MAK4391
AUTOMATIC CONTROL I

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Department of Mechanical Engineering

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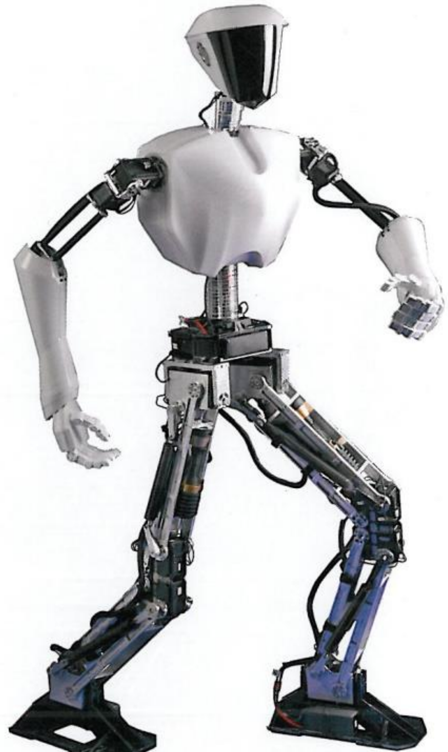
Course contents

- Basic concepts, Control systems and dynamics, Laplace Transforms, Transfer functions, Block diagrams, Time response, Stability analysis, Steady state, PID controller design

Recommended readings (references)

- ***Control Systems Engineering, N. Nise, Wiley, 4th Edt., 2004***
- ***Otomatik Kontrol sistemleri, MATLAB Destekli Analiz ve Tasarım Yaklaşımı İle, Mehmet Önder Efe***
- ***Otomatik Kontrol Sistem Dinamiği ve Denetim Sistemleri, Ibrahim Yuksel***
- ***Otomatik Kontrol Sistem Dinamiği ve Denetim Sistemleri, I Çözümlü Problemler, brahim Yuksel, Uluda***
- ***Automatic Control Systems, B. Kuo, F. Golnaraghi, Wiley, 8th Edt., 2003***
- ***Modern Control Engineering, K. Ogata, 3th Edt., Prentice-Hall, 1997***
- *Feedback Control Systems, G. F. Franklin, J. D. Powell, A. E. Naeini, Prentice-Hall, 4th Edt, 2002*
- *Modern Control Systems, R.C. Dorf, R.H. Bishop, Prentice-Hall, 10th Edt., 2005*
- *Otomatik Kontrol, Nimet Özdaş, Tahla Dinibütün, Ahmet Kuzucu, İTU, 1989*
- *Otomatik Kontrol Sistemleri, Fikret Caliskan, Birsen Yayinevi*

NORMAN S. NISE



CONTROL SYSTEMS ENGINEERING

SIXTH EDITION

WEEKS	COURSE OUTLINE	Related Preparation			Homework
1	*Basic concepts, Structure of control systems*	Book 1, Chpt. 1	Book 2, Chpt. 1	Book 3, Chpt. 1	Homework 1
2	*Laplace transforms*	Book 1, Chpt. 2	Book 2, Chpt. 2	Book 3, Chpt. 2	Homework 2
3	*Inverse Laplace transforms*	Book 1, Chpt. 2	Book 2, Chpt. 2	Book 3, Chpt. 2	Homework 3
4	*Transfer functions*	Book 1, Chpt. 2	Book 2, Chpt. 3	Book 3, Chpt. 3	Homework 4
5	*Mechanical, electrical and flow system models*	Book 1, Chpt. 2		Book 3, Chpt. 3	Homework 5
6	*Block diagrams*	Book 1, Chpt. 5		Book 3, Chpt. 3	Homework 6
7	*Block diagrams, Signal flow graph*	Book 1, Chpt. 5		Book 3, Chpt. 3	Homework 7
8	*1. Midterm*				
9	*Transient response of systems*	Book 1, Chpt. 4	Book 2, Chpt. 4	Book 3, Chpt. 5	Homework 8
10	*Steady state response of systems*	Book 1, Chpt.7	Book 1, Chpt.5	Book 3, Chpt.6	Homework 9
11	*Stability analysis, Routh-Hurwitz*	Book 1, Chpt.6		Book 3, Chpt.8	Homework 10
12	*2. Midterm*				
13	*PID controller design*		Book 1, Chpt.8	Book 3, Chpt.7	
14	*Matlab-Simulink examples*	Lecture slides			
15	*Final Exam*				

Book 1: ***Control System Engineering , Norman Nise (English)***

Book 2: ***Otomatik Kontrol sistemleri- MATLAB Destekli Analiz ve Tasarım Yaklaşımı İle, Mehmet Önder Efe (Turkish)***

Book 3: ***Otomatik Kontrol Sistem Dinamiği ve Denetim Sistemleri, Ibrahim Yuksel (Turkish)***

Course learning outcomes

- **Upon the completion of this course,**

- Learning general knowledge on automatic control and system dynamics concepts
- Modeling the physical systems on time doment
- Obtaining transfer functions and investigating stability of physical systems
- Learning control applications and realizing analysis and design methods

Attendance Policy

- **Your signatures will be taken during the class for attendance policy.**
- Also, you are expected to be at class on time.

- **Grading Policy**
- 1 Midterm (30%)
- 1Short Exam (%20)
- 1 Final (40%)
- Homework (%10)

Exam policy

- Cheating during the exams will not be tolerated. According to the *`The Code of Student Conduct` that taken from The University of Memphis, 1998-1999 Student Book*, the term `**academic misconduct**` includes, but is not limited to, all acts of **cheating** and **plagiarism**.

- **<u>The term `CHEATING` includes, but is not limited to:</u>**
    - use of any unauthorized assistance in taking quizzes, tests, or examinations.
    - dependence upon the aid of source beyond those authorized by the instructor in writing papers, preparing reports, solving problems' or carrying out other assignments.
    - the misrepresentation of papers, reports, assignments or other materials as the product of a student's sole independent effort, for the purpose of affecting the student's grade, credit, or status in the University;

Exam policy

- failing to abide by the instructions of the proctor concerning test-taking procedures; examples include, but are not limited to, talking, laughing, failure to take a seat assignment, failing to adhere to starting and stopping times, or other disruptive activity;

- influencing, or attempting to influence, any University official, faculty member, graduate student or employee possessing academic grading and/or evaluation authority or responsibility for maintenance of academic records, through the use of bribery, threats, or any other means or coercion in order to affect a student's grade or evaluation;

- **<u>The term "PLAGIARISM" includes, but is not limited to</u>**,

- the use, by paraphrase or direct quotation, of the published or unpublished work of another person without full or clear acknowledgment. It also includes the unacknowledged use of materials prepared by another person or agency engaged in the selling of term papers or other academic materials.

Lecture 1
Introduction to Control Systems

Keywords of the 1st Lecture:

- System/plant/process
- Input
- Output
- Control system:
  - Open loop control system
  - Closed loop control system
- Error(E)
- Transfer function

*What is a **Control System**?*

- A control system consists of ***subsystems*** and ***process*** (*or plants)* assembled for the purpose of controlling the outputs of the processes.

- *In its simplest form, a control system provides an output or response for a given input or stimulus…*

HOW DO I CHANGE THIS?

TO GET WHAT I WANT?

