1

Automatic Control

WEEK 1

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Grading

Midterm Exam	:	30%
Quizzes	:	30%
Final Exam	:	40%
Total	:	100%

Control Systems

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 5 = e^{3t}$$

$y(t) = ?$

TF

Open loop - Closed loop

Feedback Control

Laplace Transform

Inverse Laplace T.

1) Introduction & Basic Concepts
2) Modeling In The Frequency Domain
3) Modeling In The Time Domain
4) Time Response
5) Reduction Of Multiple Subsystems
6) Stability

Mathematical modeling of the Physical Systems

Electrical systems

Mechanical systems

→ Block Diagram of Closed Loop Sys.

↳ Electrical System

↳ Mechanical System

$\left.\right\}$ Reduction of Block Diagrams

→ Signal Flow Graphs (Mason)

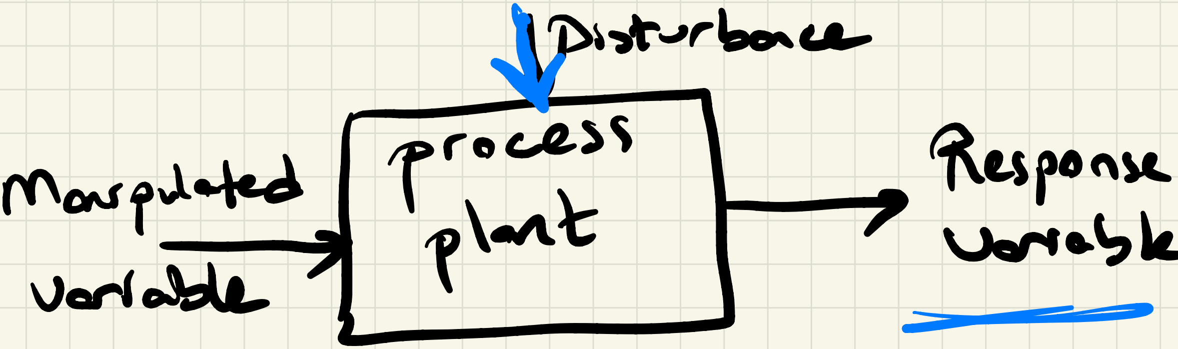
→ Stability

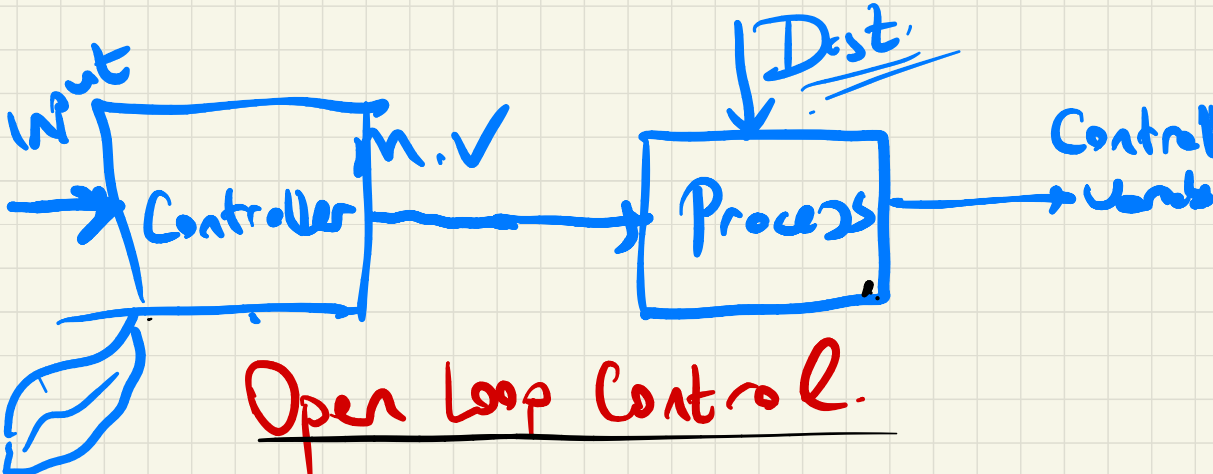
→ State-Space Analysis

What is "Control"?

- Make some object (called ***system, or plant***) behave as we desire.
- Imagine "control" around you!
  - Room temperature control
  - Car/bicycle driving
  - Voice volume control
  - "Control" (move) the position of the pointer
  - Cruise control or speed control
  - Process control
  - etc.

