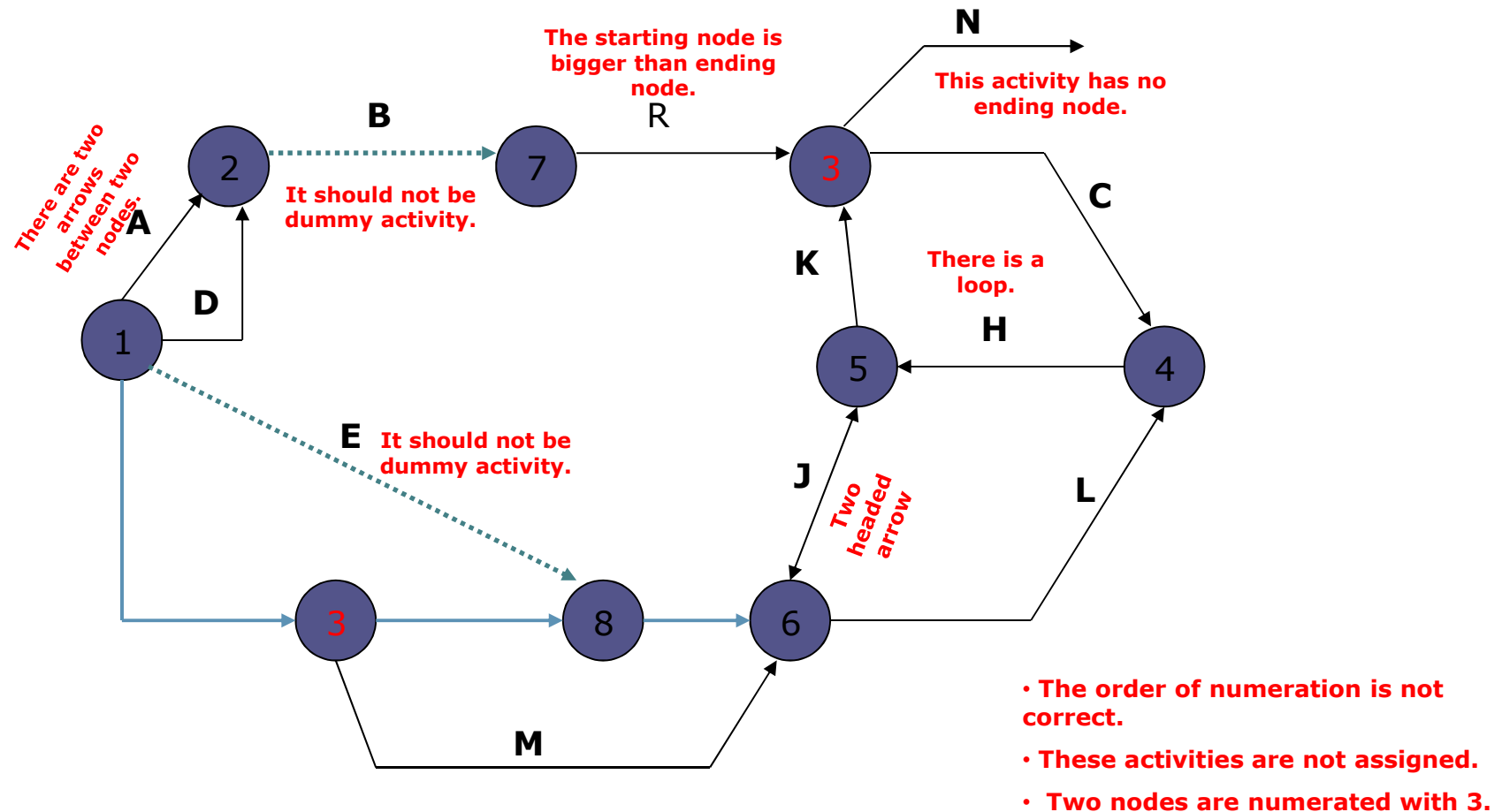
Exercise(Network)

- According to the table, draw the network diagram

ACTİVİTY	START EVENT	END EVENT
A	1	2
B	1	3
C	1	5
D	2	4
E	2	6
F	3	4
Dummy	3	5
G	3	6
H	4	6
I	5	6

Exercise: Ağ (Network)
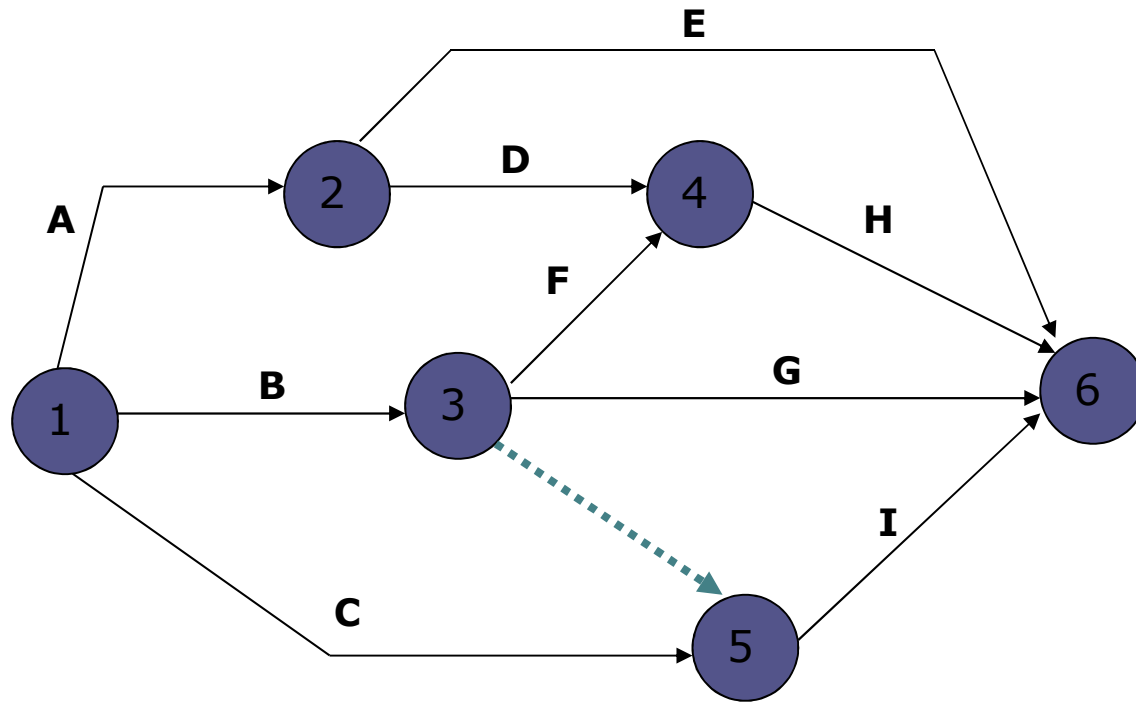
Exercise: Network

According to the data described below, draw the network diagram.

- The starting activity is A.
- C and D activities depend on Activity A.
- E and F activities depend on Activity B.
- Activity G depends on Activity C and E.
- The end activities of the project are D, F and G.

Exercise: Ağ (Network)
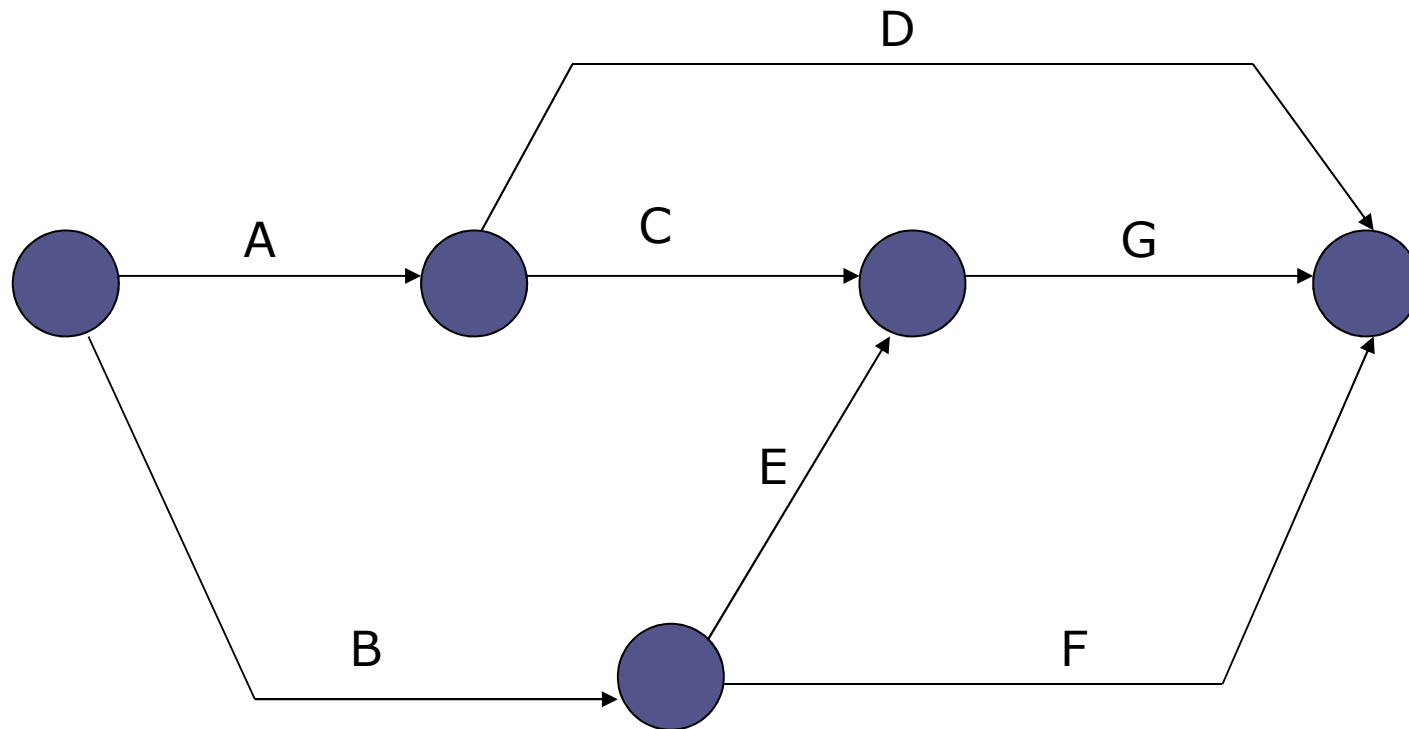
CPM Calculation

- The aim is to find the critical path, total time and which activities can be delayed.
- Critical path: A series of interconnected activities through the network diagram, which cannot be delayed without increasing the duration of the project. The critical path determines the minimum time to complete the project.
- Some of the activities in the network are not placed on the critical path, therefore these activities can be delayed without increasing the duration of the project.

CPM Calculation

- There exist two stages in CPM calculations.
  - Forward pass: Early start and early finish of each activity are calculated. The calculations start from the first event to the last event.
  - Backward pass: Late start and late finish of each activity are calculated. Forward pass calculations should be performed before perform backward pass. This pass starts from the last event to first event.

Arrow Diagram (Activity on arrow (AoA))
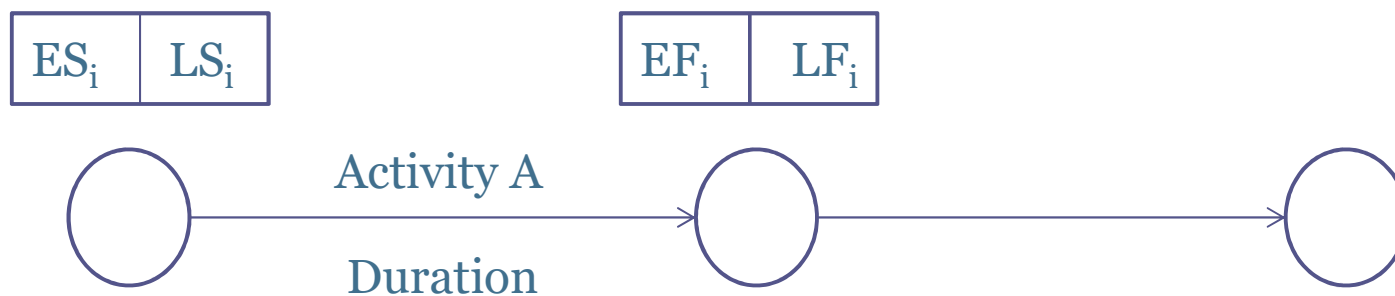
ES$_i$	LS$_i$

EF$_i$	LF$_i$

Activity A

Duration

Early Start (ES): The earliest time an activity can be started.

Early Finish (EF): The earliest time an activity can be finished and it is equal to the early start plus duration.

Late Finish (LF): The latest time an activity can be finished.

Late Start (LS): The latest time an activity can be started without delaying the completion of the project.

Forward pass

- Is used to find the least time required to complete the project and the earliest time an activity can be started according to the schedule that being developed.
- Made through the network, adding duration times to early start times of the activities.
- If 2 or more activities terminate at a junction or node, pick the larger sum value as the early start time for the activities following.

Arrow Diagram (Activity on arrow (AoA))
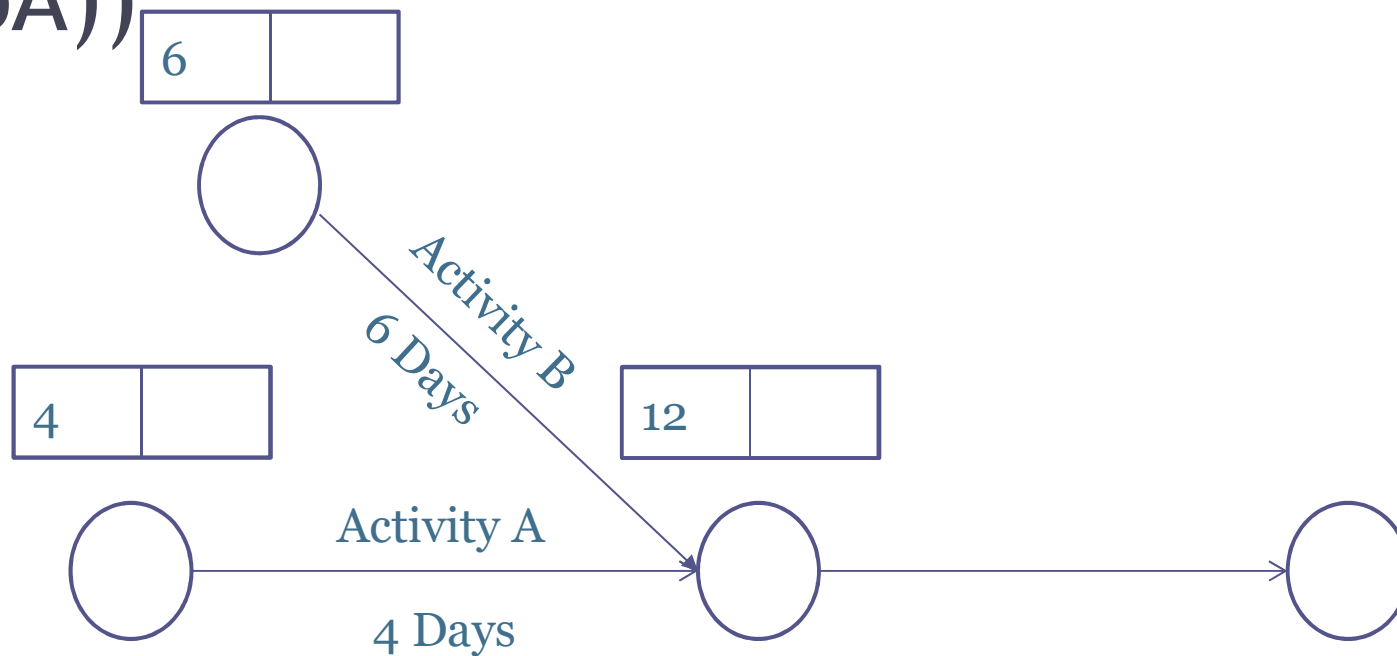
Backward pass

- Calculated to find the value of latest time an activity may start or late finish time without affecting overall project schedule.
- Subtract duration times.
- If 2 or more activities terminate at a junction or node, pick the smaller subtraction value as the late start time for the activities following.