******Make some object (called system, plant pr process) behave as we desire***

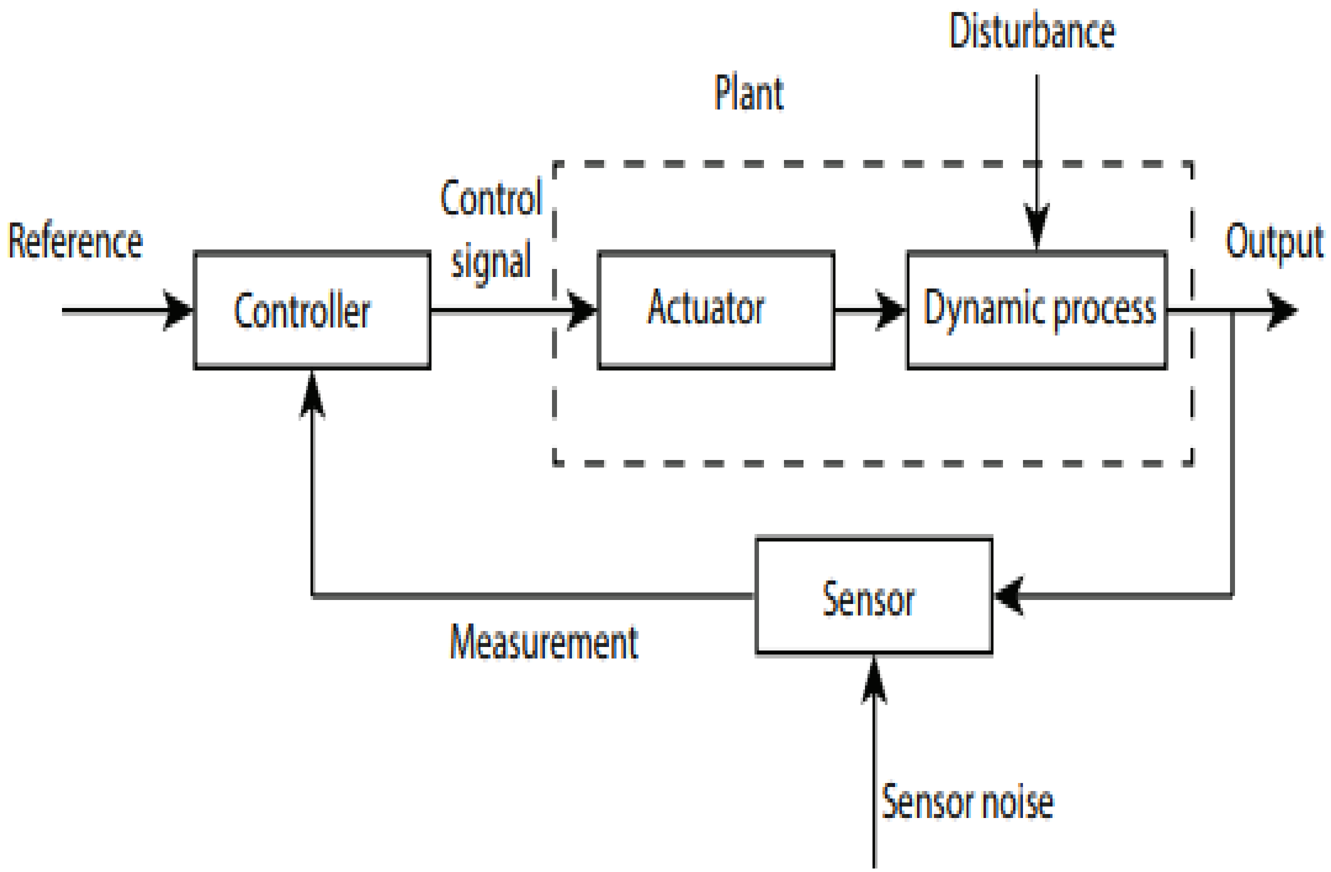
Imagine control around you…

- Room temperature control
- Car/bike driving control
- Voice volume control
- Control the position of the pointer
- Cruise control or speed control
- Process control
- Pancreas which regulates our blood sugar
- Adrenaline which is produced by our body increases along with our heart rate, causing more oxygen to be delivered to our cells
- Our eyes follow a moving object to keep it in view or we can adjust our eyes by squinting for light intensity

Reasons for using automatic control

- Reduce workload
- Avoid from dangerous situations (hot/cold places, high pressure envirenments, space, bomb removal)
- Perform tasks people can't (nanometer scale precision positioning, work inside the small space that human can not enter, ex: micro-sergey robots)
- Reduce the effects of disturbances
- Reduce the effects of plant variations
- Stabilize an unstable system
- Improve the performance of a system(time response)
- Improve the linearity of the system

Actuators:
Electrical motors (DC, brushless, step), pumps, valves, heaters



Sensors:
Temperature, pressure, flow, level, velocity-position, acceleration, force(strain/deformation)

Consider balancing a stick on your fingertip:

- (Pocess)System: Your finger+stick
- Sensor : Your eyes
- Controller: Brain+arm+muscles
- Comparison: Brain
- Desired output: 0 degree angle with vertical axis

Consider air-conditioning heating the room

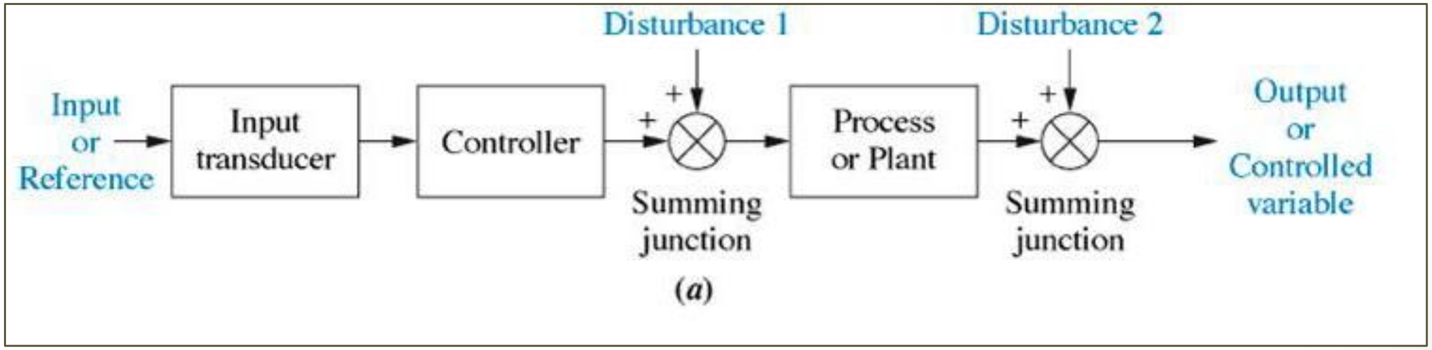
- (Pocess)System: Heater and the air in the room
- Sensor : Thermometer
- Controller: Air conditioner on/off switch
- Comparison: Logic controller in the air-conditioner that substracts the desired temperature from the measured one.
- Desired output:Desired room temperature (for instance, 22°C)

Control system configurations

We now, describe two control system configurations;

The first basic type of control is **Open** Loop which cannot compensate for any disturbances that add to the controller's driving signal(disturbance)

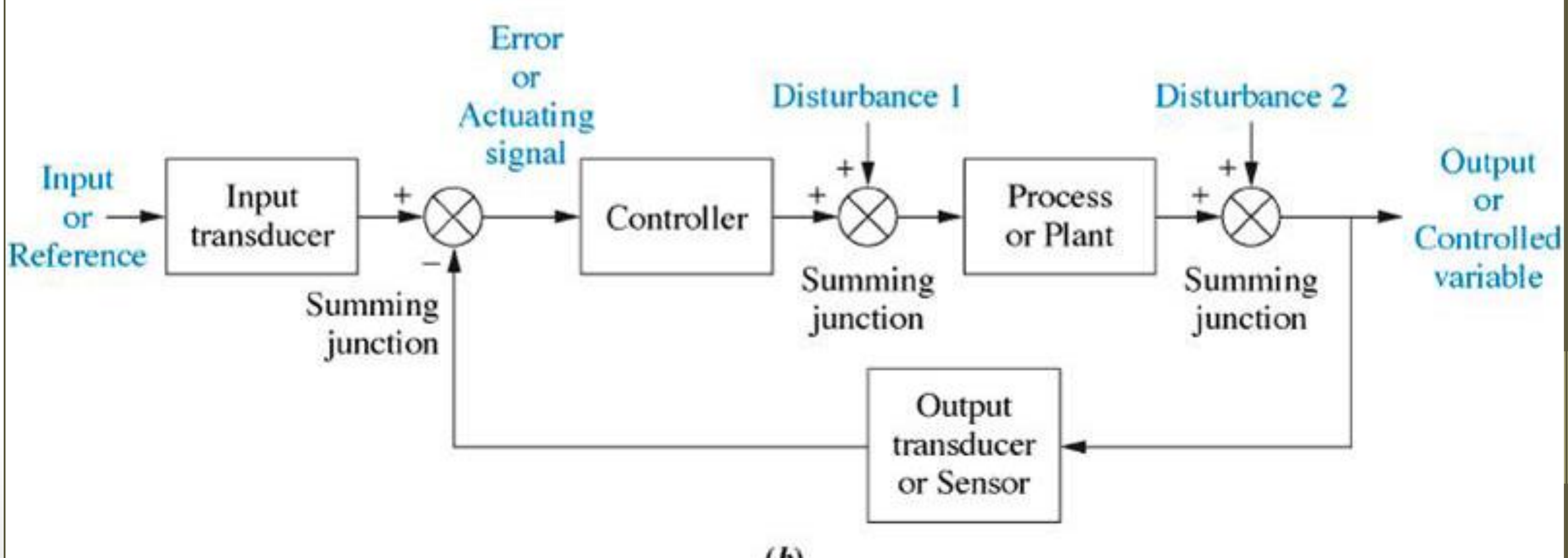
- An Open Loop Controller has actuation, but **<u>no measurement</u>**.
- ***Input transducer***(subsytem) converts the form of the input to that used by the controller. The controller drives a ***process. The summing junction*** *yields the algebraic sum of their input signals using associated signals.*