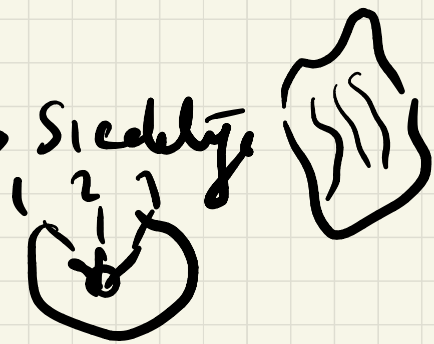
process
plant
system



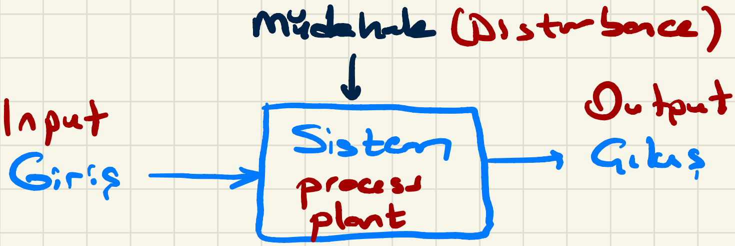


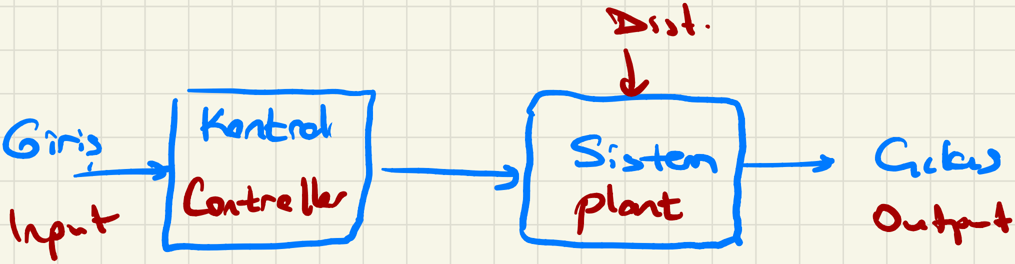
Open Loop Control

* Daktilo
* Tost Makinesi → Sıcaklık



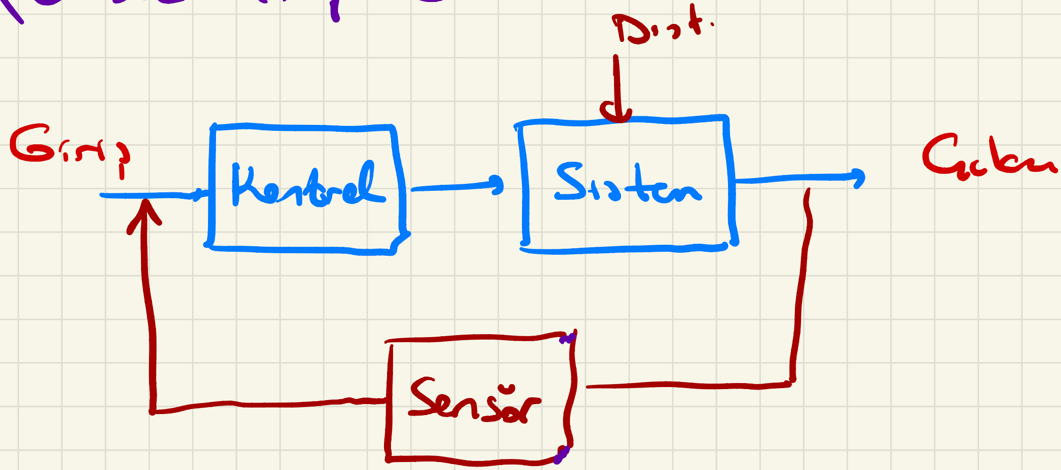
* Çamaşır Makinesi

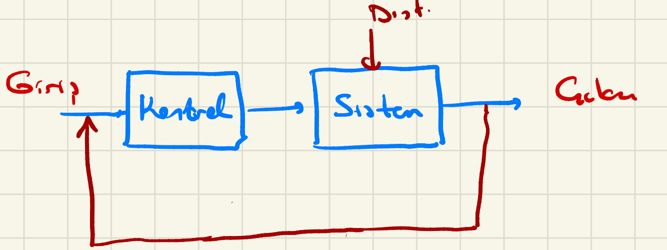


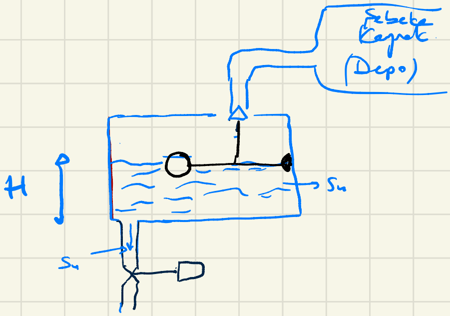


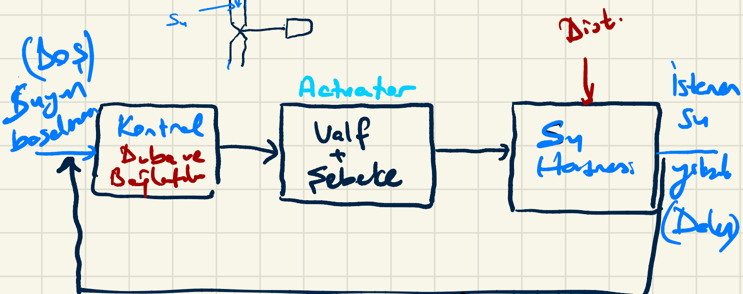
Açık Çevrim Kontrol Sistemi