Arrow Diagram (Activity on arrow (AoA))
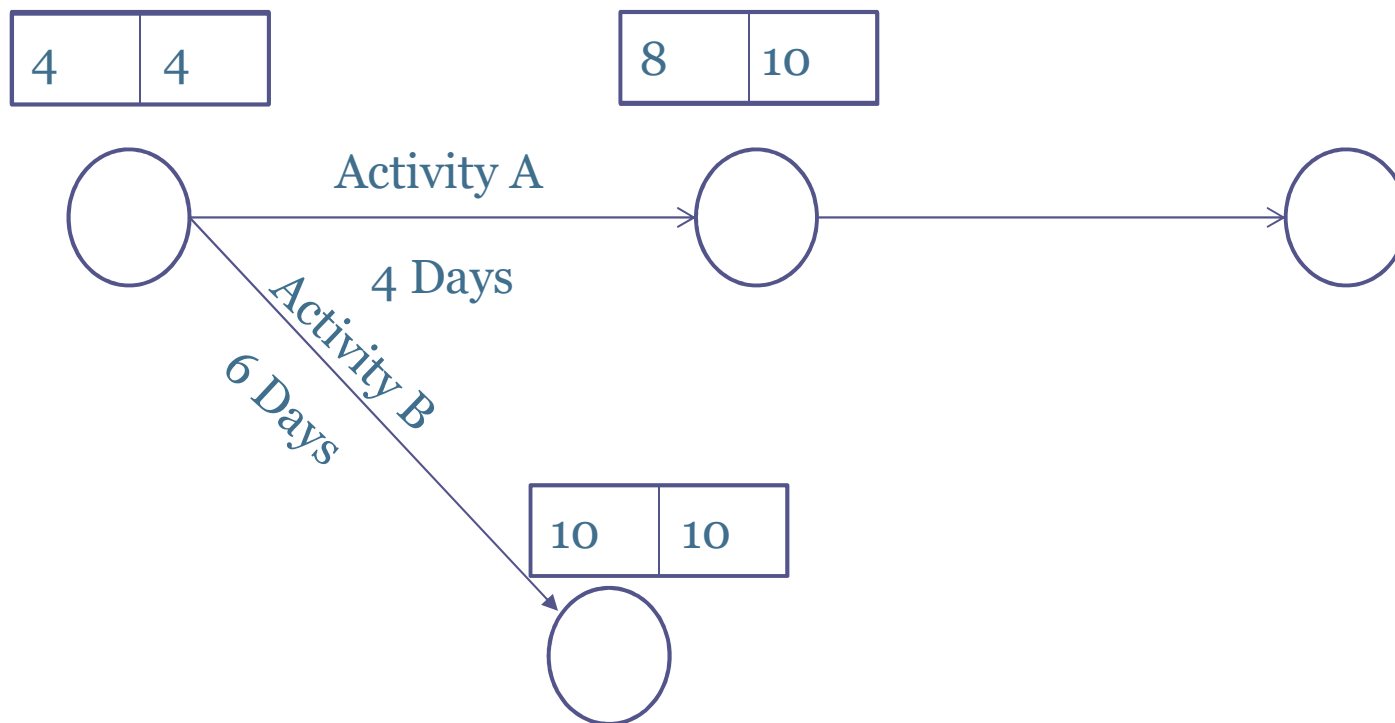
CPM Calculation

- The amount of time an activity may be delayed without delaying the completion date of project is called total float.
- Free float is the amount of time an activity may be delayed without delaying the early start time of the immediately following activity.

ES_i	LS_i

ES_j	LS_j

FF/TF
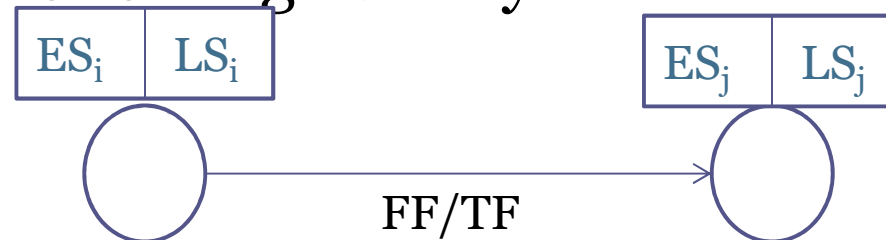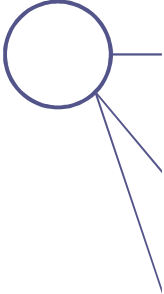
- $TF = LS_j - (ES_i + duration)$
- $FF = ES_j - (ES_i + duration)$

Exercise

- According to the table which shows a series of activities, their duration and relationships between them, draw A-o-A network.

Activity	Duration (Days)	Dependence
A	4	-
B	7	A
C	5	A
D	4	B
E	8	-
F	8	B,C,E
G	16	-

Exercise

D

Activity	Duration (Days)	Dependence
A	4	-
B	7	A
C	5	A
D	4	B
E	8	-
F	8	B,C,E
G	16	-

Exercise

TF= $LF_j - (ES_i + $ duration)
FF = $ES_j - (ES_i + $ duration)

Activity on Node (A-o-N)

- This method is preferred by the project managers because you do not have to deal with the dummy activities

- The elements of A-o-N

  ▫ Activity : is illustrated with the rectangles.

  ▫ Flow line

```
┌──────────┐              ┌──────────┐
│ Activity │─────────────▶│ Activity │
│    A     │     ╲        │    B     │
└──────────┘      ╲       └──────────┘
                   ╲
                    ▶ Flow Line
```
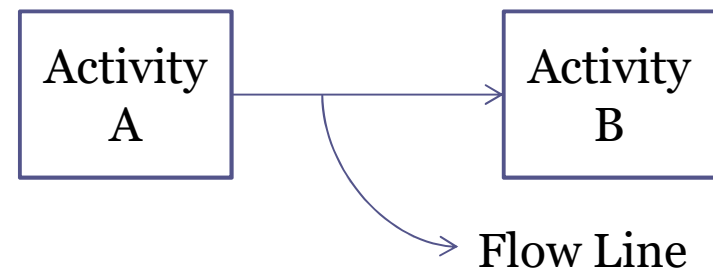
Activity on Node (A-o-N)
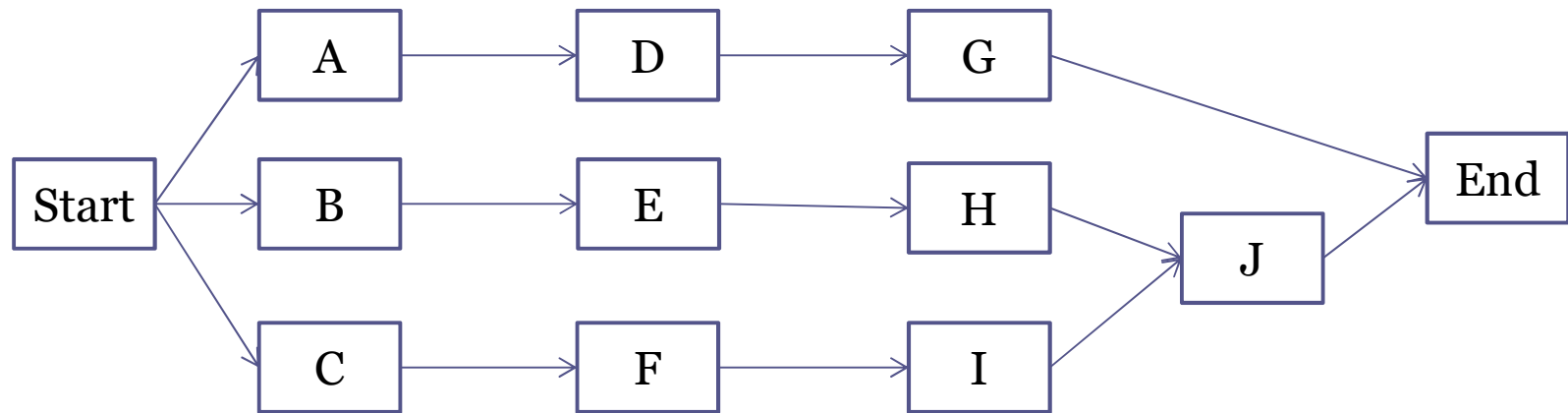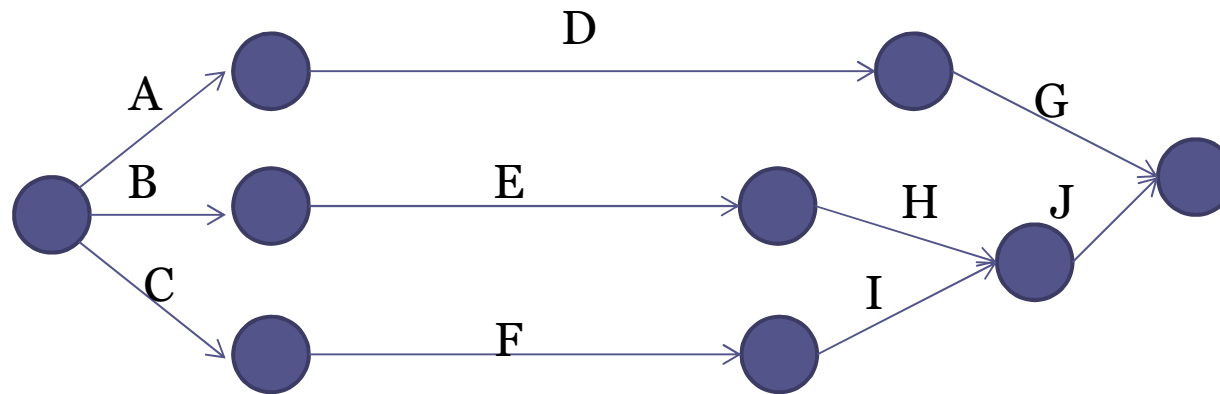
Network Diagram (Activity on Node (AoA))

Explanations	A-o-N

- B depends on A

```
[A] ———→ [B]
```

- C depends on A and B/ A and

  B starts at the same time

```
[A]
      ↘
        [C]
      ↗
[B]
```

- C depends on A/ D depends on

  A and B

```
[A] ———→ [C]
     ↘  ↗
[B] ———→ [D]
```
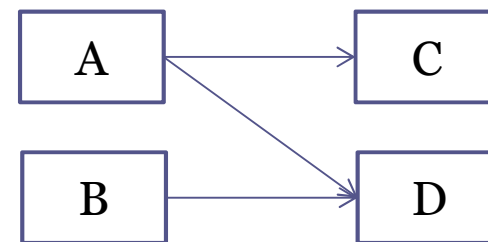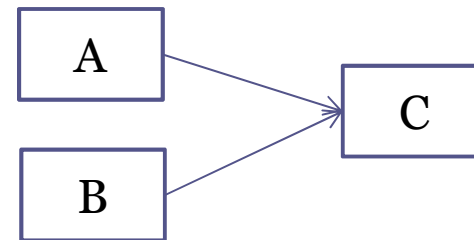
Network Diagram (Activity on Node (AoA))

Explanations	A-o-N

- C depends on A and B/D

  depends on A and B

- K depends on A / L depends

  on A and B/ M depends on A,
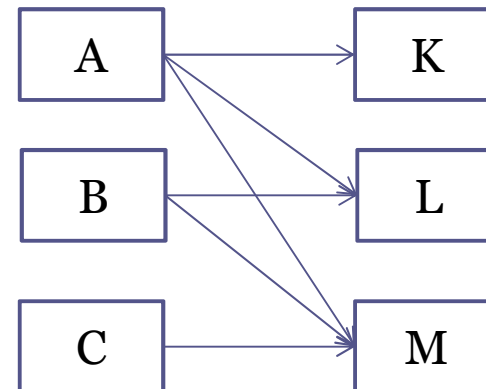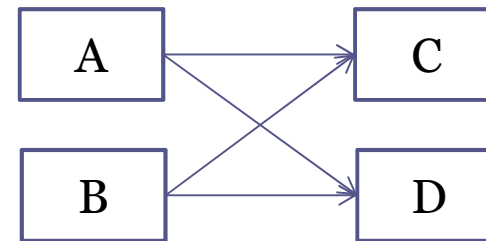
  B and C.

Network Diagram (Activity on Node (AoN))

Explanations	A-o-N

- K depends on A / L depends

  on A and B/ M depends in B

  and C.
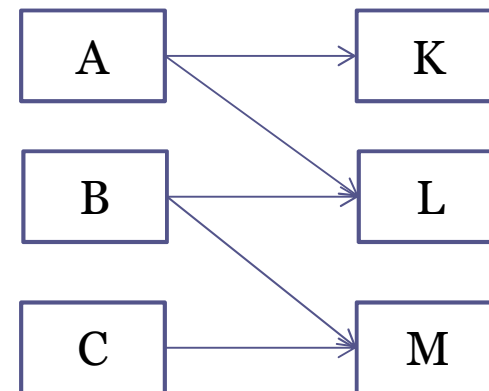
Network Diagram (Activity on arrow (AoA))

Explanations	A-o-A

- K depends on A / L depends on A and B/ M depends on B and C.
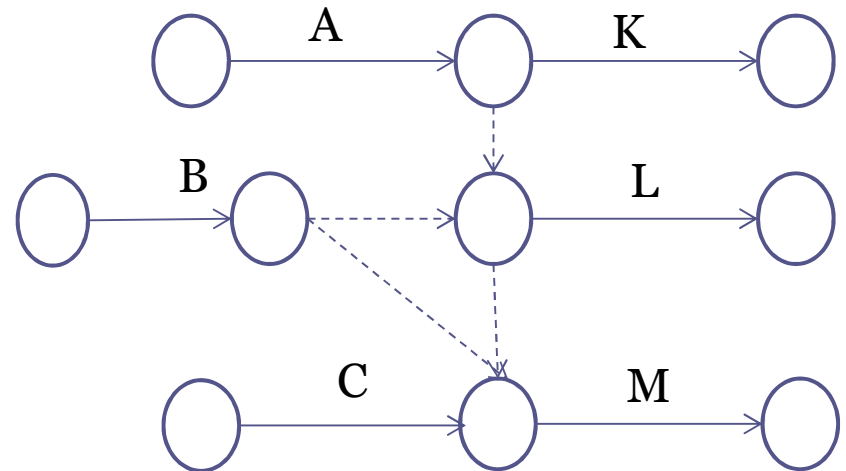
Network Diagram (Activity on Node (AoA))

- Notation of Activities

ES	TF	EF
	Activity Name	
LS	Durat.	LF

CPM Calculation

- There exist two stages in CPM calculations.
  - Forward pass: Early start and early finish of each activity are calculated. The calculations start from the first event to the last event.
  - Backward pass: Late start and late finish of each activity are calculated. Forward pass calculations should be performed before perform backward pass. This pass starts from the last event to first event.