Closed-Loop (Feedback control) Systems

The second type of control is **Closed-Loop.**

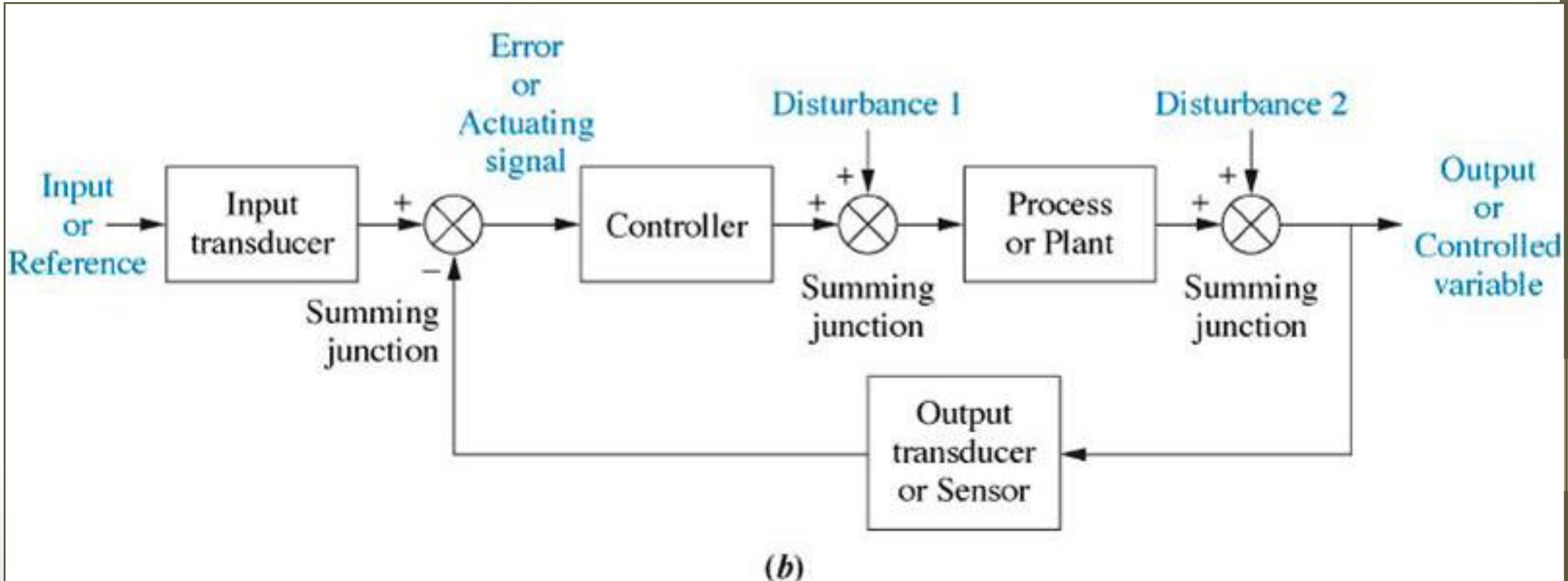
- The disadvantages of open-loop systems, namely sensitivity to disturbances and inability to correct for these disturbances, may be overcome in closed-loop systems.



(b)

Closed-Loop (Feedback control) Systems

- The *input transducer* converts the form of the input to the form used by the controller. An *output transducer*, or sensor, measures the output response and converts it into the form used by the controller.



(*b*)

Closed-Loop (Feedback control) Systems

- The input transducer converts the form of the input to the form used by the controller. An output transducer or sensor measures the output response and converts it into the form used by the controller.
- For example if the controller uses electrical signals to operate the valves of a temperature control system, the input position and the output temperature are converted to electrical signals.
- The input position can be converted to a voltage by a *potentiometer*, a variable resistor, and the output temperature can be converted to a voltage by a *thermistor*, a device whose electrical resistance changes wi temperature.

Closed-Loop (Feedback control) Systems

- The first *summing junction* algebraically adds the signal from the input to the signal from the output, which arrives via the *feedback path,* the return path from the output to the summing junction. In figure the output signal is subtracted from the input signal. The result is generally called the *actuating signal.*
- However, in systems where both the input and output transducers have *unity gain,* the actuating signal's value is equal to the actual difference between the input and output. Under this condition the actuating signal is called *the error.*
- The closed loop systems compensates for disturbances by measuring the output response, feeding that measurement back through a feedback path, and comparing that response to the input at summing junction. If there is no difference , the system does not drive the plant, since the plant's response is already the desired response.

Comparison for open and closed-loop systems

Closed loop systems have the obvious advantage of greater accuracy than open loop systems.

- They are less sensitive to noise, disturbances and changes in the environment.
- Transient response and steady state error can be controlled more conveniently.

On the other hand,

- Closed loop systems are more complex and expensive

Comparison for open and closed-loop systems

- A standard open-loop toaster is simple and inexpensive.

- A closed-loop toaster oven is more expensive and complex since it has to measure both color (through light reflectivity) and humidity inside the toaster oven.

Thus,

- The control system engineer must consider

the trade-off between the simplicity and low cost of an open loop system and the accuracy and higher cost of a closed loop system.

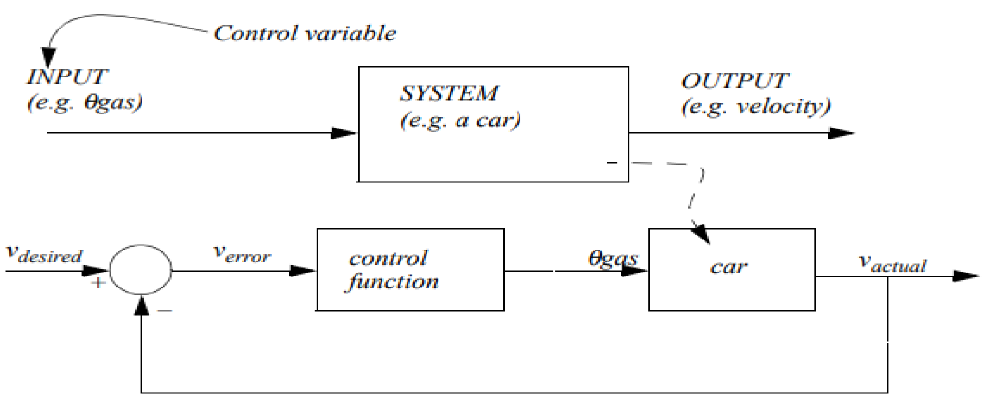
Briefly,

- Open-loop systems do not monitor or correct the output for disturbances; however, they are simpler and less expensive than closed-loop systems.

- Closed-loop systems monitor the output and compare it to input. If error is detected, the system corrects the output and hence corrects the effects of disturbances.