(Open loop Control System)

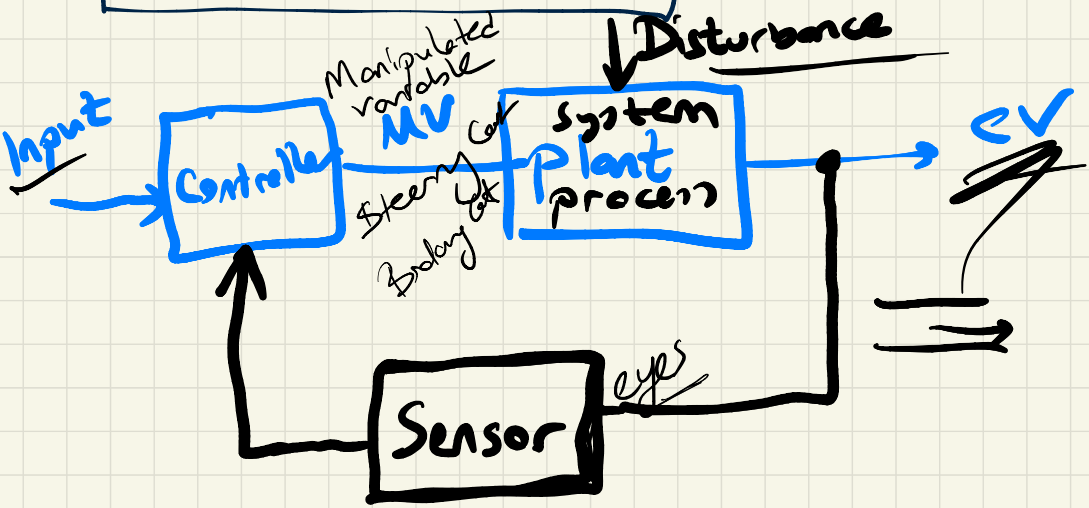
Kapalı Çevrim Kontrol Sistemi
(Closed loop Control System)





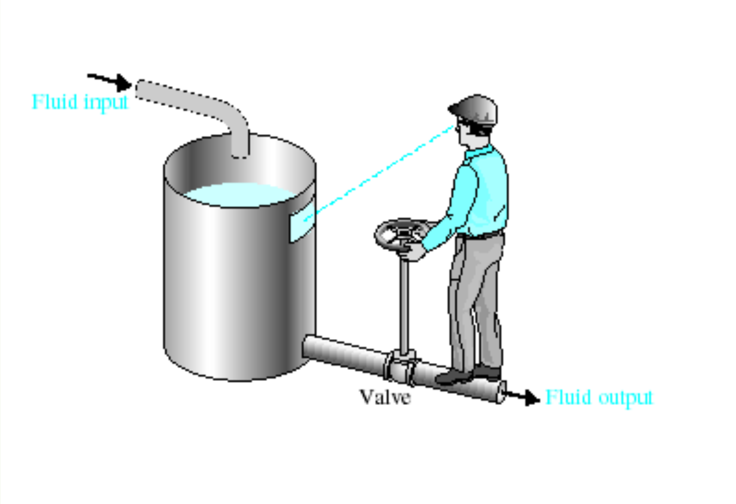






Closed Loop Systems

Feedback Systems



A manual control system for regulating the level of fluid in a tank by adjusting the output valve. The operator is watching the level of fluid through a port in the side of the tank.