Exercise

- According to the table which shows a series of activities, their duration and relationships between them, draw A-o-N network.

Activity	Duration (Days)	Dependence
A	4	-
B	7	A
C	5	A
D	4	B
E	8	-
F	8	B,C,E
G	16	-

Exercise

Activity	Duration (Days)	Dependence
A	4	-
B	7	A
C	5	A
D	4	B
E	8	-
F	8	B,C,E
G	16	-

Start

End

Start: 0 | 0 | 0

A: 0 | 4 | 4
B: 4 | 11 | 7
C: 4 | 9 | 5
D: 11 | 15 | 4
E: 0 | 8 | 8
F: 11 | 19 | 8
G: 0 | 16 | 16
End: 19 | 19 | 0
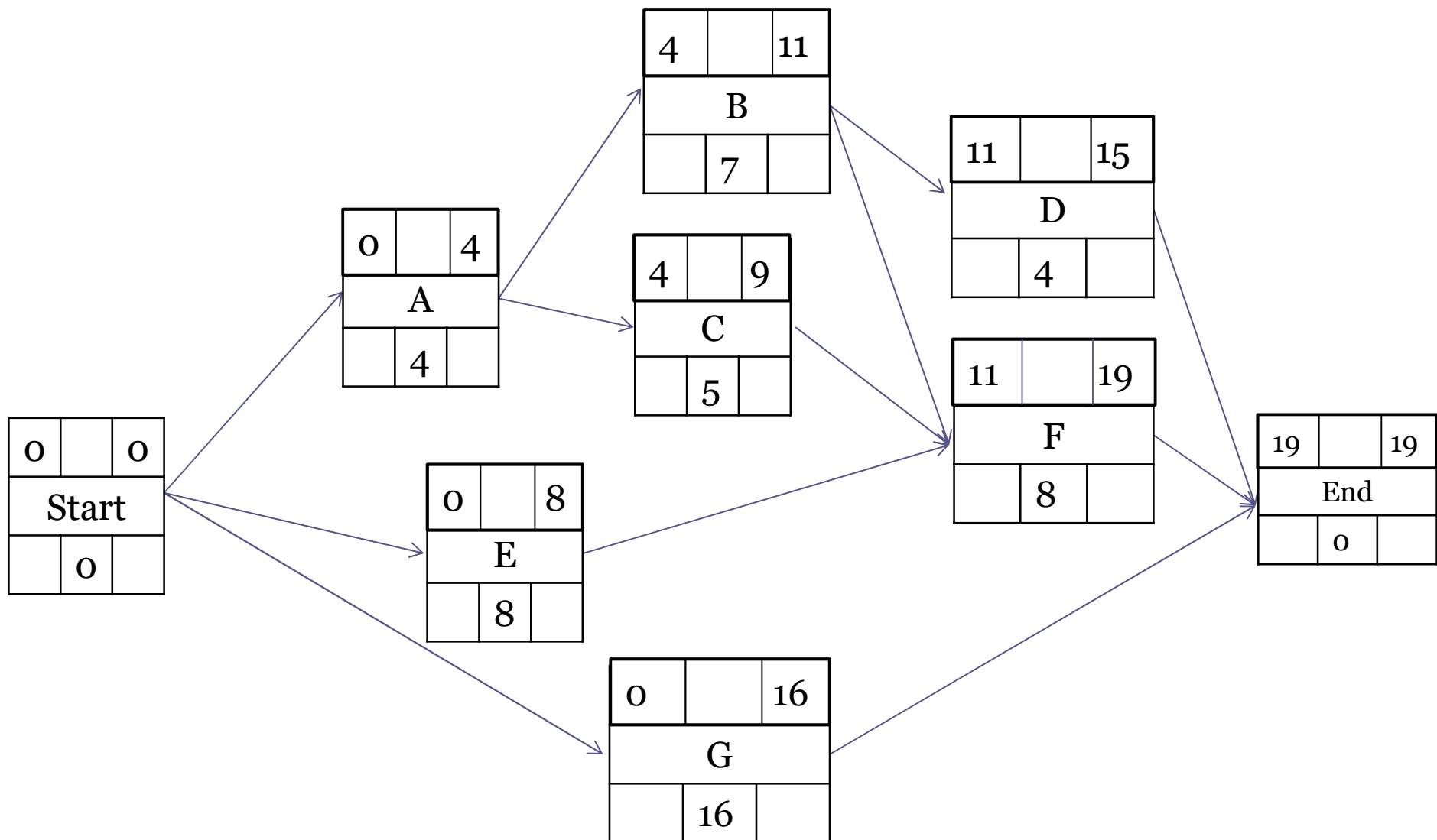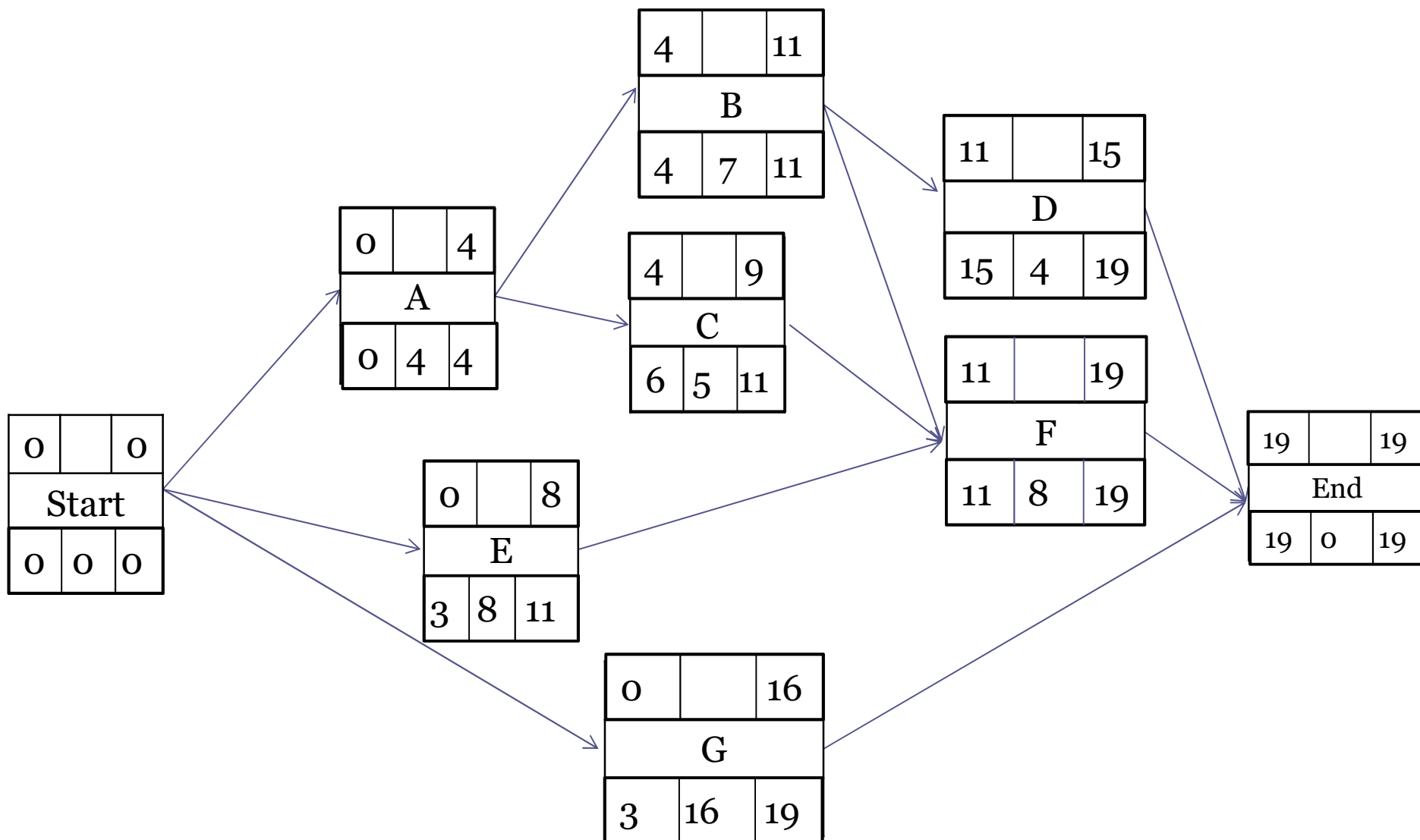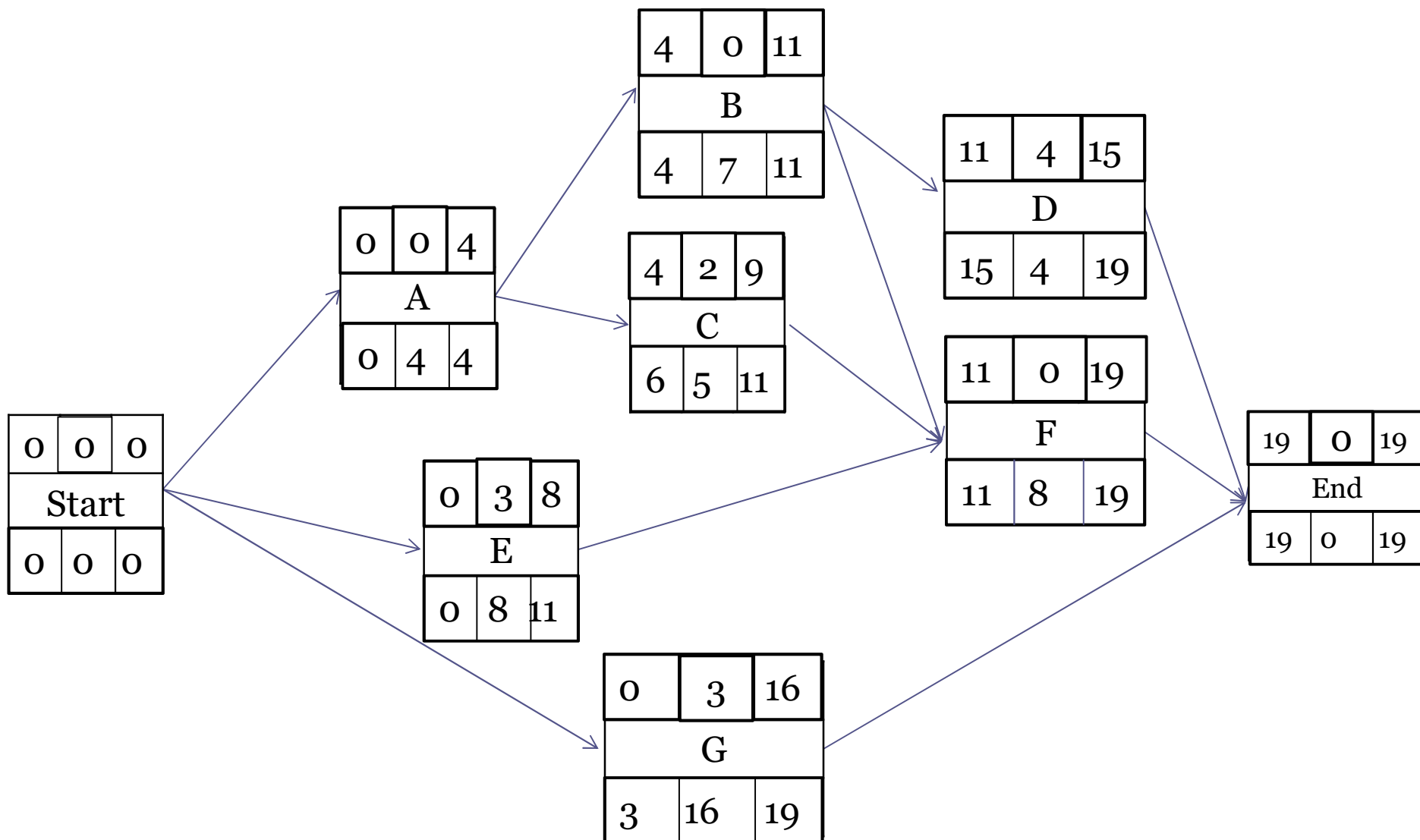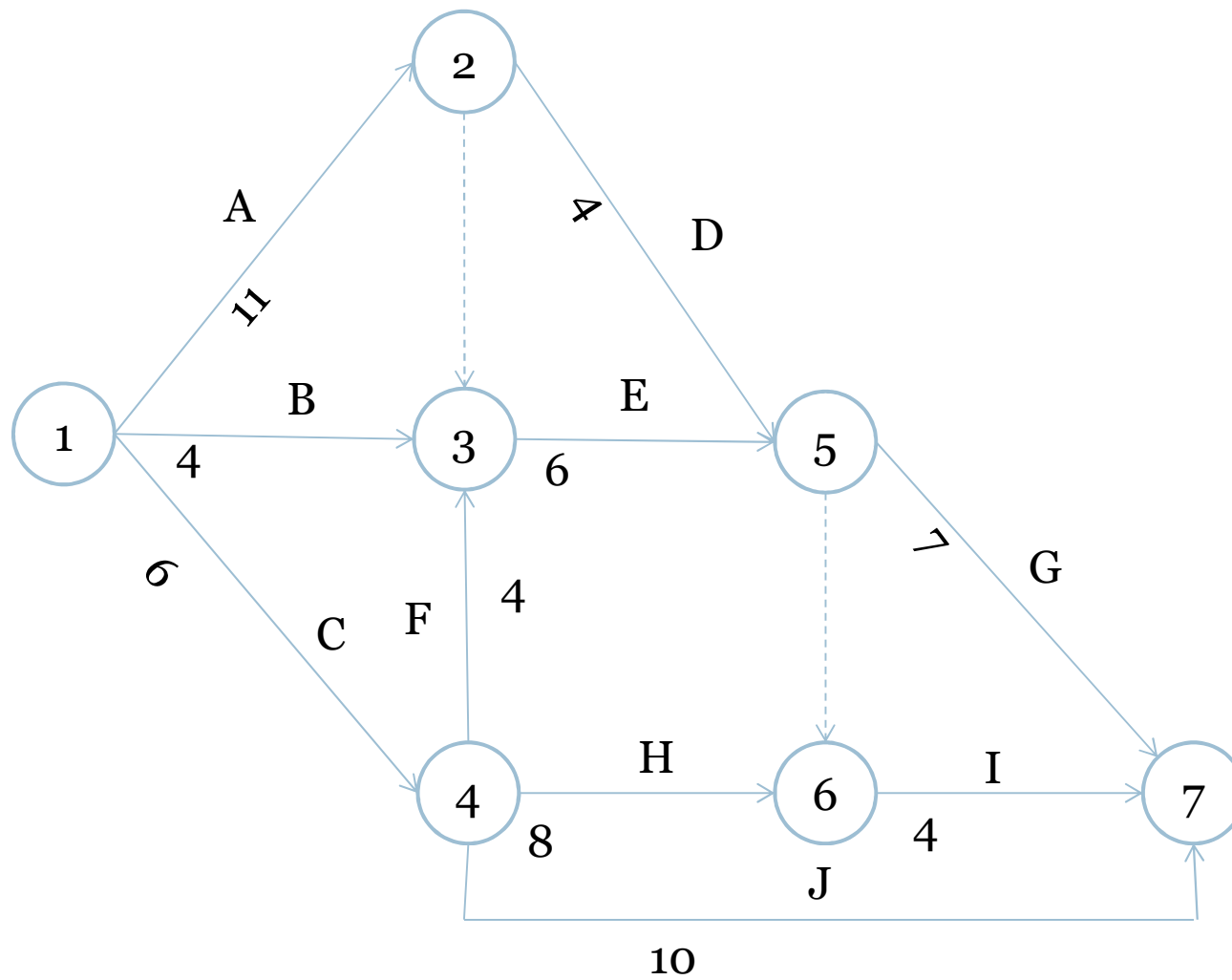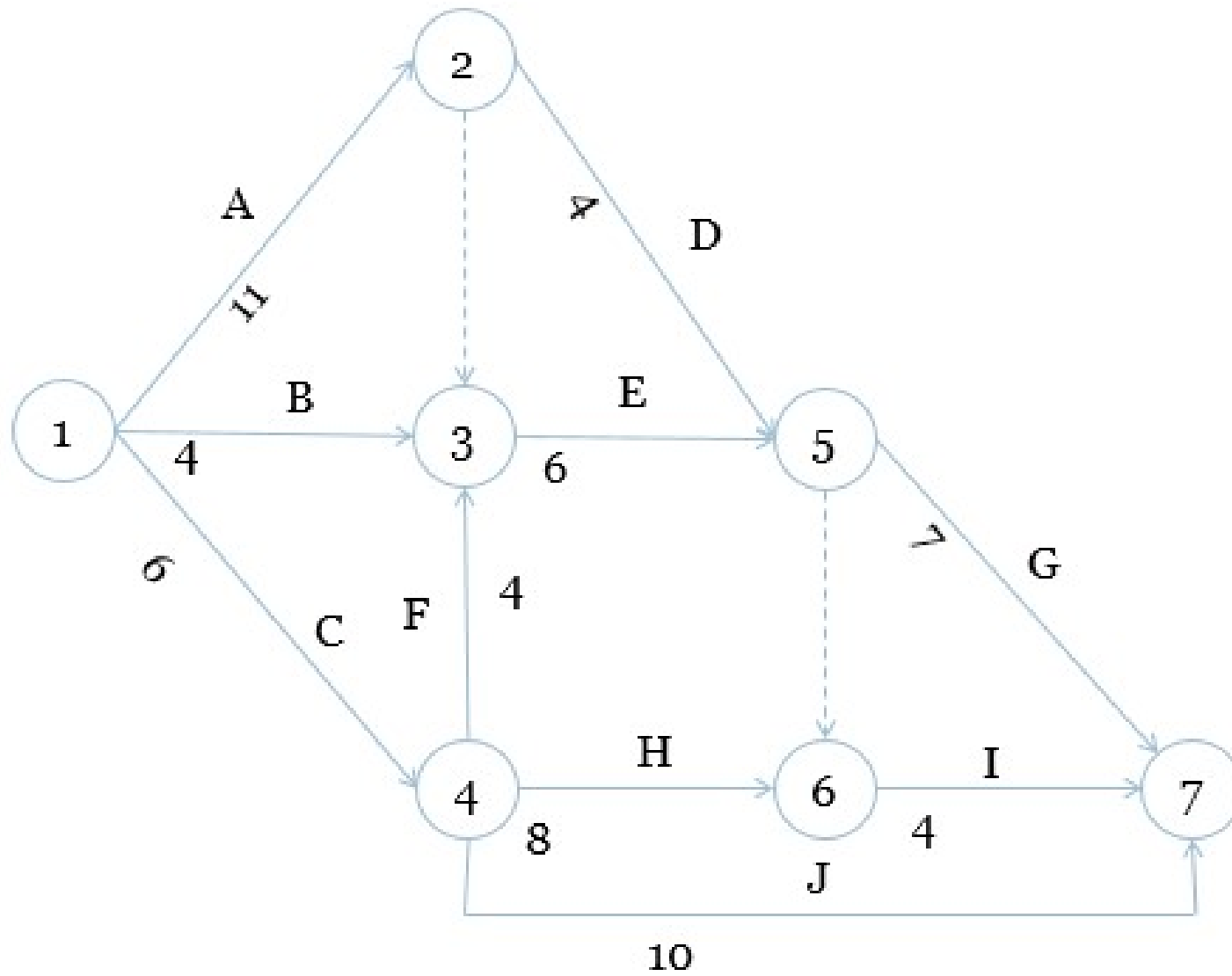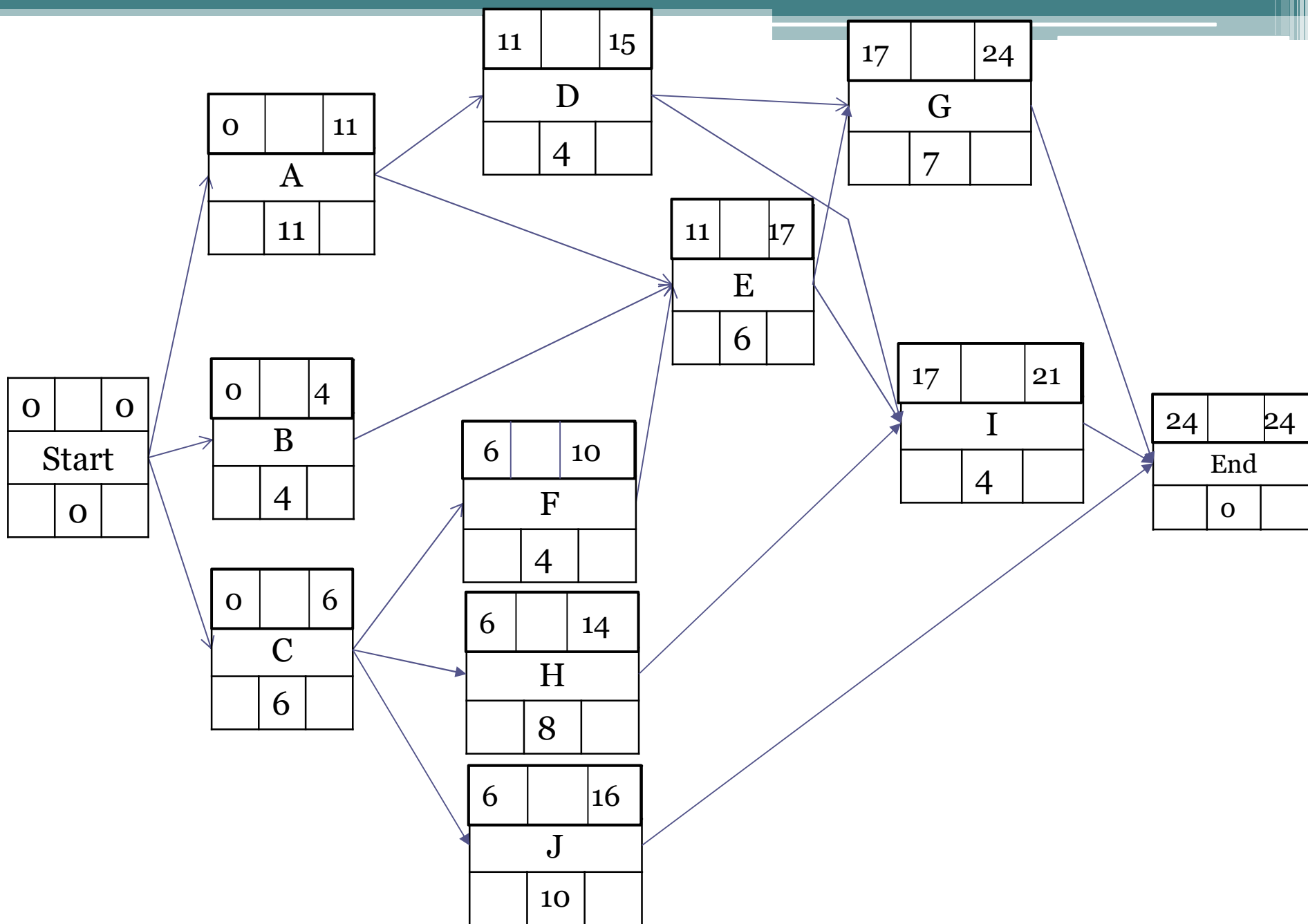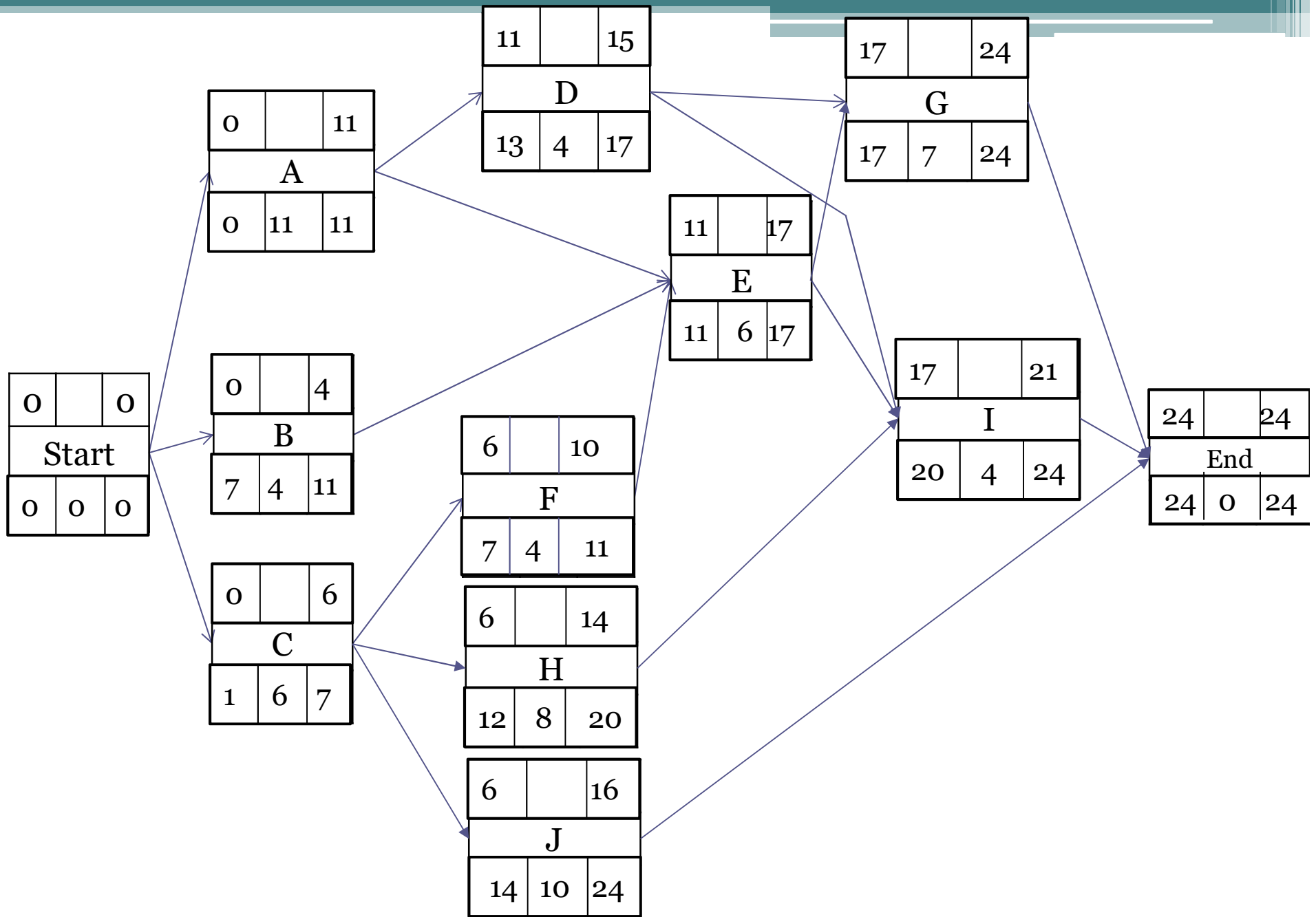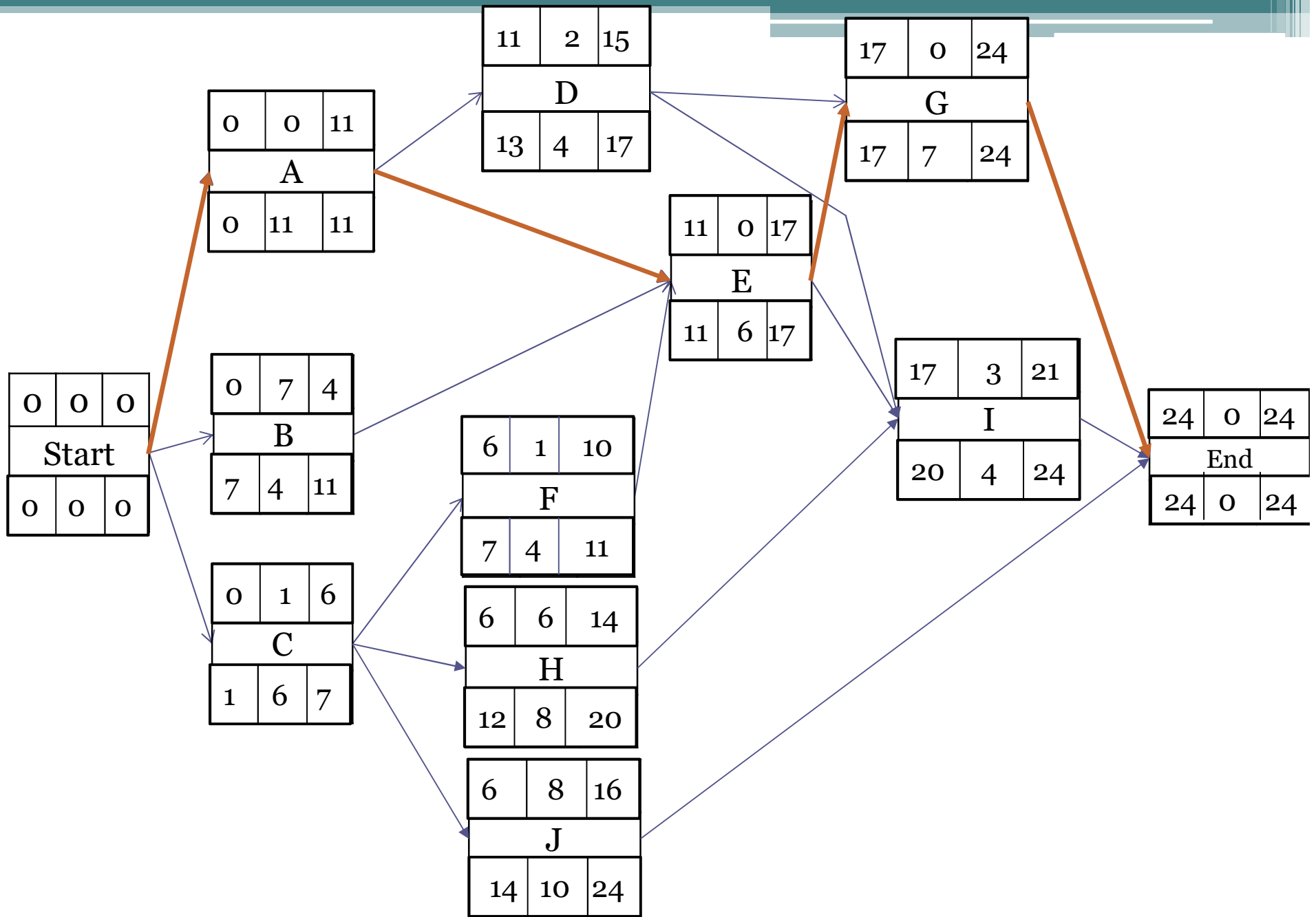
Exercise