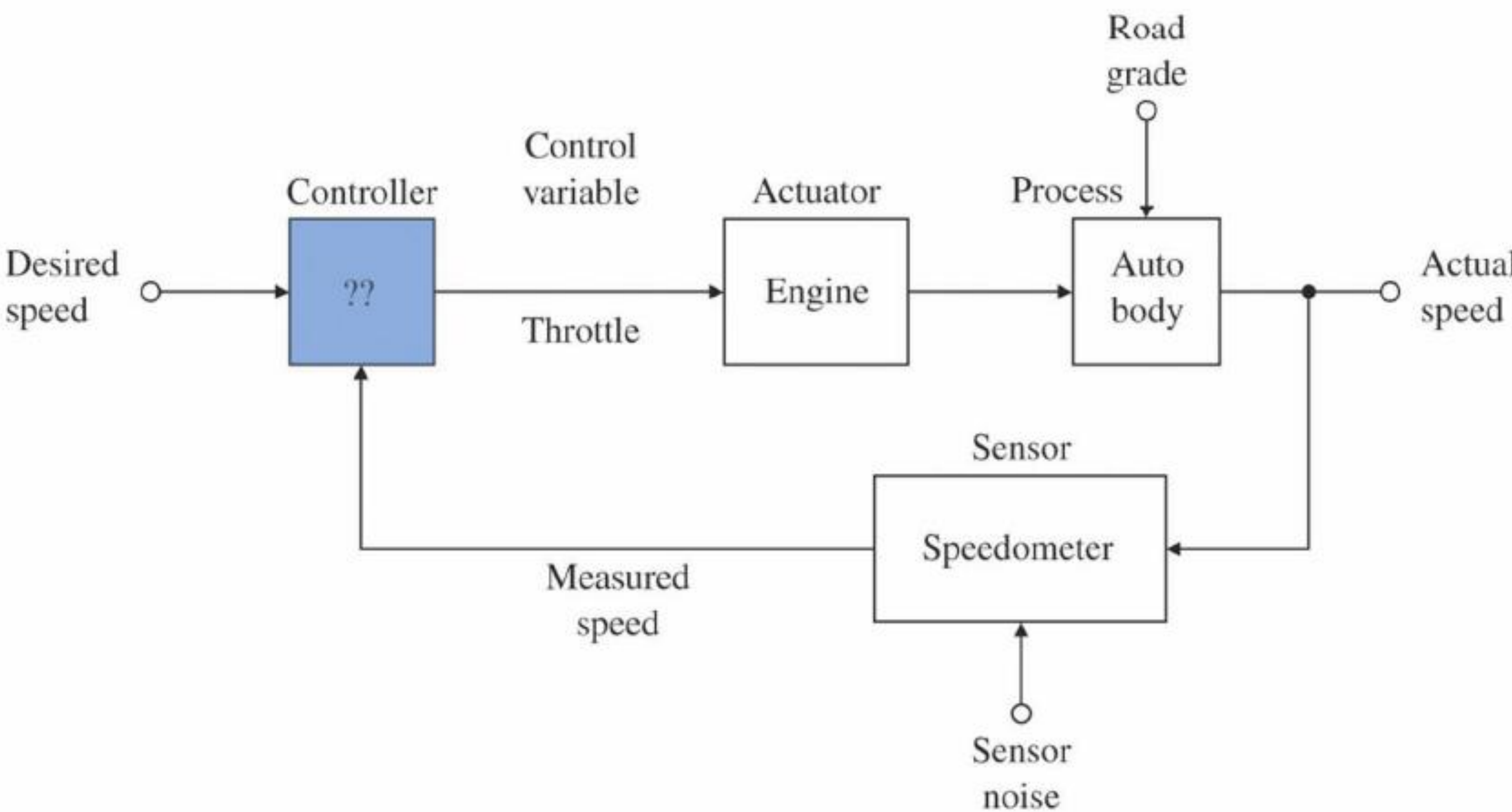
Let's discuss the definitions of open loop and closed loop control system by an exampe

- The figure shows a transfer function block for a car. The input, or control variable is the gas pedal angle. The system output, or result, is the velocity of the car. In standard operation the gas pedal angle is controlled by the driver. When a cruise control system is engaged the gas pedal must automatically be adjusted to maintain a desired velocity setpoint. To do this a control system is added, in this figure it is shown inside the dashed line. In this control system the output velocity is subtracted from the setpoint to get a system error. The subtraction occurs in the summation block (the circle on the left hand side). This error is used by the controller function to adjust the control variable in the system. Negative feedback is the term used for this type of controller



Note: The arrows in the diagram indicate directions so that outputs and inputs are unambiguous. Each block in the diagram represents a transfer function.

Component block diagram of automobile cruise control



- There are two main types of feedback control systems: **negative feedback** and **positive feedback**.

- In a positive feedback control system the setpoint and output values are added. *(Pozitif geri beslemede çıkış girişe aynı yönde etki eder.)*
- A Positive Feedback loop is less common, this is when a change starts, the nervous system recognizes the change, and then stimulates more hormones to accelerate the change.

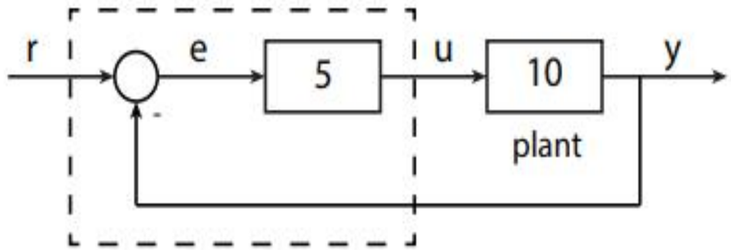
- In a negative feedback control the setpoint and output values are subtracted. As a rule negative feedback systems are more stable than positive feedback systems. Negative feedback also makes systems more immune to random variations in component values and inputs. (*Negatif geri beslemede çıkıştaki değişimler girişe ters yönde etki eder)*

- A Negative feedback loop is the most common – this occurs when there is a change in the body (i.e. blood glucose increases), the nervous system detects the change, and stimulates an anti-dote hormonal response. This reverses the effect back to homeostasis, and gives you a negative feedback loop.

Not:

- Negatif geri beslemede daima giriş ile çıkışın bir farkı alınır ve bu fark (-) yada (+) değerli olabilir. Denetim organına bir hata girişi olarak iletilen bu değer, yukarıda da açıklandığı gibi çıkışın istenen değere getirilmesi ni ve bu değerde sabit tutulmasını sağlar. Negatif geri besleme endüstriyel sistemlerin en önemli özelliğidir ve daima hatayı en küçük değerde tutmaya veya sıfır yapmaya çalışır.

- Pozitif geri beslemede çıkış girişe aynı yönde etki eder. Buna göre çıkışta herhangi bir artış meydana gelecek olursa bu giriş ile toplanarak hata sinyalinde bir artış dolayısıyla da denetim sinyalinde bir artış meydana getirir. Bu sistemde çıkışı daha da arttıracak yönde bir etki yaratır. Sonuçta artış sistemin fiziksel sınırlamalarına kadar devam eder ve sistem denetlenebilirliğini kaybeder. Pozitif geri besleme iç döngüler hariç bir kapalı döngü sisteminde kullanılamaz.



Note that the control gain (5) is arbitrary - we'll figure out how to chose it later.

How well do these control systems work?

Look at transfer functions from r to y:

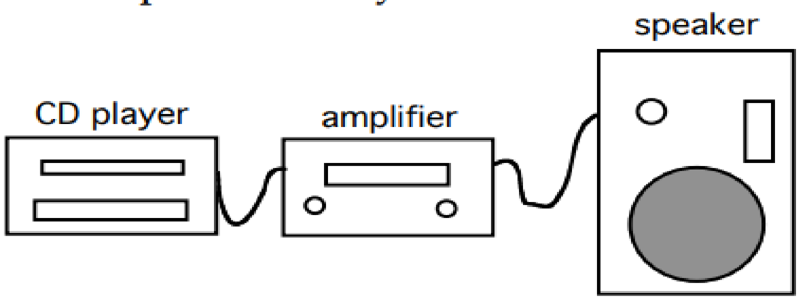
	Open Loop	Closed Loop
$\frac{y}{r}$	$\frac{1}{10} \cdot 10 = 1$	$\frac{5 \cdot 10}{1+5 \cdot 10} = \frac{50}{51} \approx 0.98$

We want $\frac{y}{r} = 1$, so at first glance, it looks like open-loop is better than closed-loop. However, consider what happens if it turns out our plant model was wrong (or changes), so that really $G = 15$. Then

	Open Loop	Closed Loop
$\frac{y}{r}$	$\frac{1}{10} \cdot 15 = 1.5$	$\frac{5 \cdot 15}{1+5 \cdot 15} = \frac{75}{76} \approx 0.9868$

That is, if the plant gain changes by 50%, the transfer function of the open-loop system will vary by 5-%. However, the transfer function of the closed-loop system will vary by only 0.66% (in this case).

- Example: stereo system



  - CD player→ amplifier→ speakers
  - CD player: $G_1(s) = \dfrac{X_1(s)}{H(s)}$
    - Input: *H(s)* from CD laser reader
    - Output: CD voltage $X_1(s)$
  - Power amplifier: $G_2(s) = \dfrac{X_2(s)}{X_1(s)}$
    - Input: CD output voltage $X_1(s)$
    - Output: amp voltage $X_2(s)$
  - Speakers: $G_3(s) = \dfrac{X(s)}{X_2(s)}$
    - Input: amp voltage $X_2(s)$
    - Output: sound, acoustic pressure $X(s)$

Advantages of control systems?