What is “Automatic Control”?

- Not manual!
- Why do we need automatic control?
  - Convenient (room temperature control, laundry machine)
  - Dangerous (hot/cold places, space, bomb removal)
  - Impossible for human (nanometer scale precision positioning, work inside the small space that human cannot enter, huge antennas control, elevator)
  - It exists in nature. (human body temperature control)
  - High efficiency (engine control)
- Many examples of automatic control around us

Why Automatic Control ?

Automatic control of many day to day tasks relieves the human beings from performing repetitive manual operations. Automatic control allows optimal performance of dynamic systems, increases productivity enormously, removes drudgery of performing same task again and again.

TERMINOLOGY

Plant or Process: System to be controlled

Inputs: Excitations (known, unknown) to the system

Outputs: Responses of the system

Sensors: They measure system variables (excitations, responses, etc.)

Actuators: They drive various parts of the system.

Controller: Device that generates control signal

Control Law: Relation or scheme according to which the control signal is generated

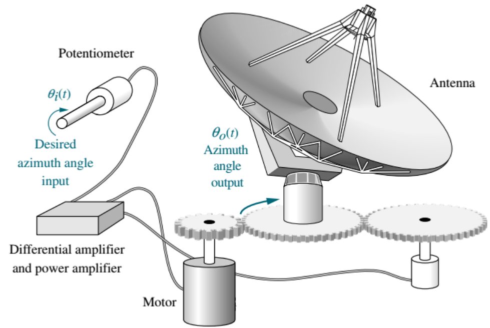
Control System: Plant + controller, at least
(Can include sensors, signal conditioning, etc.)

Feedback Control: Control signal is determined
according to plant "response"

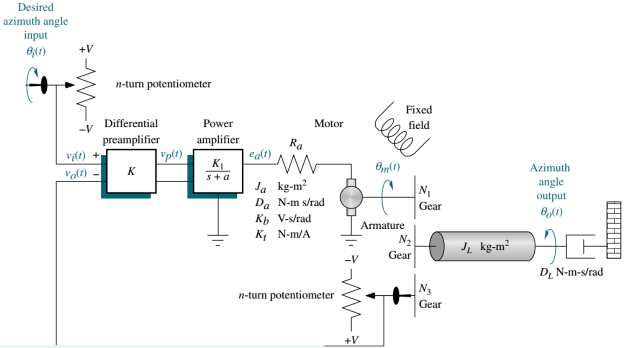
Open-loop Control: No feedback of plant response to controller