Exercise
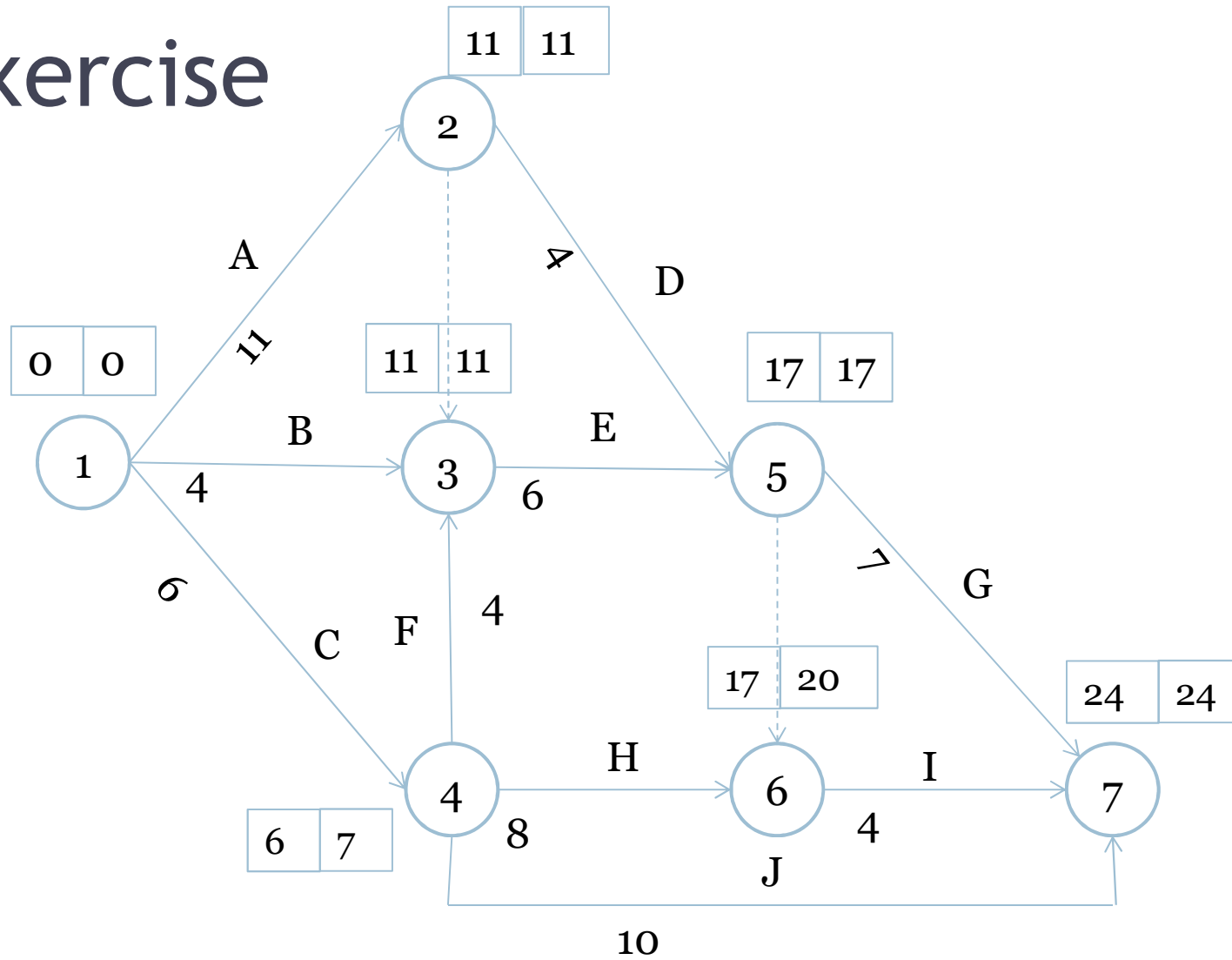
ÖRNEK

- According to the data about the activities and their durations given in following table, a network diagram is drawn. According to this table, find the critical path by performing CPM calculations.

Activities	Duration (month)
1-2	5
1-3	7
1-4	9
2-4	6
2-6	12
3-5	6
4-5	5
4-6	8
4-7	9
5-8	13
6-7	4
7-8	6

$TF = LF_x - (ES_x + duration)$

$FF = EF_x - (ES_x + duration)$

Critical Path; *The path which have TF = 0 = FF. There can be more than one critical path*

Activities	Months
1-2	5
1-3	7
1-4	9
2-4	6
2-6	12
3-5	6
4-5	5
4-6	8
4-7	9
5-8	13
6-7	4
7-8	6

TF=2 FF=2

TF=0 FF=0

TF=0 FF=0

TF=0 FF=0

TF=2 FF=2

TF=0 FF=0

TF=3 FF=3

TF=0 FF=0

TF=3 FF=0

TF=0 FF=0

TF=3 FF=3

CP1= (1-2), (2-4), (4-6), (6-7), (7-8)

CP2= (1-2), (2-4), (4-5), (5-8)
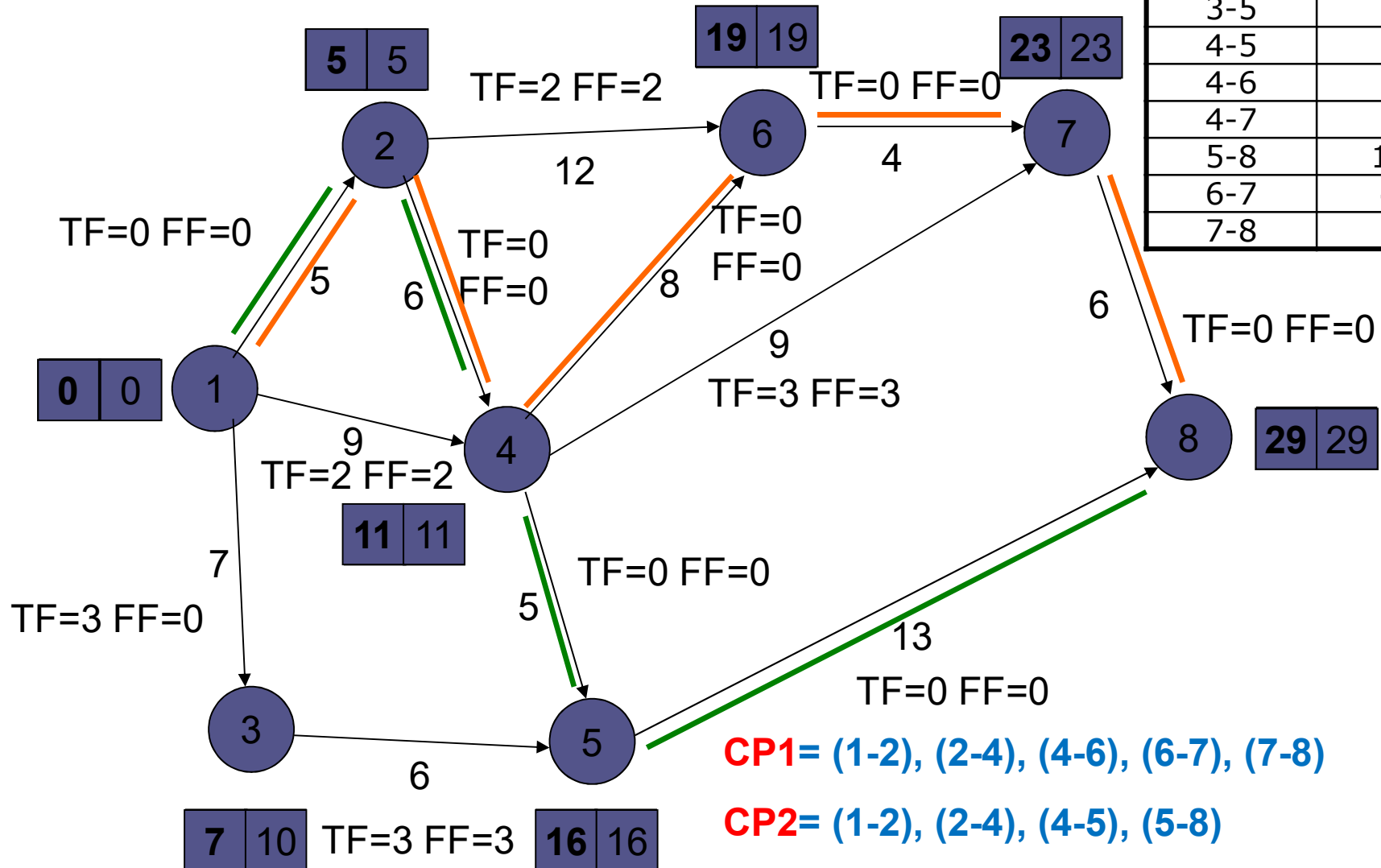
Crashing

- If the schedule developed does not allow the project to complete when desired, the actions should be taken to decrease the total project duration. This is known as crashing the schedule.
  - Relieve employees of other responsibilities.
  - Reallocate resource obtained from noncritical activities.
  - Assigning more resources to the project activities.
  - Allowing additional overtime.
- Crashing only works, when adding more resources will lead to a faster completion of a project.

Network Crashing

Utility Curve: shows the cost increase with crashing the schedule
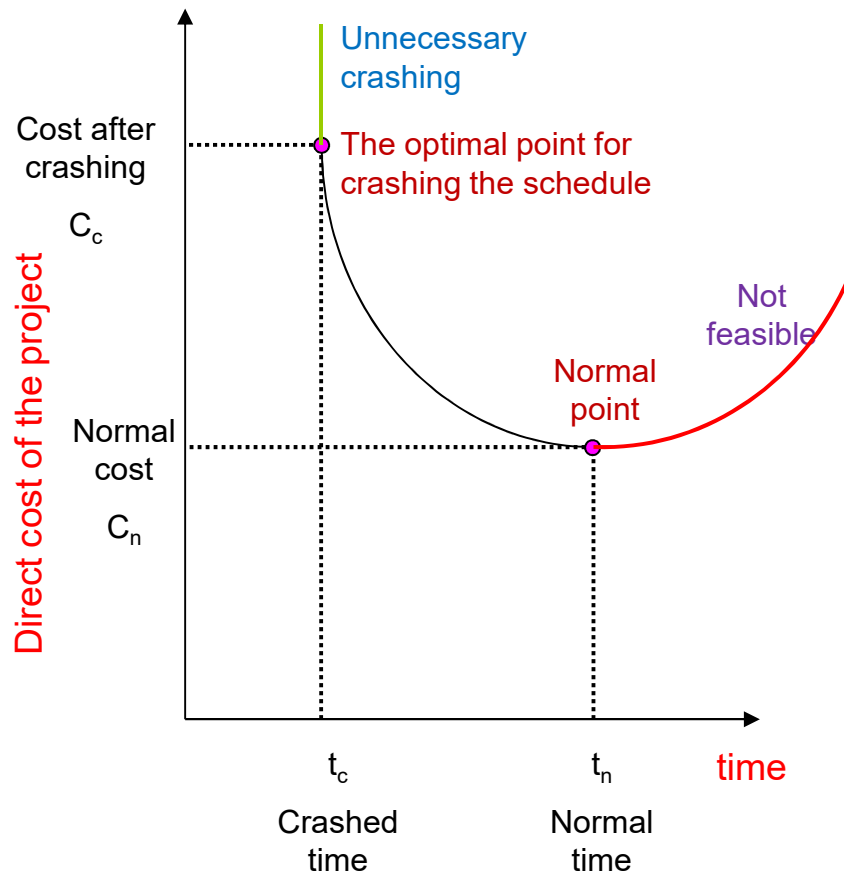
Theoretical Curve
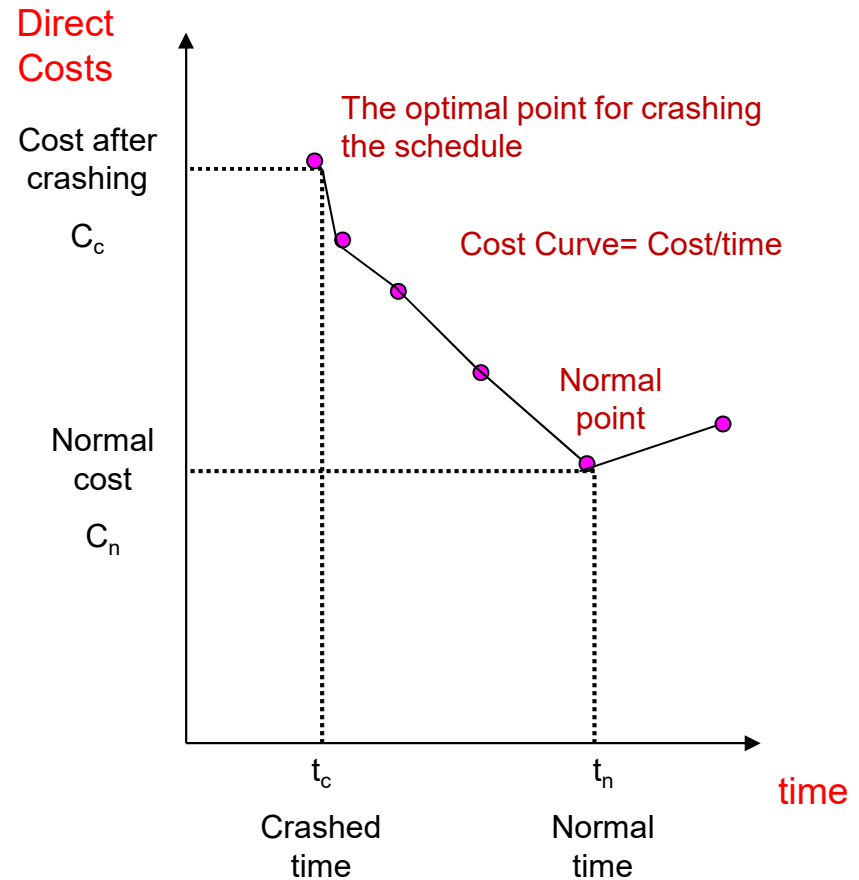
Practical Curve

Example

Excavation activity will be performed for a pipeline. According to the data provided below, draw the utility curve of the crashing.