It makes possible to move large equipment with precision that would otherwise be impossible.

- We can point huge antennas toward the farthest reaches of the universe to pick up faint radio signals (controlling these antennas by hand is impossible)
- Elevators carry us quickly to our destination automatically stopping at the right floor
- We alone could not provide the power required for the load and the speed; motors provide the power and control systems regulate the position and speed

Advantages of controlled systems

We build control systems for;

- Power amplification
- Remote control
- Convenience of input form
- Compensation for disturbances

Advantages of controlled systems

Power amplification

- ***A radar antenna***; positioned by the low-power rotation of a knob at the input, requires a large amount of power for its output rotation. A control system can produce the needed power amplification, or power *gain*.

Advantages of controlled systems

Remote control

- Robots; designed by control system principles can compensate for human disabilities.
- A remote-controlled robot arm can be used to pick up material in a radioactive environments.
- Robots for mine disposal

Advantages of controlled systems

Convenience of input form

- In a temperature control system, the input is a position on a thermostat. The output is heat. Thus, a convenient position input yields a desired thermal output.

Advantages of controlled systems

Compensation for disturbances

Systems	Variables
Thermal systems	Temperature
Mechanical systems	Position and velocity
Electrical systems	Voltage, current or frequency

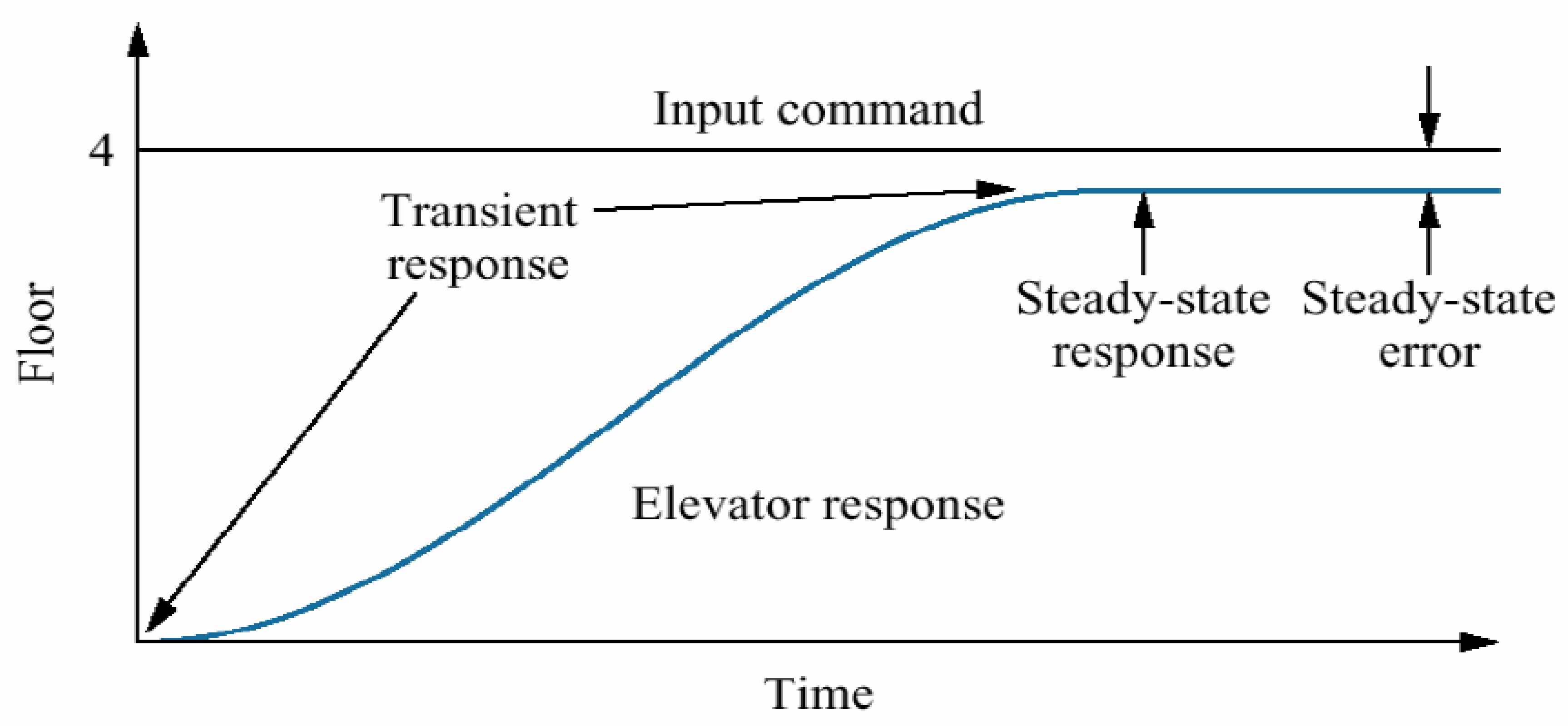
Response Characteristics and System Configurations

- A control system provides an output or response for a given input or stimulus.
- The input represents a desired response; the output is the actual response.

- For example:
- When the fourth floor button of an elevator is pushed on the ground floor, the elevator rises on the fourth floor with a speed and floor leveling accuracy designed for passenger comfort.

Response Characteristics and System Configurations

The elevator response



Response Characteristics and System Configurations

- The push of the fourth floor button is the *input command* and is represented by *the step command.*
- The input represents what we would like the output to be after the elevator to be after the elevator has stopped; the elevator itself follows the displacement described by the curve marked *elevator response*.

Response Characteristics and System Configurations

- Two factors make the output different from the input. First, compare the instantaneous change of the input against the gradual change of the input in the figure. Physical entities cannot change their state (such as position or velocity) instantaneously. The state changes through a path that is related to the physical device and the way it acquires or dissipates energy. Thus, the elevator undergoes a gradual change as it rises from the first floor to the fourth floor. We call this part of the response *transient response.*

Response Characteristics and System Configurations

- After the transient response, a physical system approaches its *steady-state* response, which is its approximation to the commanded or desired response. For the elevator example, this response occurs when the elevator reaches the fourth floor.
- The accuracy of the elevator's leveling with the floor is a second factor that could make the output different from the input. We call this difference *steady-state error.*
- Often, *steady-state* error is inherent in the designed system , and the control system engineer determines whether or not that error leads to significant degradation of system functions.
- For example, in a system tracking a satellite, some steady-state error can be tolerated as long as it is small enough to keep the satellite close to the center of the tracking radar beam. **However,** for a robot inserting a memory chip onto a board, the steady-state error must be zero.

The Design Process

- Control system analysis and design focuses on three primary objectives.

1. Producing the desired transient response
2. Reducing steady-state response
3. Achieving stability *(is independent from input and disturbance)*

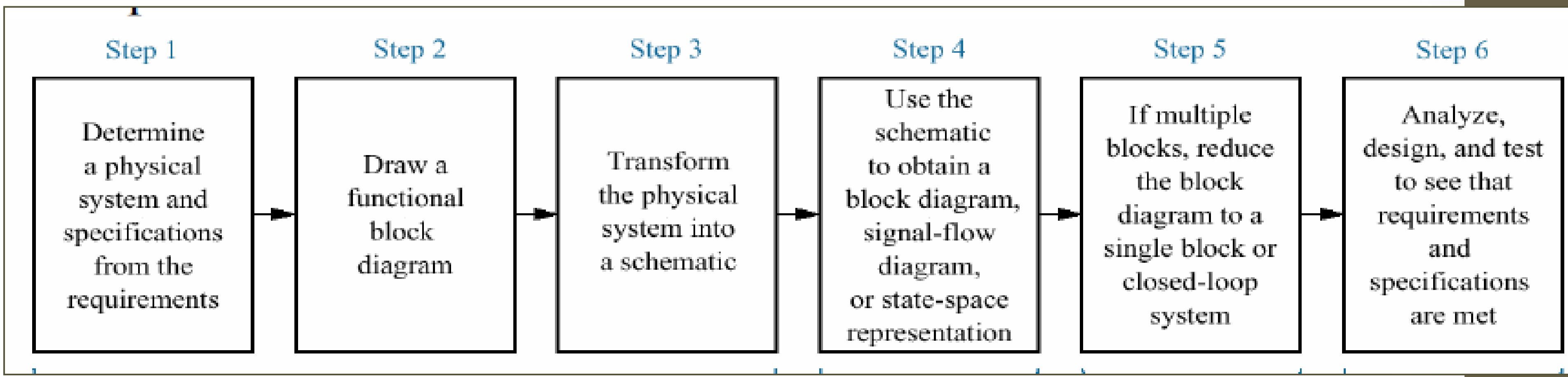
- A system must be stable in order to produce the proper transient and steady-state response. *Transient response is important because it affects the speed of the system* and influences human patience and comfort, not to mention mechanical stress. *Steady-state response determines the accuracy of the control system*; it governs how closely the output matches the desired response.