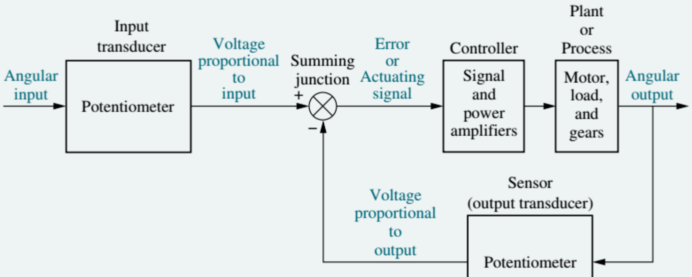
Antenna Azimuth Position Control System

Layout

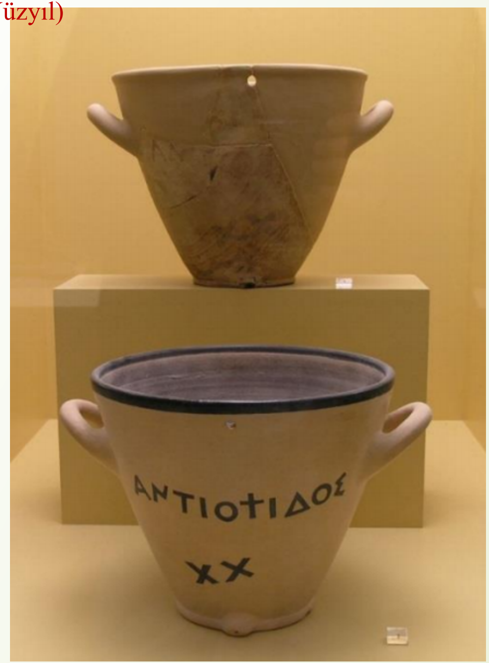


Schematic



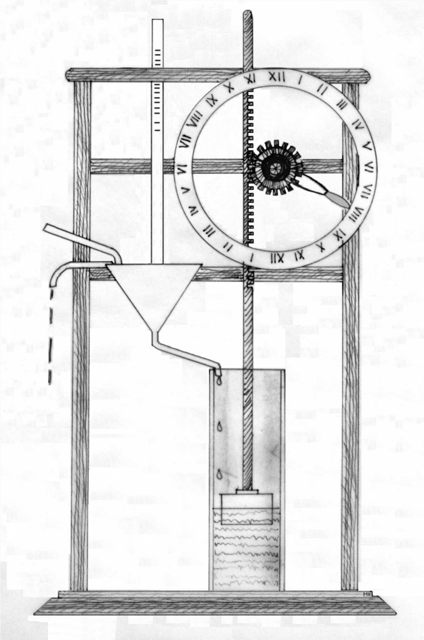


Küçük Ölçekli Su Saati (Atina, M.Ö. 5. Yüzyıl)



Büyük Ölçekli Su Saati (Rodos, M.Ö. 50)

Ctesibius's 285–222 BC clepsydra (water thief)



In both Greek and Roman times, this type of clepsydra was used in courts for allocating periods of time to speakers. In important cases, when a person's life was at stake for example, it was filled. But, for more minor cases, it was only partially filled. If proceedings were interrupted for any reason, such as to examine documents, the hole in the clepsydra was stopped with wax until the speaker was able to resume his pleading.

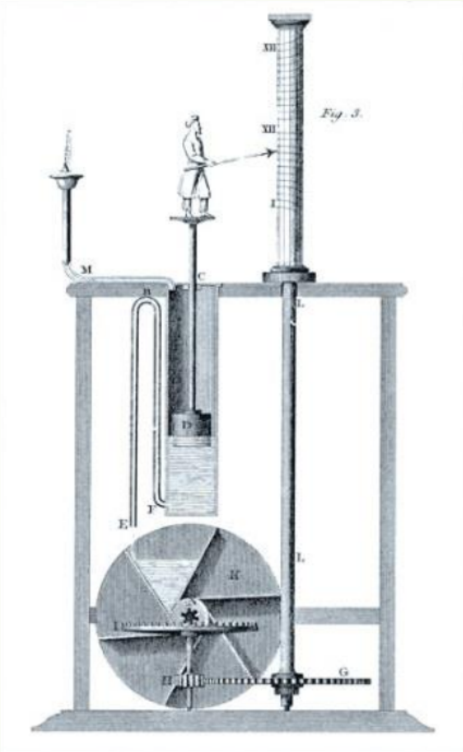
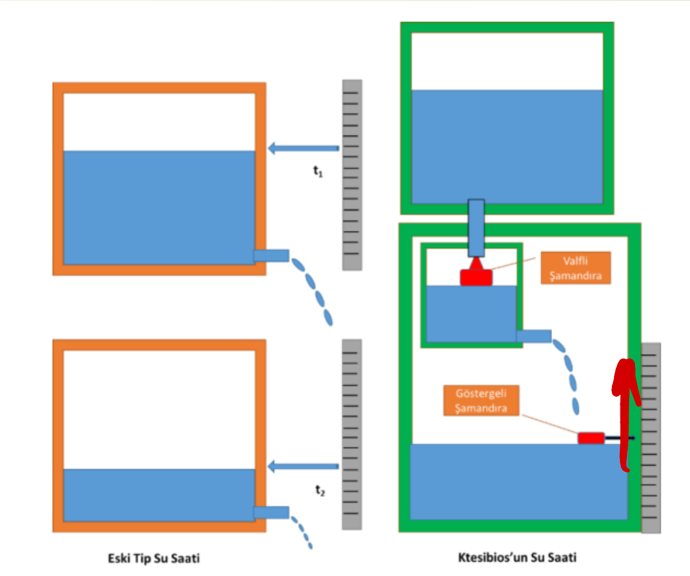
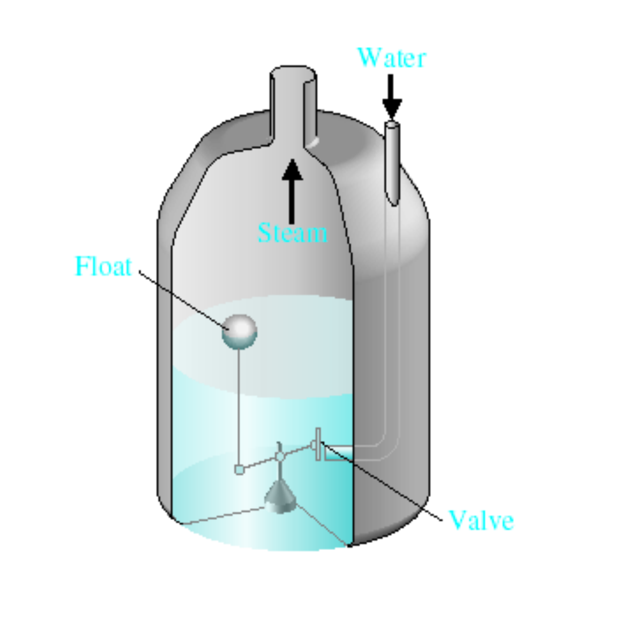