- Normal working periods
  - 8 hours/day-5 days/week
- Normal cost
  - 4 units/day
- Normal working conditions
  - 300 man-days-Equip with 10 workers
- If there are 2 shifts:
  - Price for first shift: 4 units / day
  - Price for second shift : 5 units / day
- If there are 3 shifts :
  - Price for first shift :4 units / day
  - Price for second shift :5 units / day
  - Price for third shift :6 units / day
- The other alternative is increasing the number of workers to 30 for all three shifts. However, this alternative increases the necessary man day to 360 man - day.

Data

- **Normal working duration**
  - 8 hours/day- 5 days/week
- **Normal cost**
  - 4 units/day
- **Normal working conditions**
  - 300 man-days - Team comprises of 10 man

❖ **If there are 2 shifts:**
  - Price for first shift: 4 units/ day
  - Price for second shift : 5 units/ day

❖ **If there are 3 shifts :**
  - Price for first shift :4 units / day
  - Price for second shift :5 units/ day
  - Price for third shift :6 units / day

❖ The other alternative is increasing the number of workers to 30 for all three shifts. However, this alternative increases the necessary man day to 360 man - days.

SOLUTION:

- **Normal duration= tn=300/10= 30 days**
- **Normal cost=cn=10*30*4= 1200 units**

➤ **For 2 shifts;**
- **Duration of 2 shifts =300/20=15 days**
- **Cost of 2 shifts   =10*15*4=600 units**

  =10*15*5=750 units

  Total = 1350 units

➤ **For 3 shifts;**
- **Duration of 3 shifts =300/30   = 10 days**
- **Cost of 3 shifts   =10*10*4 = 400 units**

  =10*10*5 = 500 units

  =10*10*6 = 600 units

  Total = 1500 units

➤ **Other alternative;**
- **Duration   = 360 / (3*30) =4 days**
- **Cost of crashing   = 30*4*4 = 480 units**

  = 30*4*5 = 600 units

  = 30*4*6 = 720 units

  Total = 1800 units

- At first stage, the activity can be completed within the normal time and cost.

- If the activity is requested to be completed with the shorter time, the work can be crashed with assigning more resources which leads to increase in the cost of the activity.

- This is called as crashing.
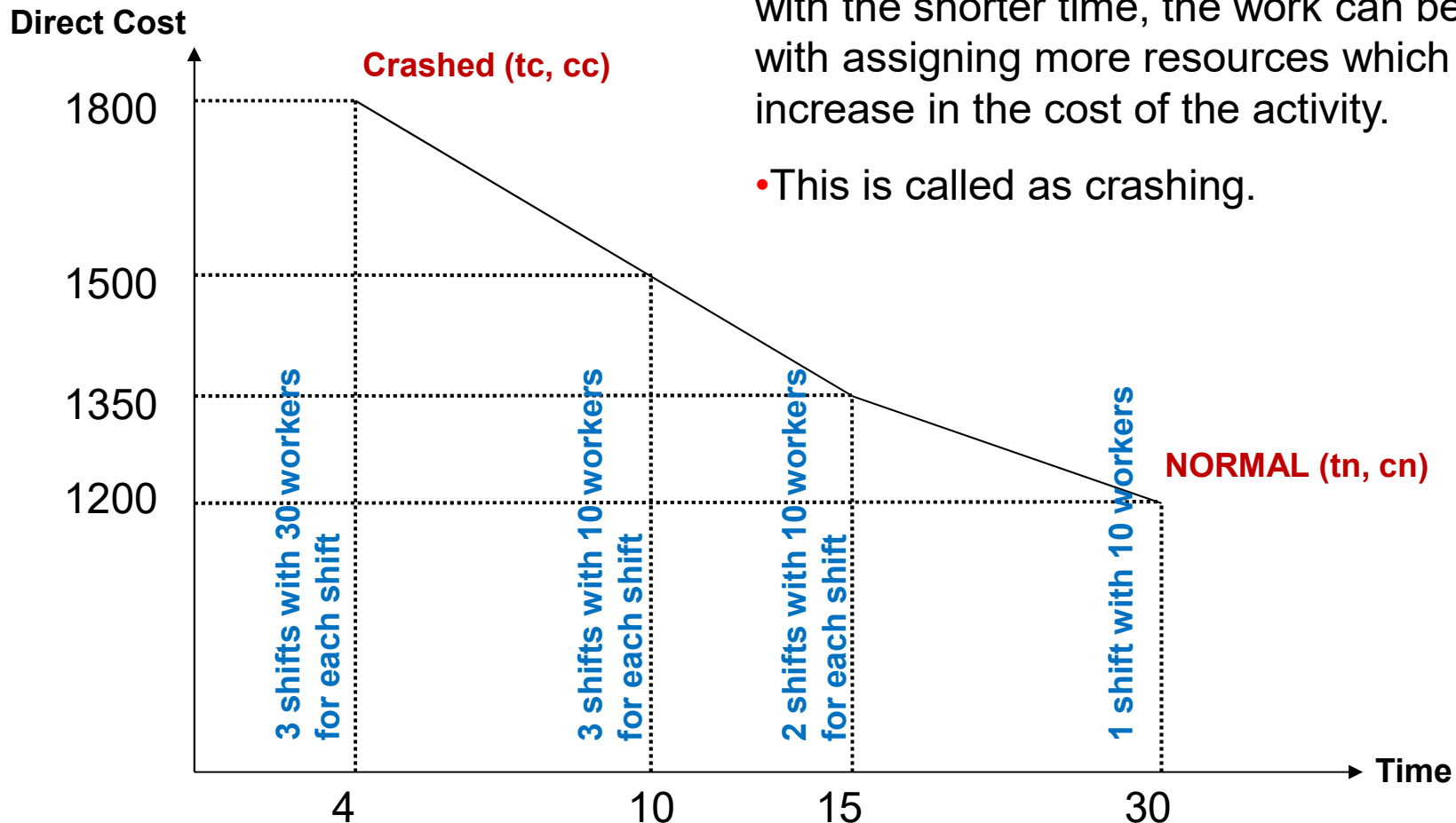
Project Crashing – The points should be considered in cost analysis

1) The activities placed on the critical path is crashed.

2) The crashing is evaluated by considering the unit price.

3) Firstly, the activities having lowest costs are crashed.

4) The limitations specific to the project are evaluated.

Example

- A subcontractor, performing infrastructure works, wants to finish their job as soon as possible. So, the subcontractor wants to know how much the cost of the job will increase when crashing is applied to the job. Calculate the required cost by using the table below.

Activity	Normal Duration (month)	Crashed Duration (month)	The unit cost of crashing TL / month
1-2	5	4	2000
1-3	3	3	-
2-3	3	2	2500
2-4	5	4	2000
2-5	4	4	-
3-4	3	2	3500
3-6	5	5	-
4-7	10	8	7500
5-7	6	6	-
6-7	4	4	-

Act.	Normal Duration (month)	Crashed Duration (month)	The unit cost of crashing TL / month
1-2	5	4	2000
1-3	3	3	-
2-3	3	2	2500
2-4	5	4	2000
2-5	4	4	-
3-4	3	2	3500
3-6	5	5	-
4-7	10	8	7500
5-7	6	6	-
6-7	4	4	-

I. Crashing:

Act.	Normal Duration (month)	Crashed Duration (month)	The unit cost of crashing TL / month
1-2	5	4	2000
1-3	3	3	-
2-3	3	2	2500
2-4	5	4	2000
2-5	4	4	-
3-4	3	2	3500
3-6	5	5	-
4-7	10	8	7500
5-7	6	6	-
6-7	4	4	-

Cost: 2000 TL

II. Crashing:

Act.	Normal Duration (month)	Crashed Duration (month)	The unit cost of crashing TL / month
1-2	5	4	2000
1-3	3	3	-
2-3	3	2	2500
2-4	5	4	2000
2-5	4	4	-
3-4	3	2	3500
3-6	5	5	-
4-7	10	8	7500
5-7	6	6	-
6-7	4	4	-

Cost: 2500 TL

Act.	Normal Duration (month)	Crashed Duration (month)	The unit cost of crashing TL / month
1-2	5	4	2000
1-3	3	3	-
2-3	3	2	2500
2-4	5	4	2000
2-5	4	4	-
3-4	3	2	3500
3-6	5	5	-
4-7	10	8	7500
5-7	6	6	-
6-7	4	4	-

III. Crashing:

TF= 4

FF= 0

TF= 4

FF= 4

TF= 0

FF= 0

TF= 0 FF= 0

4*

4

6

4*

TF= 0

FF= 0

2*

TF= 0 FF= 0

TF= 0 FF= 0

10

3

TF= 3

FF= 3

2*

5

4

TF= 3

FF= 0

TF= 3

FF= 3

4 **4**

8 **12**

0 **0**

8 **8**

18 **18**

6 **6**

11 **14**
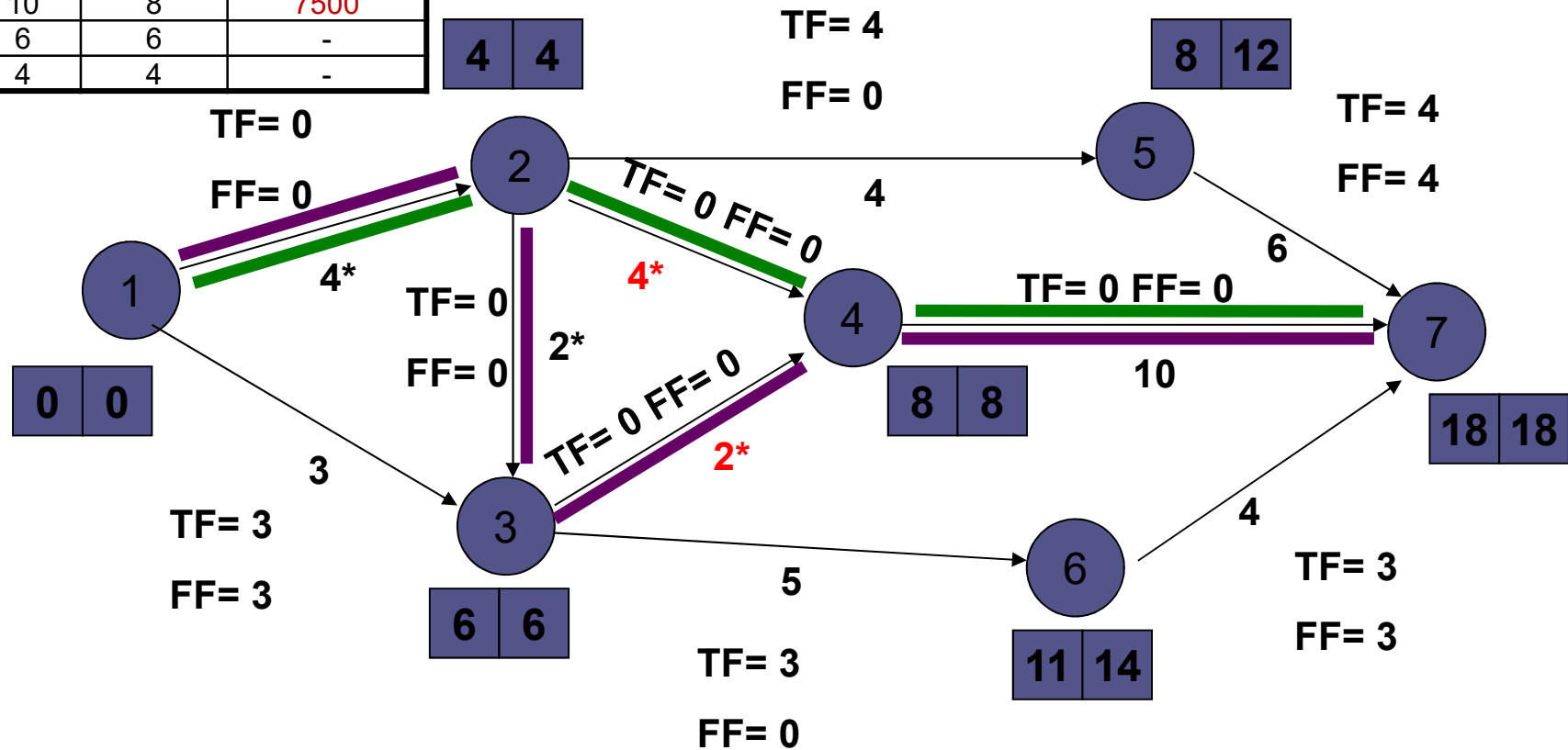
Cost: 2000 TL + 3500 TL = 5500 TL

Act.	Normal Duration (month)	Crashed Duration (month)	The unit cost of crashing TL / month
1-2	5	4	2000
1-3	3	3	-
2-3	3	2	2500
2-4	5	4	2000
2-5	4	4	-
3-4	3	2	3500
3-6	5	5	-
4-7	10	8	7500
5-7	6	6	-
6-7	4	4	-

