

**SUGGESTED QUESTION**

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NOTE CHART

Q2) Evaluate $\int_0^\infty \frac{1}{(x+1)\sqrt{x}} dx$

SOLUTION:

To evaluate this integral, split it at a convenient point such that

$$\int_0^\infty \frac{1}{(x+1)\sqrt{x}} dx = \underbrace{\int_0^c \frac{1}{(x+1)\sqrt{x}} dx}_{\text{Type2}} + \underbrace{\int_c^\infty \frac{1}{(x+1)\sqrt{x}} dx}_{\text{Type1}}, \text{ where } c > 0.$$

Take $c = 1$ for convenience:

$$\left(\int \frac{1}{(x+1)\sqrt{x}} dx = 2 \int \frac{du}{u^2 + 1} = 2 \arctan \sqrt{x} + C \right) \quad \begin{matrix} u = \sqrt{x} \Rightarrow x = u^2 \\ du = 2udu \end{matrix}$$

$$\int_0^1 \frac{1}{(x+1)\sqrt{x}} dx = 2 \lim_{k \rightarrow 0^+} \arctan \sqrt{x} \Big|_k^1 = 2 \lim_{k \rightarrow 0^+} \left(\frac{\pi}{4} - \arctan \sqrt{k} \right) = \frac{\pi}{2}$$

$$\int_1^\infty \frac{1}{(x+1)\sqrt{x}} dx = 2 \lim_{b \rightarrow \infty} \arctan \sqrt{x} \Big|_1^b = 2 \lim_{b \rightarrow \infty} \left(\arctan \sqrt{b} - \frac{\pi}{4} \right) = \frac{\pi}{2}$$

$$\text{So } \int_0^\infty \frac{1}{(x+1)\sqrt{x}} dx = \underbrace{\int_0^1 \frac{1}{(x+1)\sqrt{x}} dx}_{\frac{\pi}{2}} + \underbrace{\int_1^\infty \frac{1}{(x+1)\sqrt{x}} dx}_{\frac{\pi}{2}} = \boxed{\pi}$$