Fig. 3.
M
B
C
L
D
E
F
K
L
G
H

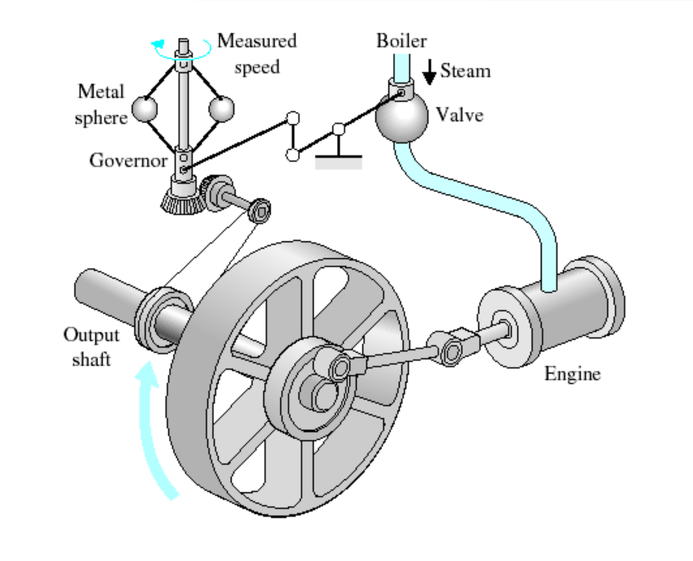

t_1
t_2
Valfli Şamandıra
Göstergeli Şamandıra
Eski Tip Su Saati
Ktesibios'un Su Saati

Polzunov’s water-level float regulator 1765



The first historical feedback system claimed by Russia was developed by Polzunov in 1765. Polzunov’s water-level float regulator employs a float that rises and lowers in relation to the water level, thereby controlling the valve that covers the water inlet in the boiler.

Watt’s Flyball Governor (18th century)



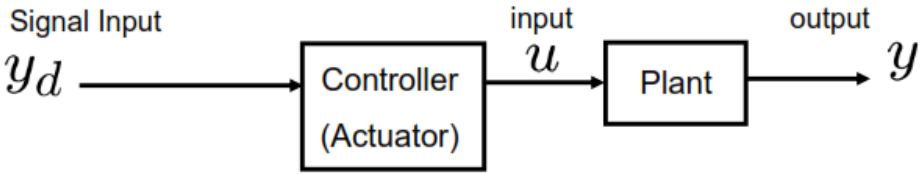
James Watt designed his first governor in 1788 following a suggestion from his business partner Matthew Boulton. It was a conical pendulum governor and one of the final series of innovations Watt had employed for steam engines. James Watt never claimed the centrifugal governor to be an invention of his own. Centrifugal governors were used to regulate the distance and pressure between millstones in windmills since the 17th century. It is therefore a misunderstanding that James Watt is the inventor of this device.

Open-Loop Systems: A system in which the output has no effect on the control action is known as an open loop control system. For a given input the system produces a certain output. If there are any disturbances, the out put changes and there is no adjustment of the input to bring back the output to the original value. A perfect calibration is required to get good accuracy and the system should be free from any external disturbances. No measurements are made at the output.