IV. Crashing:

TF= 2 FF= 0

TF= 0 FF= 0

TF= 3 FF= 3

TF= 0 FF= 0

TF= 0 FF= 0

TF= 2 FF= 2

TF= 0 FF= 0

TF= 1 FF= 0

TF= 1 FF= 1
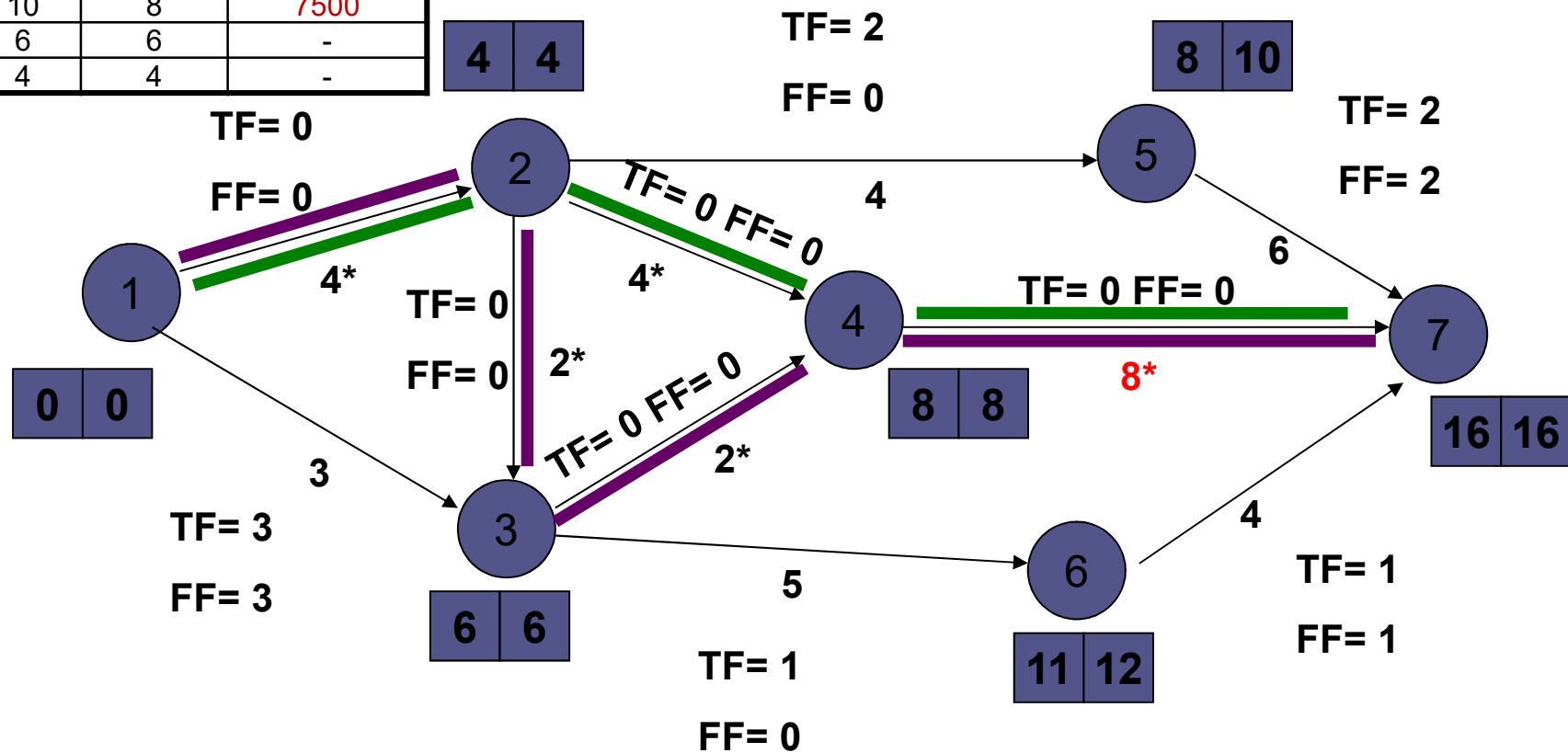
Cost: 2 x 7500 TL = 15000 TL

- I. Crashing: 2 000 TL / 1 Month
- II. Crashing: 2 500 TL / 1 Month
- III. Crashing: 5 500 TL / 2 Months
- IV. Crashing: 15 000 TL / 2 Months

Total Cost: 25 000 TL / 5 Months (!)

NOTE: Although the total amount of crashing on the activities is 6 months, the work is crashed only 5 months.

Example

- A construction company is planning to bid a project, whose duration is one year and bidding price is 170 million USD.
- According to the data given below, decide whether the construction company should bid the project or not.

Activity	Normal Duration (Month)	Normal Cost (Million $)	Duration after crashing (Month)	Total cost of the activity after crashing (Million $)
1-2	3	9	2	15
1-3	2	5	1	10
2-4	5	5	3	20
3-4	4	20	2	40
3-5	5	24	2	57
3-6	-	-	-	-
4-5	2	20	1	30
4-6	4	30	2	48
5-6	6	24	2	56

The total cost of normal duration= 137 million $

Activity	Normal Duration (Month)	Crashed Duration (Month)	Crashed Time (Month)	The total cost of the crashed activity (Million $)	Normal Cost (Million $)	Crashing Cost (Million $)	Crashing Unit Cost (Million $ / Month)
1-2	3	2	1	15	9	6	6
1-3	2	1	1	10	5	5	5
2-4	5	3	2	58	33	15	7.5
3-4	4	2	2	40	20	20	10
3-5	5	2	3	57	24	33	11
3-6	-	-	-	-	-	-	-
4-5	2	1	1	30	20	10	10
4-6	4	2	2	48	30	18	9
5-6	6	2	4	56	24	32	8

Activity	Normal Duration (Month)	Crashed Duration (Month)	Crashed Time (Month)	The Cost of the Crashed Activity (Million $)	Normal Cost (Million $)	Crashing Cost (Million $)	Crashing Unit Cost (Million $ / Month)
1-2	3	2	1	15	9	6	6
1-3	2	1	1	10	5	5	5
2-4	5	3	2	58	33	15	7.5
3-4	4	2	2	40	20	20	10
3-5	5	2	3	57	24	33	11
3-6	-	-	-	-	-	-	-
4-5	2	1	1	30	20	10	10
4-6	4	2	2	48	30	18	9
5-6	6	2	4	56	24	32	8

I. Crashing:

Cost: 6 Million $

Activity	Normal Duration (Month)	Crashed Duration (Month)	Crashed Time (Month)	The Cost of the Crashed Activity (Million $)	Normal Cost (Million $)	Crashing Cost (Million $)	Crashing Unit Cost (Million $ / Month)
1-2	3	2	~~1~~	15	9	6	6
1-3	2	1	1	10	5	5	5
2-4	5	3	~~2~~ 1	58	33	15	7.5
3-4	4	2	2	40	20	20	10
3-5	5	2	3	57	24	33	11
3-6	-	-	-	-	-	-	-
4-5	2	1	1	30	20	10	10
4-6	4	2	2	48	30	18	9
5-6	6	2	4	56	24	32	8

II. Crashing:

Cost: 7.5 Million $

Activity	Normal Duration (Month)	Crashed Duration (Month)	Crashed Time (Month)	The Cost of the Crashed Activity (Million $)	Normal Cost (Million $)	Crashing Cost (Million $)	Crashing Unit Cost (Million $ / Month)
1-2	3	2	~~1~~	15	9	6	6
1-3	2	1	1	10	5	5	5
2-4	5	3	~~2~~ 1	58	33	15	7.5
3-4	4	2	2	40	20	20	10
3-5	5	2	3	57	24	33	11
3-6	-	-	-	-	-	-	-
4-5	2	1	1	30	20	10	10
4-6	4	2	2	48	30	18	9
5-6	6	2	4	56	24	32	8

III. Crashing:

Cost: 7.5 Million $

Activity	Normal Duration (Month)	Crashed Duration (Month)	Crashed Time (Month)	The Cost of the Crashed Activity (Million $)	Normal Cost (Million $)	Crashing Cost (Million $)	Crashing Unit Cost (Million $ / Month)
1-2	3	2	1	15	9	6	6
1-3	2	1	1	10	5	5	5
2-4	5	3	2	58	33	15	7.5
3-4	4	2	2	40	20	20	10
3-5	5	2	3	57	24	33	11
3-6	-	-	-	-	-	-	-
4-5	2	1	1	30	20	10	10
4-6	4	2	2	48	30	18	9
5-6	6	2	4	56	24	32	8

III. Crashing:

Cost: 12.5 Million $

II. Crashing:

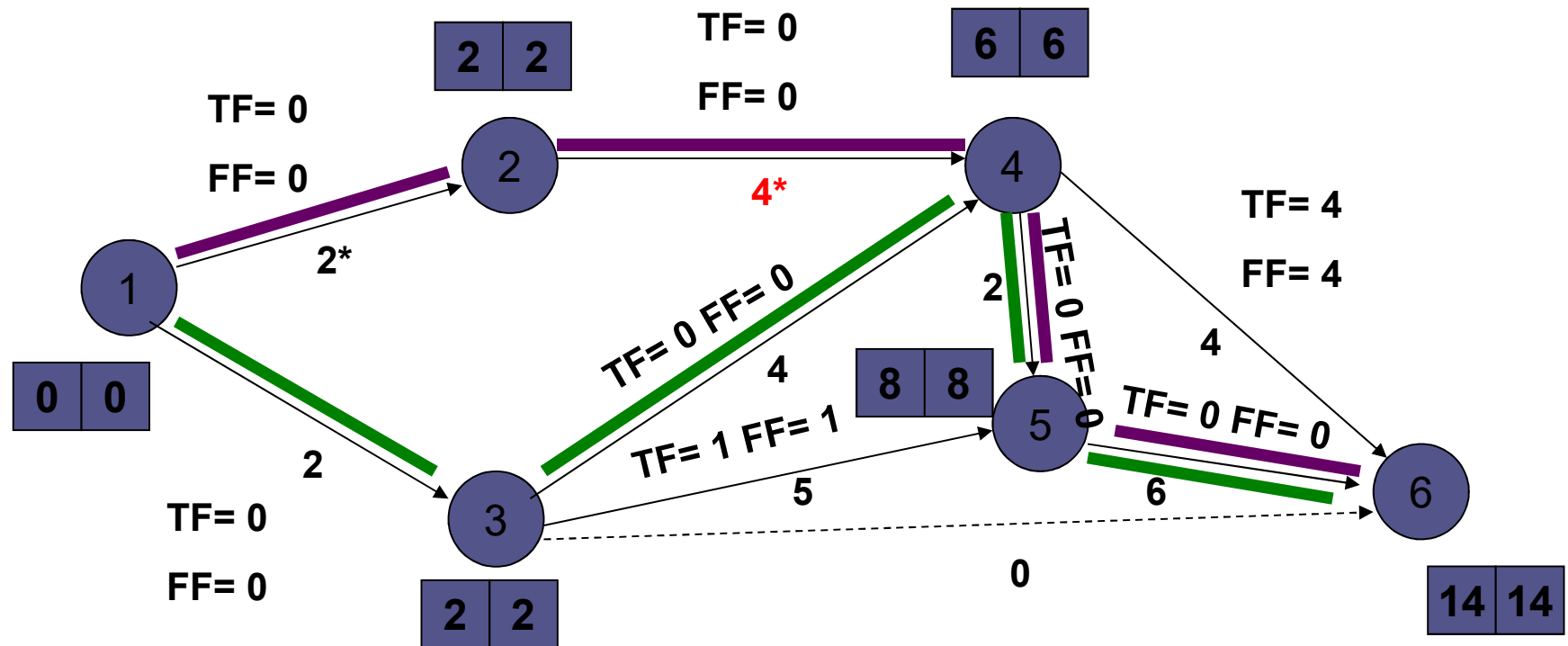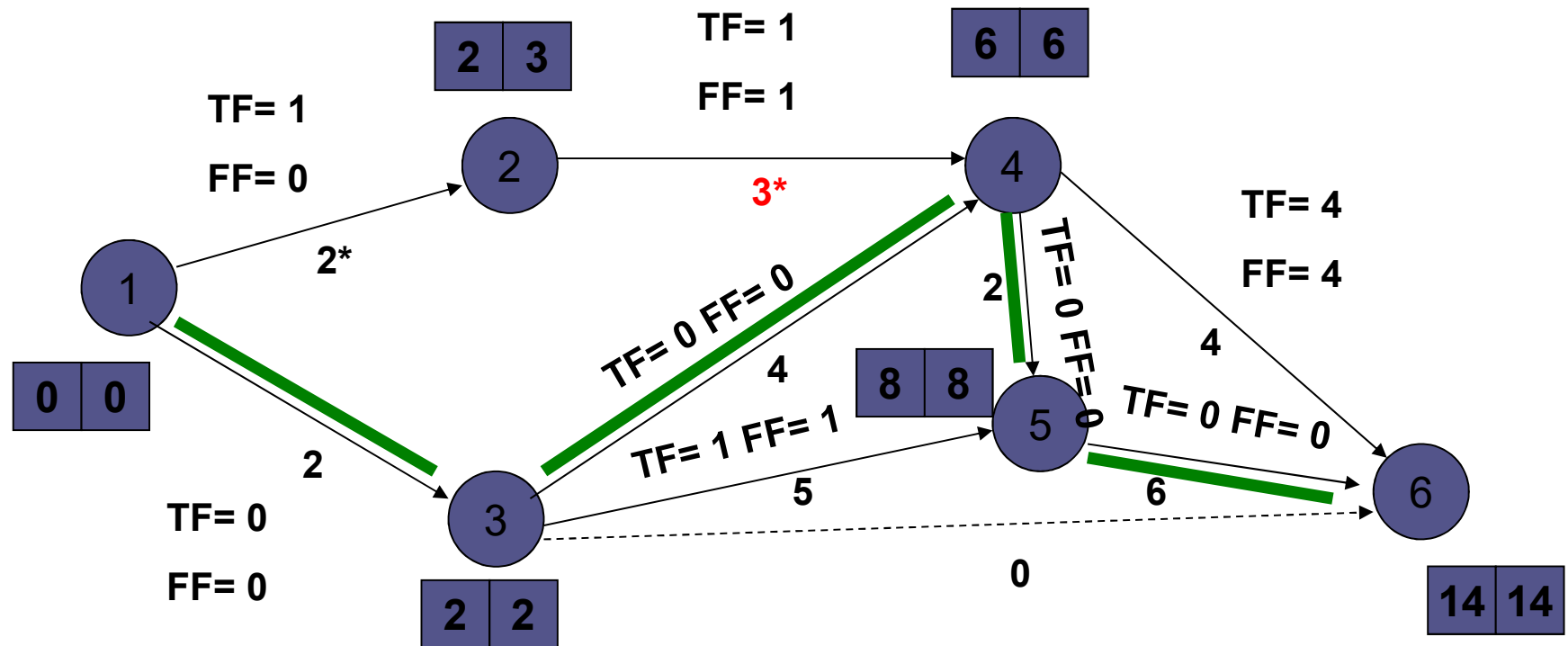| Activity	Normal Duration (Month)	Crashed Duration (Month)	Crashed Time (Month)	The Cost of the Crashed Activity (Million $)	Normal Cost (Million $)	Crashing Cost (Million $)	Crashing Unit Cost (Million $ / Month)
1-2	3	2	~~1~~	15	9	6	6
1-3	2	1	1	10	5	5	5
2-4	5	3	~~2~~ 1	58	33	15	7.5
3-4	4	2	2	40	20	20	10
3-5	5	2	3	57	24	33	11
3-6	-	-	-	-	-	-	-
4-5	2	1	1	30	20	10	10
4-6	4	2	2	48	30	18	9
5-6	6	2	~~4~~ 3	56	24	32	8