The Design Process

- The design of a control system follows these steps:

- ***Step 1:*** Determine a physical system and specifications from requirement
- ***Step 2:*** Draw a functional block diagram
- ***Step 3:*** Represent the physical system as a schematic
- ***Step 4:*** Use the schematic to obtain a mathematical model, such as a block diagram
- ***Step 5:*** Reduce the block diagram
- ***Step 6:*** Analyze and design the system to meet specified requirements and specifications that include stability, transient response, and steady-state performance

The Design Process



Antenna Azimuth (Example for position control system)

Position control systems find widespread applications in antennas, robot arms and computer disk drives.

How to change the azimuth angle precisely according to the motion?

Manually through the gears?

Too slow and imprecise..

Antenna Azimuth
(Example for position control system)

Use POT to convert angle command into voltage

Unrealistic arrangement! WHY?
Answer: Large motor requires a lot of power

So,

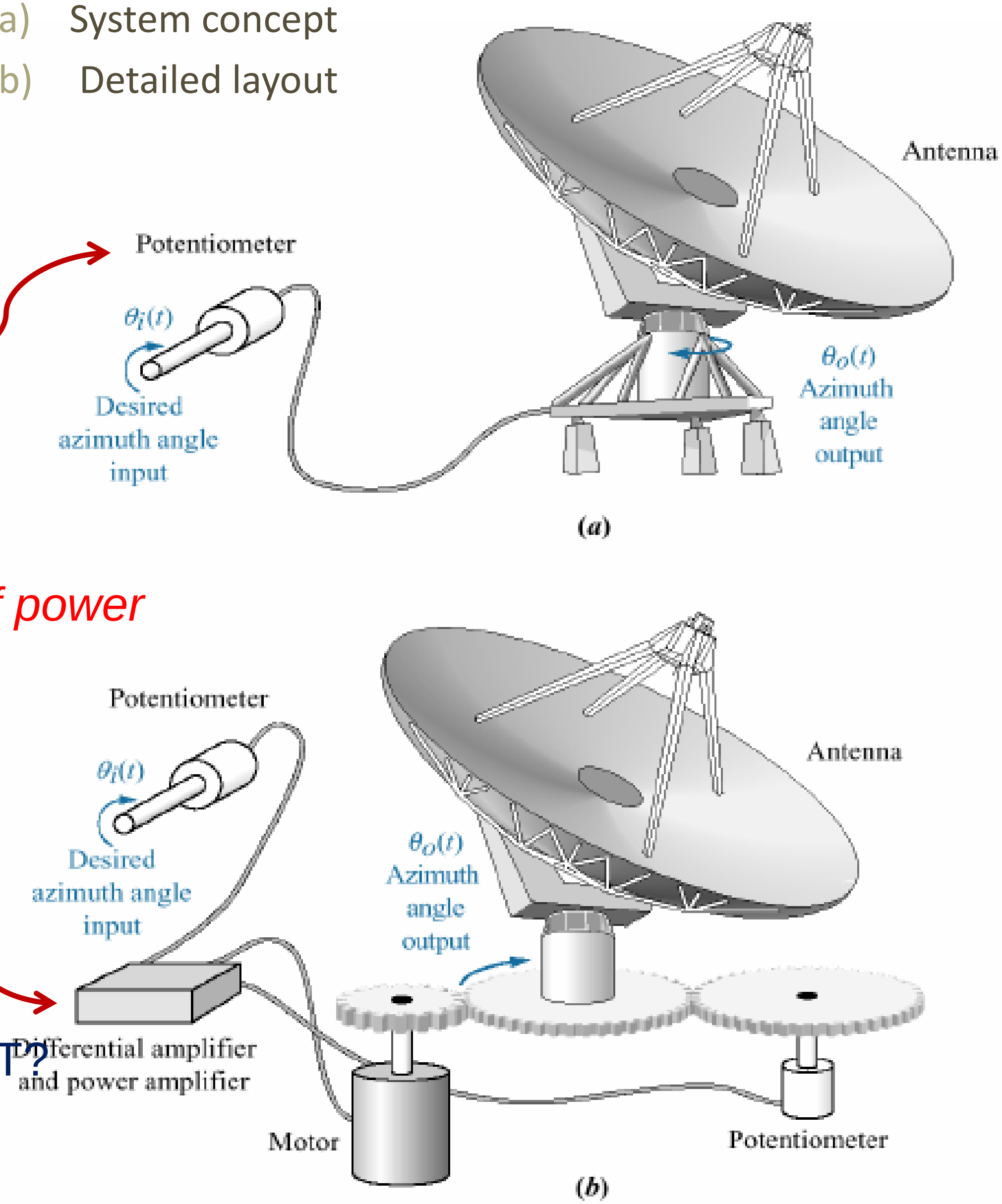
Use Power amplifier to magnify POT output voltage

STILL SOMETHING MISSING! WHAT?
Answer: Error Correction

"feedback"

Use another gear/pot to check whether actually the output is following the input. If not then use the difference/error to drive the system

a) System concept
b) Detailed layout

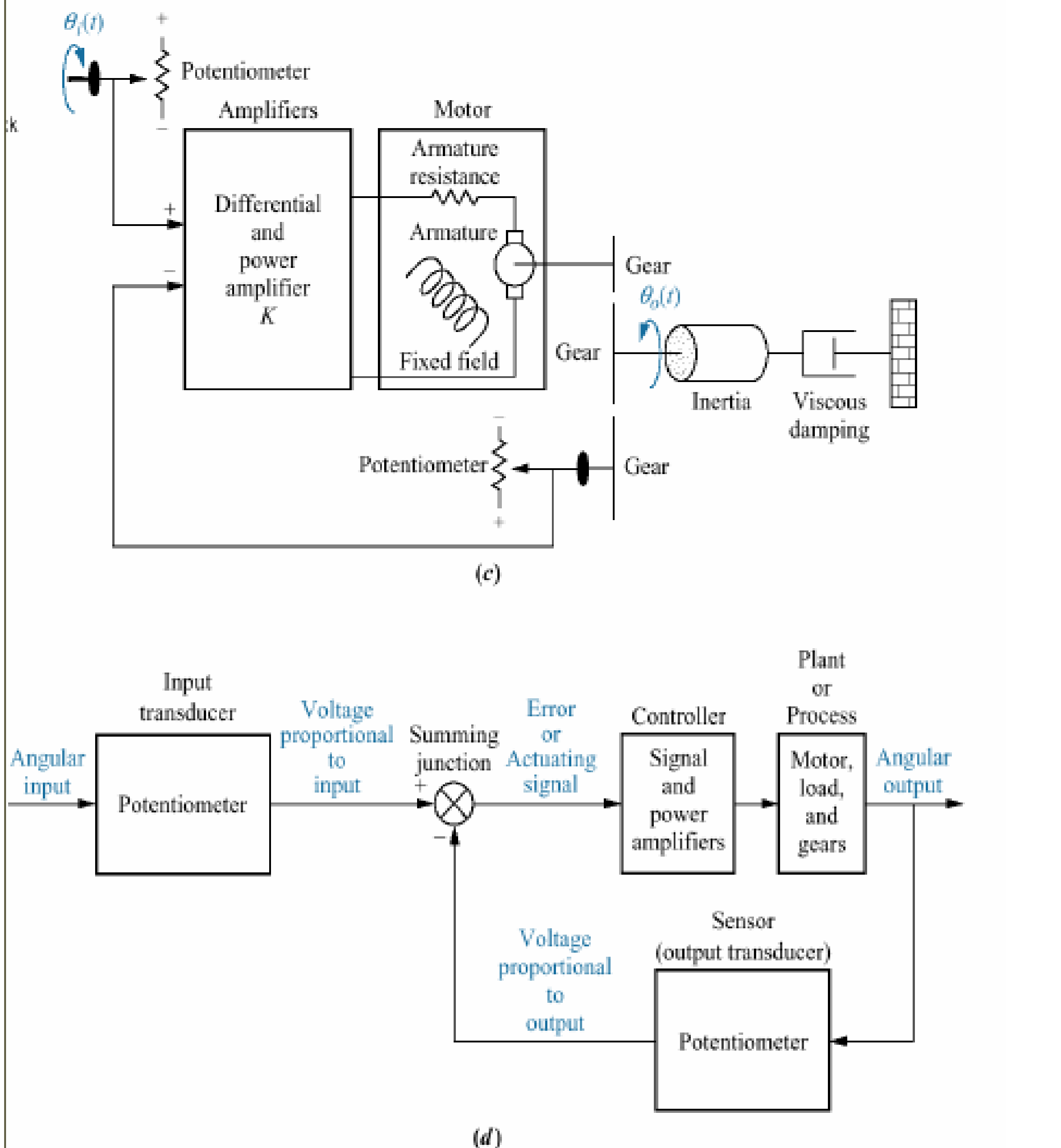


Antenna Azimuth (Example for position control system)

c) Schematic

d) Functional block diagram

- The input command is an angular displacement. The potentiometer converts the angular displacement into a voltage. Similarly, the output displacement is converted to a voltage by the potentiometer in the feedback path. The signal and power amplifiers boost the difference between the input and output voltages. This amplified actuating signal drives the plant.

