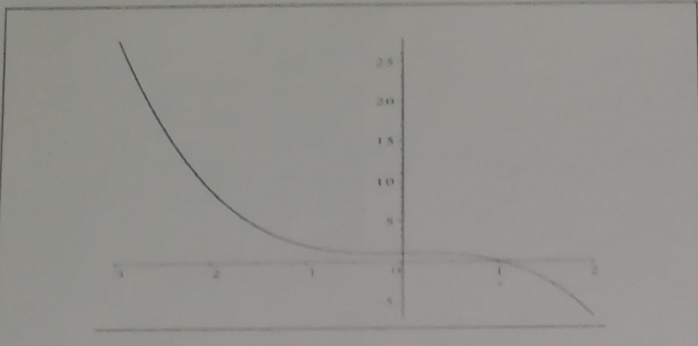
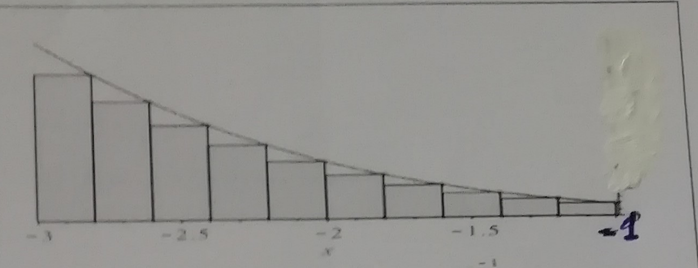
[35p] Q1) Express the definite integral

$$\int_{-3}^{-1}\left(1-x^{3}\right)dx$$

as a limit of Riemann sums and evaluate the limit.
(Use the lower Riemann sums with n subintervals of equal length)

SOLUTION:





A lower Riemann sum approximation of $\int_{-3}^{-1} f(x)\,dx$, where $f(x) = -x^3 + 1$ and the partition is uniform.

Since $f(x)=1-x^3$ is decreasing on the interval $\left[-3,-1\right]$, the minumum value on each subinter occurs at the right endpoints.

$$\boxed{\lim_{n\to\infty} L(f,P_n) = \lim_{n\to\infty} U(f,P_n) = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x_i = \int_a^b f(x)\,dx}$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x\,,\quad \Delta x=\frac{b-a}{n},\quad x_i=a+i\Delta x,\quad i=0\ldots n\,.$$

$$\Delta x=\frac{-1-(-3)}{n}=\frac{2}{n},\quad x_i=-3+i\frac{2}{n},\quad i=0\ldots n.$$

$$f(x_i)=1-\left(x_i\right)^3=1-\left(-3+i\frac{2}{n}\right)^3=28-\frac{54}{n}i+\frac{36}{n^2}i^2-\frac{8}{n^3}i^3$$

$$\int_{-3}^{-1}\left(1-x^3\right)dx=\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x=\lim_{n\to\infty}\sum_{i=1}^{n}\left(28-\frac{54}{n}i+\frac{36}{n^2}i^2-\frac{8}{n^3}i^3\right)\frac{2}{n}$$

$$=\lim_{n\to\infty}\left(28n-\frac{54}{n}\left(\frac{n(n+1)}{2}\right)+\frac{36}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)-\frac{8}{n^3}\left(\frac{n(n+1)}{2}\right)^2\right)\frac{2}{n}$$

$$=\lim_{n\to\infty}\left(28n-27(n+1)+6\frac{(n+1)(2n+1)}{n}-2\frac{(n+1)^2}{n}\right)\frac{2}{n}$$

$$=\lim_{n\to\infty}\left(28-27\left(1+\frac{1}{n}\right)+6\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)-2\left(1+\frac{1}{n}\right)^2\right)2$$

$$=\left(28-27+12-2\right)2$$

$$\boxed{=22}$$