III. Crashing:

Cost: 8 Million $

Activity	Normal Duration (Month)	Crashed Duration (Month)	Crashed Time (Month)	The Cost of the Crashed Activity (Million $)	Normal Cost (Million $)	Crashing Cost (Million $)	Crashing Unit Cost (Million $ / Month)
1-2	3	2	~~1~~	15	9	6	6
1-3	2	1	1	10	5	5	5
2-4	5	3	~~2~~ 1	58	33	15	7.5
3-4	4	2	2	40	20	20	10
3-5	5	2	3	57	24	33	11
3-6	-	-	-	-	-	-	-
4-5	2	1	1	30	20	10	10
4-6	4	2	2	48	30	18	9
5-6	6	2	~~4~~ 2	56	24	32	8

IV. Crashing:
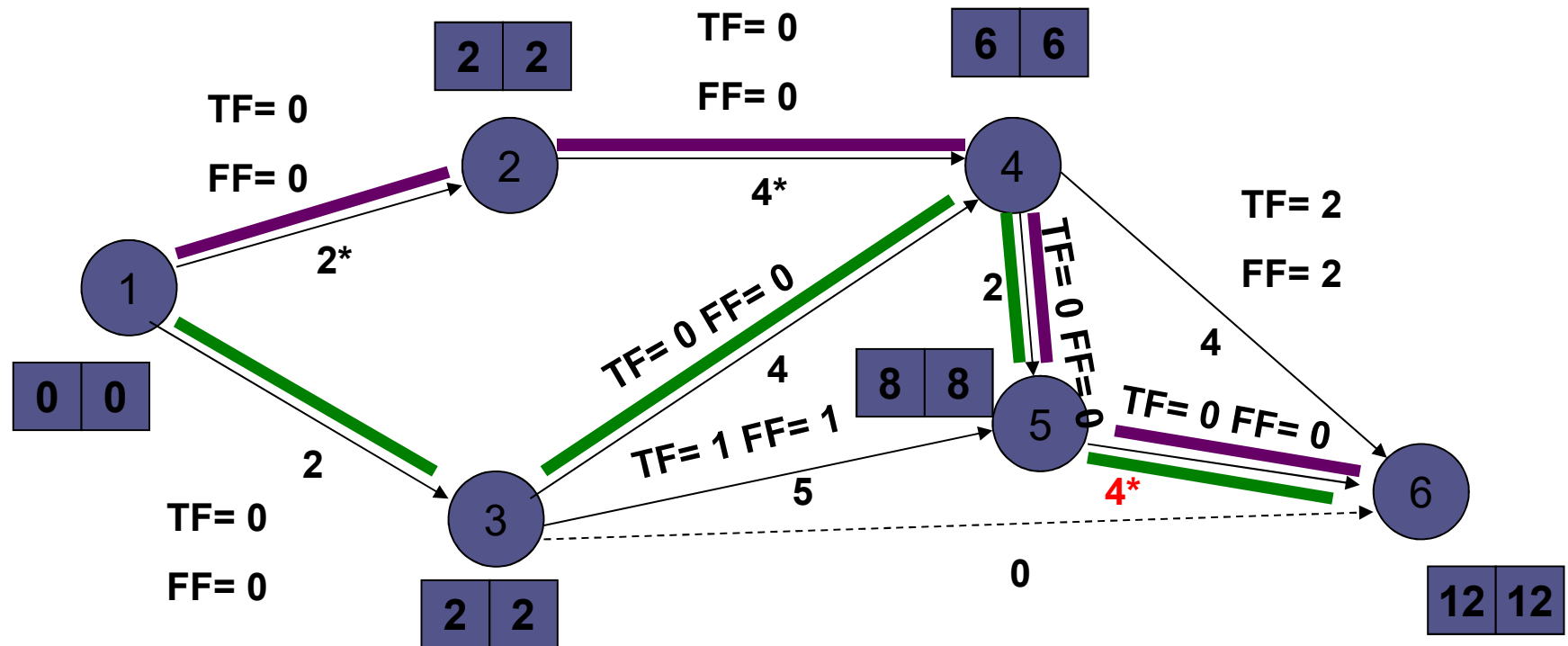
Cost: 8 Million $

- The Cost of the project: 137 Million $
- I. Crashing   :   6 Million $
- II. Crashing   :   7.5 Million $
- III. Crashing   :   8 Million $
- IV. Crashing   :   8 Million $

  The final cost of the project after crashing; 166.5 Million $ / 12 Month

Example 4:

- A project wants to be completed five weeks earlier. The data provided by the planning department is shown in the table. According to the table, please find the normal completion project duration, normal project cost, crashed completion time, cost increase and total cost of project after crashing.

Activity	Normal Duration (weeks)	Normal Cost ($)	Crashed Duration (weeks)	Crashed unit cost ($/week)
1-2	2	8000	1	5000
2-3	6	5000	5	2000
2-4	4	9000	2	1500
3-4	4	15000	1	2500
3-5	5	12000	4	3000
3-6	6	18000	4	2000
4-6	3	14000	1	7000
4-8	6	9000	3	4000
5-7	8	18000	7	4000
6-9	11	20000	9	3500
7-9	7	8000	6	8000
8-9	8	12000	6	6000

i ──A──▶ j
t

ESi LSi   ESj LSj

$TF = LS_j - (ES_i + duration)$
$FF = ES_j - (ES_i + duration)$

8 | 8   TF= 0   13 | 13   TF= 0   21 | 21

3 ──FF= 0──▶ 5 ──FF= 0──▶ 7

5   8

TF= 0 FF= 0
6

TF= 3 FF= 1
6

TF= 2 FF= 0
4

TF= 0   TF= 2 FF= 2   TF= 0 FF= 0
7

FF= 0   6   11

1 ──▶ 2   15 | 17   9

2   TF= 8 FF= 6   TF= 2 FF= 0   28 | 28

0 | 0   2 | 2   4   3   TF= 2 FF= 2

4   8

12 | 14   TF= 2 FF= 0

6

8

18 | 20

Activity	Normal Duration (weeks)	Normal Cost ($)	Crashed Duration (weeks)	Crashed unit cost ($/week)
1-2	2	8000	1	5000
2-3	6	5000	5	2000
2-4	4	9000	2	1500
3-4	4	15000	1	2500
3-5	5	12000	4	3000
3-6	6	18000	4	2000
4-6	3	14000	1	7000
4-8	6	9000	3	4000
5-7	8	18000	7	4000
6-9	11	20000	9	3500
7-9	7	8000	6	8000
8-9	8	12000	6	6000

I. Crashing:

$$TF = LS_j - (ES_i + duration)$$
$$FF = ES_j - (ES_i + duration)$$

Cost: 2 000 $ / week

Activity	Normal Duration (weeks)	Normal Cost ($)	Crashed Duration (weeks)	Crashed unit cost ($/week)
1-2	2	8000	1	5000
2-3	6	5000	5	2000
2-4	4	9000	2	1500
3-4	4	15000	1	2500
3-5	5	12000	4	3000
3-6	6	18000	4	2000
4-6	3	14000	1	7000
4-8	6	9000	3	4000
5-7	8	18000	7	4000
6-9	11	20000	9	3500
7-9	7	8000	6	8000
8-9	8	12000	6	6000

II. Crashing:

$$TF = LS_j - (ES_i + duration)$$
$$FF = ES_j - (ES_i + duration)$$

Cost: 3 000 $ / week

Activity	Normal Duration (weeks)	Normal Cost ($)	Crashed Duration (weeks)	Crashed unit cost ($/week)
1-2	2	8000	1	5000
2-3	6	5000	5	2000
2-4	4	9000	2	1500
3-4	4	15000	1	2500
3-5	5	12000	4	3000
3-6	6	18000	4	2000
4-6	3	14000	1	7000
4-8	6	9000	3	4000
5-7	8	18000	7	4000
6-9	11	20000	9	3500
7-9	7	8000	6	8000
8-9	8	12000	6	6000

III. Crashing:

$ES_i | LS_i$

$ES_j | LS_j$

$TF = LS_j - (ES_i + duration)$

$FF = ES_j - (ES_i + duration)$

Cost: 4 000 $ / week

Activity	Normal Duration (weeks)	Normal Cost ($)	Crashed Duration (weeks)	Crashed unit cost ($/week)
1-2	2	8000	1	5000
2-3	6	5000	5	2000
2-4	4	9000	2	1500
3-4	4	15000	1	2500
3-5	5	12000	4	3000
3-6	6	18000	4	2000
4-6	3	14000	1	7000
4-8	6	9000	3	4000
5-7	8	18000	7	4000
6-9	11	20000	9	3500
7-9	7	8000	6	8000
8-9	8	12000	6	6000

IV. Crashing:

i --A--> j
t

ESi LSi   ESj LSj

$TF = LS_j - (ES_i + duration)$

$FF = ES_j - (ES_i + duration)$

| 6 | 6 | TF= 0 | | 10 | 10 | TF= 0 | | 17 | 17 |

3   FF= 0   5   FF= 0   7

TF= 0

FF= 0

TF= 0 FF= 0

TF= 1 FF= 1

4*

6

5* 4

7*

TF= 0 FF= 0

TF= 0 FF= 0

TF= 0 FF= 0

1   FF= 0   2

1*

| 0 | 0 |

| 1 | 1 |

TF= 5 FF= 5

4

6   TF= 0 FF= 0

11

| 13 | 13 |

9

| 24 | 24 |

TF= 0 FF= 0

3

4

| 10 | 10 |

TF= 0 FF= 0

6

TF= 0 FF= 0

8

7

TF= 0 FF= 0

8

| 16 | 16 |

Cost: 5 000 $ / week

Activity	Normal Duration (weeks)	Normal Cost ($)	Crashed Duration (weeks)	Crashed unit cost ($/week)
1-2	2	8000	1	5000
2-3	6	5000	5	2000
2-4	4	9000	2	1500
3-4	4	15000	1	2500
3-5	5	12000	4	3000
3-6	6	18000	4	2000
4-6	3	14000	1	7000
4-8	6	9000	3	4000
5-7	8	18000	7	4000
6-9	11	20000	9	3500
7-9	7	8000	6	8000
8-9	8	12000	6	6000

V. Crashing:

$TF = LS_j - (ES_i + duration)$

$FF = ES_j - (ES_i + duration)$

Cost: 2 500 $ / week

8 000 $ / week
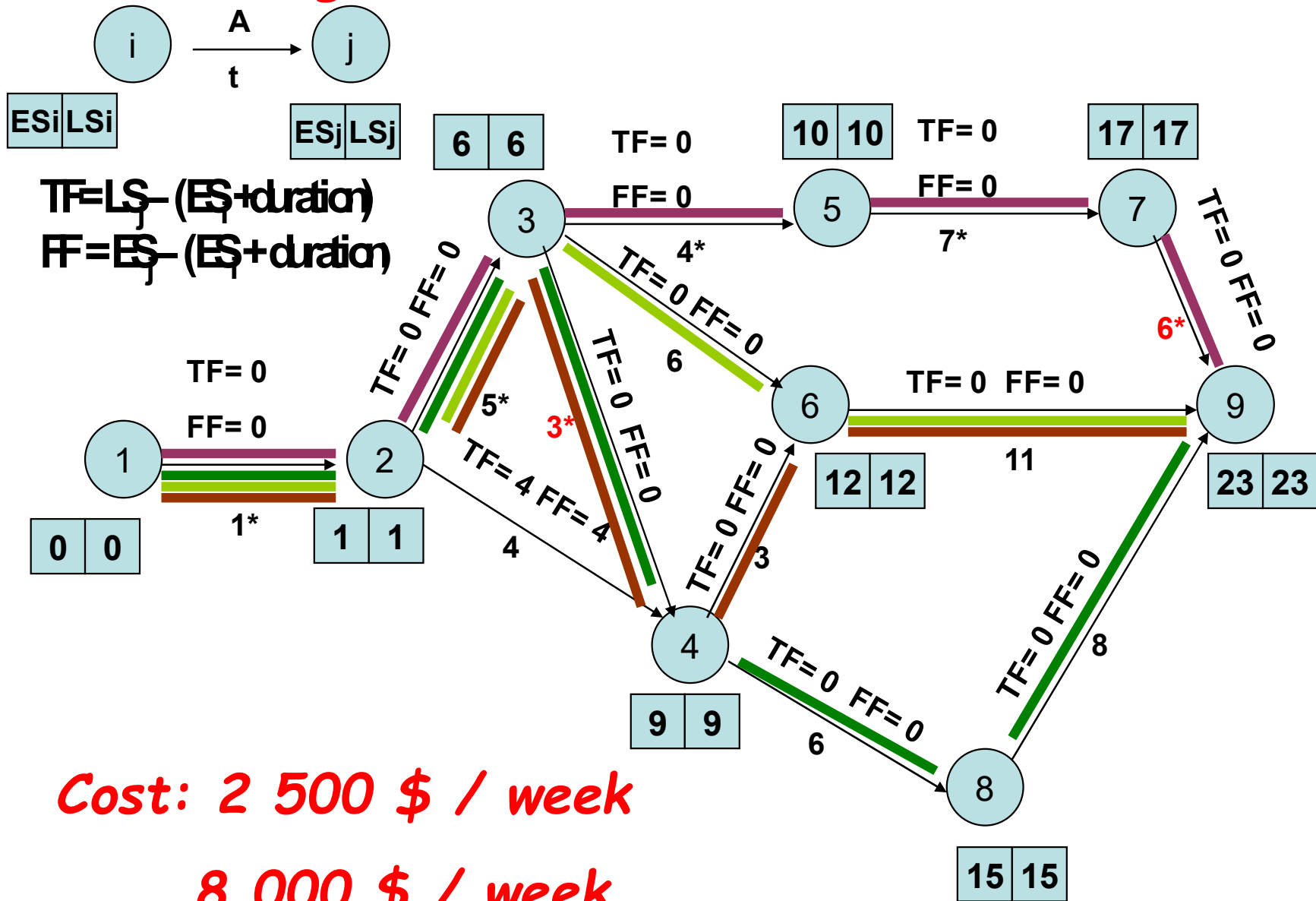
- **Cost increment:** 1. Crashing ; 2 000$ (2-3 / 1W)
                              2. Crashing ; 3 000$ (3-5 / 1W)
                              3. Crashing ; 4 000$ (5-7 / 1W)
                              4. Crashing ; 5 000$ (1-2 / 1W)
                              5. Crashing ; 2 500$ (3-4 / 1W)
                              5. Crashing ; 8 000$ (7-9 / 1W)
- **Total Cost Increment   =   24 500$ / 5 Weeks**
- **Normal Cost   = 148 000$ (28 Weeks)**
- Total Cost after Crashing= 172 500$ (23 Weeks)