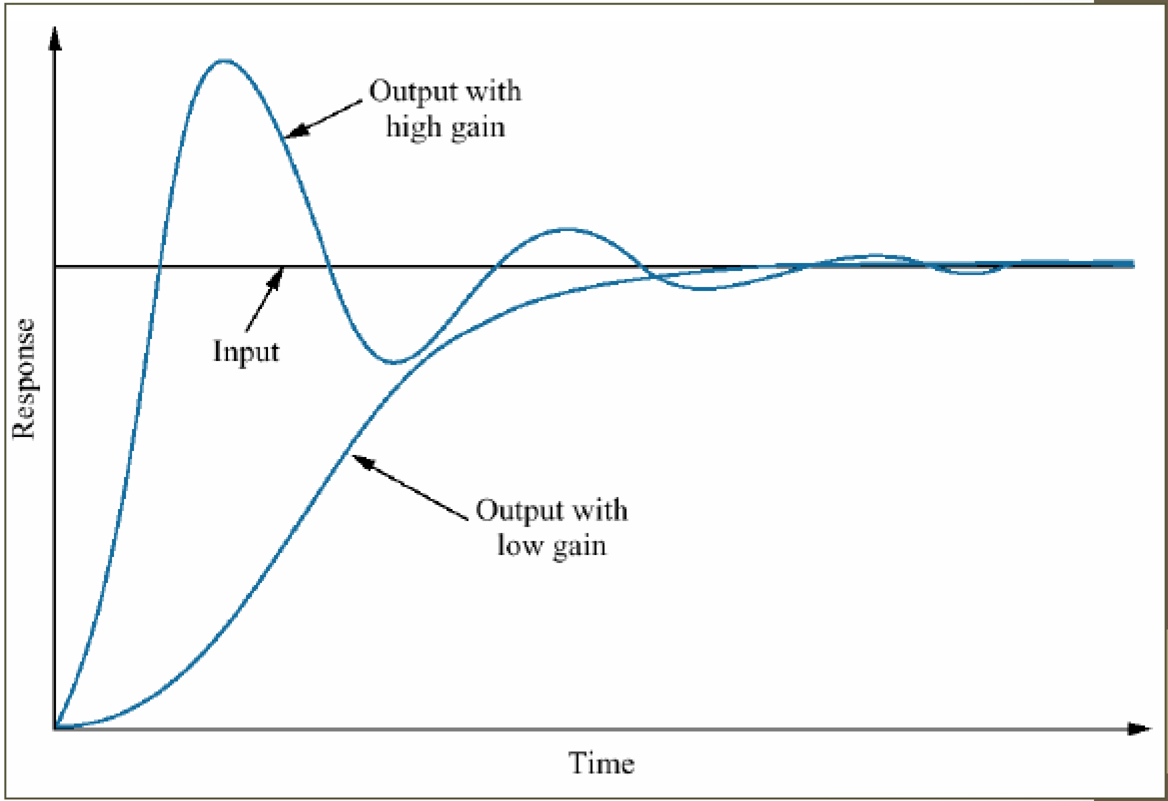


Antenna Azimuth (Example for position control system)

- When the input and output match, the output will be zero and the motor will not turn.
- Thus, the motor is driven when the output and input do not match. The greater the difference between the input and output, the larger the motor input voltage, and the faster the motor will turn.

Antenna Azimuth
(Example for position control system)

Response of a position control system, showing effect of high and low controller gain on the output response.



High gain

Test waveforms used in control systems

- Impulse function response
- (billard cue-billard ball)..response for the case that initial conditions are zero
- Football, pass and shoot the ball..response for the case that initials values are not zero.

TABLE 1.1 Test waveforms used in control systems

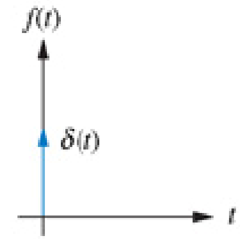
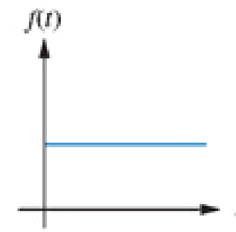
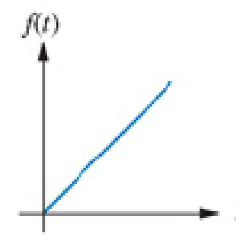
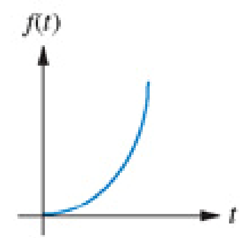
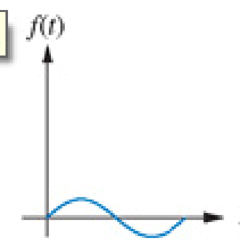
Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)\,dt = 1$		Transient response Modeling
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error