Disadvantage of open-loop systems is that they are poorly equipped to handle disturbances or changes in the conditions which may reduce its ability to complete the desired task.

Open-Loop Control

- Open-loop Control System
  - Toaster, microwave oven, shooting a basketball



  - Calibration is the key!
  - Can be sensitive to disturbances

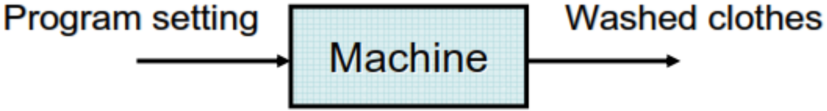
Example: Toaster

- A toaster toasts bread, by setting timer.

- Objective: make bread golden browned and crisp.
- A toaster does not measure the color of bread during the toasting process.
- For a fixed setting, in winter, the toast can be white and in summer, the toast can be black (Calibration!)
- A toaster would be more expensive with sensors to measure the color and actuators to adjust the timer based on the measured color.

Example: Laundry machine

- A laundry machine washes clothes, by setting a program.



- A laundry machine does not measure how clean the clothes become.
- Control without measuring devices (sensors) are called ***open-loop control***.

We can define the main characteristics of an “Open-loop system” as being:

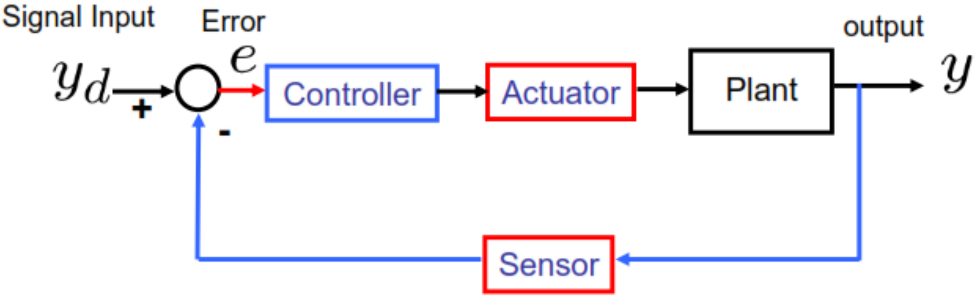
- There is no comparison between actual and desired values.
- An open-loop system has no self-regulation or control action over the output value.
- Each input setting determines a fixed operating position for the controller.
- Changes or disturbances in external conditions does not result in a direct output change. (unless the controller setting is altered manually)

Closed-Loop Systems: These are also known as feedback control systems. A system which maintains a prescribed relationship between the controlled variable and the reference input, and uses the difference between them as a signal to activate the control, is known as a feedback control system.

The output or the controlled variable is measured and compared with the reference input and an error signal is generated. This is the activating signal to the controller which, by its action, tries to reduce the error. Thus the controlled variable is continuously fedback and compared with the input signal. If the error is reduced to zero, the output is the desired output and is equal to the reference input signal.

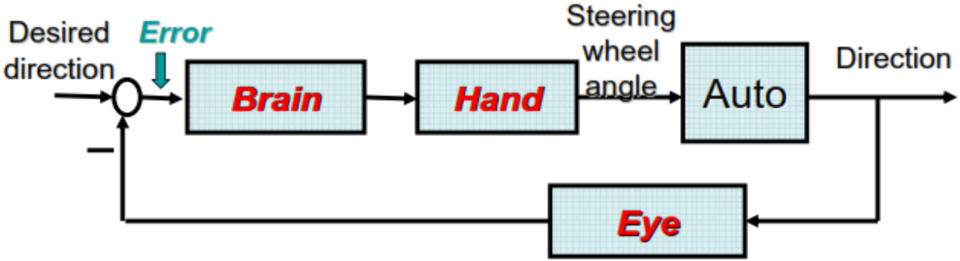
Closed-Loop (Feedback) Control

- Compare actual behavior with desired behavior
- Make corrections based on the error
- The sensor and the actuator are key elements of a feedback loop
- Design control algorithm