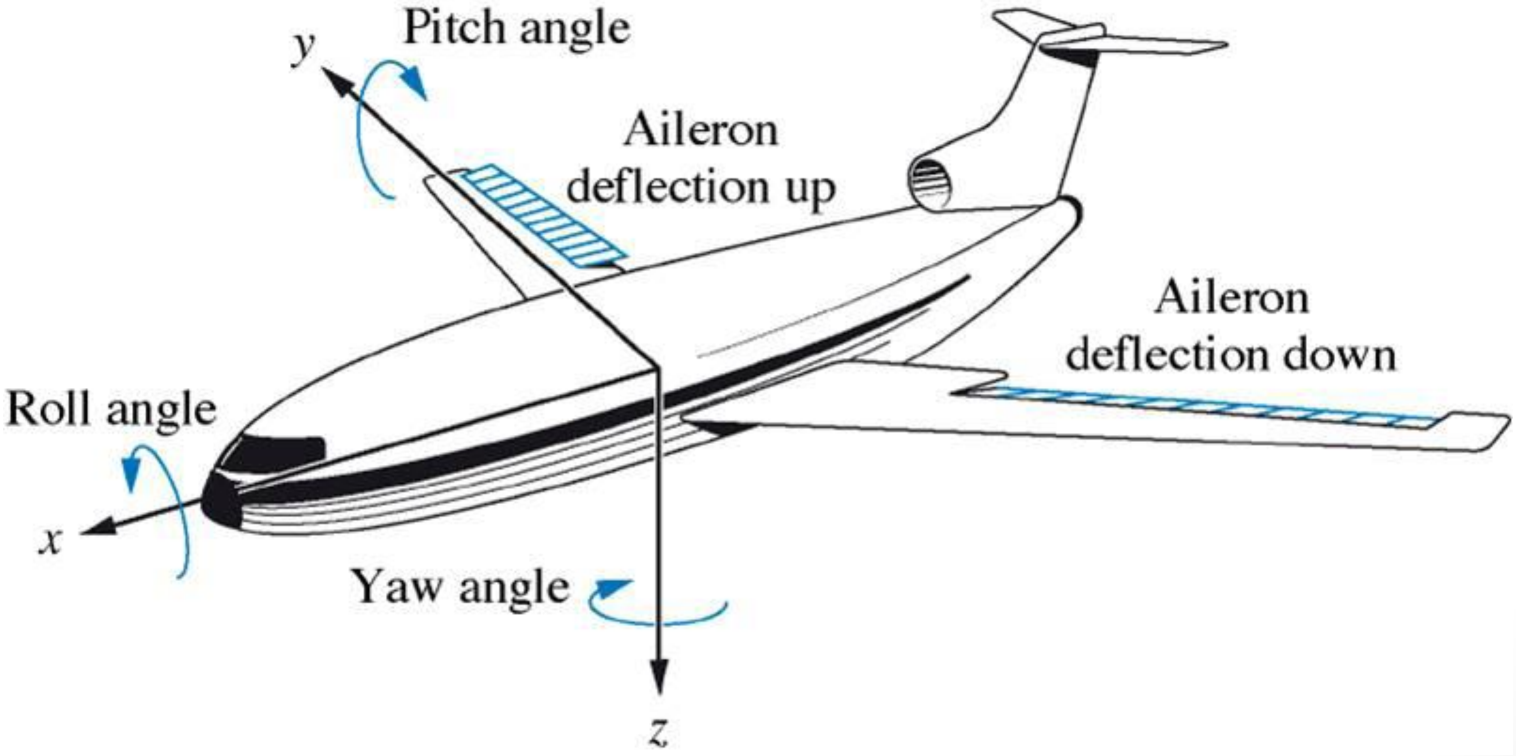
table_01_01

- **Test input signals** are used, both analytically and during testing, to verify the design.It is neither necessarily practical nor illuminating to choose complicated input signals to analyze a system's performance. Thus, the engineer usually selects standard test inputs.These inputs are impulses, steps, ramps, parabolas, and sinusoids, as shown in Table
- An ***impulse*** is infinite at $t = 0$ and zero elsewhere. The area under the unit impulse is 1. An approximation of this type of waveform is used to place initial energy into a system so that the response due to that initial energy is only the transient response of a system. From this response the designer can derive a mathematical model of the system.

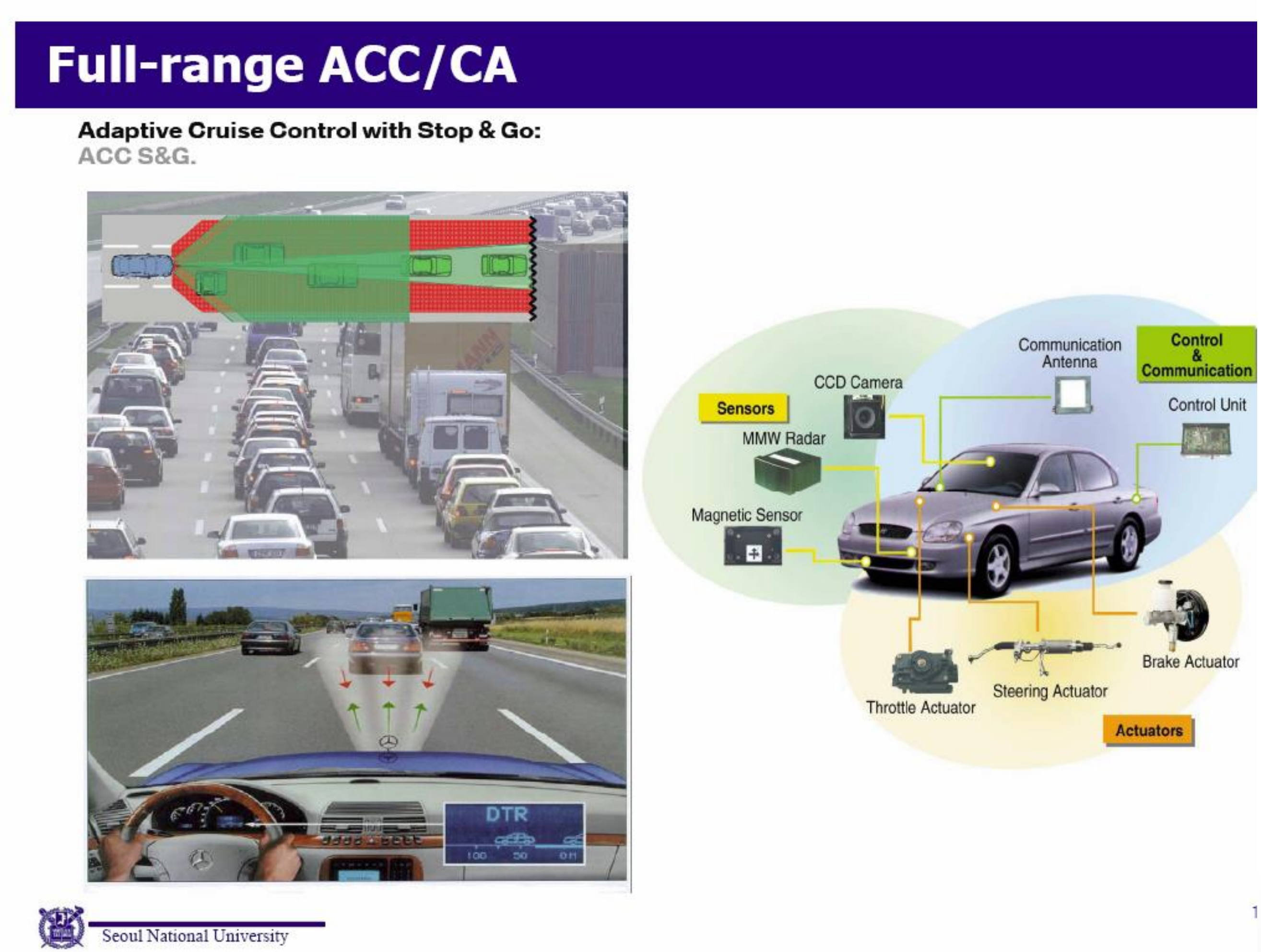
- A ***step*** input represents a *constant command,* such as position, velocity, or acceleration. Typically, the step input command is of the same form as the output. For example, if the system's output is position, as it is for the antenna azimuth position control system, the step input represents a desired position, and the output represents the actual position. If the system's output is velocity, as is the spindle speed for a video disc player, the step input represents a constant desired speed, and the output represents the actual speed. The designer uses step inputs because both the transient response and the steady-state response are clearly visible and can be evaluated

- The ***ramp*** input represents a *linearly increasing command.* For example, if the system's output is position, the input ramp represents a linearly increasing position, such as that found when tracking a satellite moving across the sky at constant speed. If the system's output is velocity, the input ramp represents a linearly increasing velocity. The response to an input ramp test signal yields additional information about the steady-state error. The previous discussion can be extended to *parabolic* inputs, which are also used to evaluate a system's steady-state error.

Examples of systems



Examples of systems:



Summary

Analysis and Design Objectives

Let's define our analysis and design objectives

1.) Transient Response : We analyze the system for its existing transient response. We then adjust parameters or design components to yield a desired transient response. (this is our first analysis and design objective)

2.) Steady-State Response : We are concerned about the accuracy of steady-state response. We analyze system's steady-state error, and then design corrective action to reduce steady-state error. (this is our second analysis and design objective)

Summary

3.) Stability : Discussion of transient response and steady state error is moot if the system does not have *stability*! For a linear sytem, we can write;

Total response = Natural response + Forced response

For a control systems to be useful, the natural response must eventually approach to zero, thus leaving only the forced response. If the natural response approaches to zero, we can say the system is "stable"

End of Lecture 1

Lecture 2

• The Laplace Transform

- It is an operational method for solving linear differential equations. It converts many common functions such as sinusoidal functions, damped sinusoidal functions and exponential functions into algebraic functions of a complex variable ***s***.
- Operations such as differentiation and integration can be replaced by algebraic operations in the complex plane. Thus, a linear dif. Equation can be transformed into an algebraic eq. in a a complex variable ***s***.
- If the algebraic equation in s is solved for the dependent variable, then the solution of the Dif. Eq. (the inverse Laplace transform of the dependent variable) may be found by use of a Laplace transform table.

Advantages of Laplace Transform

- It allows the use of graphical techniques for predicting the system performance without actually solving system Dif. Eqs.

- When one solves the dif. Eqs. Both transient component and steady-state component of the solution can be obtained simultaneously.

TABLE 2.1 **Laplace transform table**

table_02_01

Item no.	$f(t)$	$F(s)$
1.	$\delta(l)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$l^n u(t)$	$\frac{n1}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

TABLE 2.2 **Laplace transform theorems**

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) \;= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] \;= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] \;= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] \;= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] \;= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] \;= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] \;= s^2F(s) - sf(0-) - \dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] \;= s^nF(s) - \sum_{k=1}^{n} s^{n-k} f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{1} f(\tau)d\tau\right] \;= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) \;= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) \;= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]