Ex: Automobile direction control

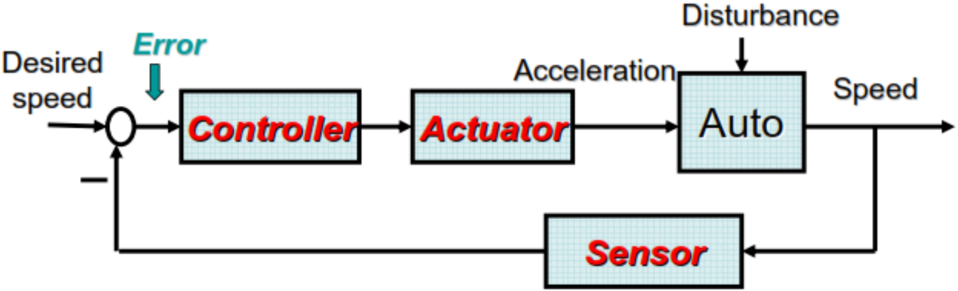
- Attempts to change the direction of the automobile.



- Manual closed-loop (***feedback***) control.
- Although the controlled system is “Automobile”, the ***input*** and the ***output*** of the system can be different, depending on ***control objectives***!

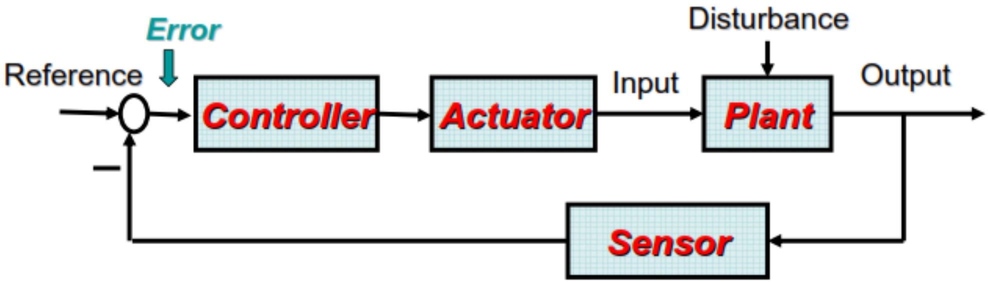
Ex: Automobile cruise control

- Attempts to maintain the speed of the automobile.



- Cruise control can be both manual and automatic.
- Note the similarity of the diagram above to the diagram in the previous slide!

Basic elements in feedback control systems



Control system design objective

To design a controller s.t. the output follows the reference in a "satisfactory" manner even in the face of disturbances.

Open Loop vs Closed Loop Control Systems

Open Loop Systems

Advantages

1. They are simple and easy to build.
2. They are cheaper, as they use less number of components to build.
3. They are usually stable.
4. Maintenance is easy.

Disadvantages

1. They are less accurate.
2. If external disturbances are present, output differs significantly from the desired value.
3. If there are variations in the parameters of the system, the output changes.

Open Loop vs Closed Loop Control Systems

Closed Loop Systems

Advantages

1. They are more accurate.

2. The effect of external disturbance signals can be made very small.

3. The variations in parameters of the system do not affect the output of the system

4. Speed of the response can be greatly increased.

Disadvantages

1. They are more complex and expensive

2. They require higher forward path gains.

3. The systems are prone to instability. Oscillations in the output many

OTOMATİK KONTROL

DERS 2

13. 10. 2023

MATEMATİKSEL TEMELLER

- Karmaşık sayılar ve değişkenler $\sqrt{-1} = i = j$
- Laplace Trans. — Ters Laplace Trans.
- Adi Dif. Denklem Çözümü

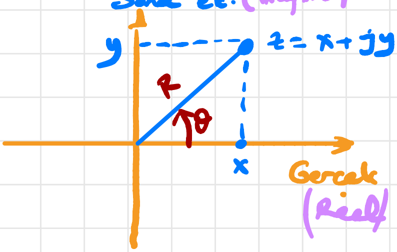
?) $$\frac{d^2x(t)}{dt^2} - 2\frac{dx(t)}{dt} + x(t) = e^t \qquad x(t) = ?$$

$$x(0) = -2 \qquad \frac{dx(t)}{dt} = -3$$

$$i_c = C\cdot\frac{dv_c(t)}{dt}$$

Karmaşık Sayılar

$z = x + jy$ $\qquad j = \sqrt{-1}$



R : z nin büyüklüğü (magnitude)

θ : Faz

$$R = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$z = x + jy = R\angle\theta$$

$$z = R\cos\theta + jR\sin\theta$$

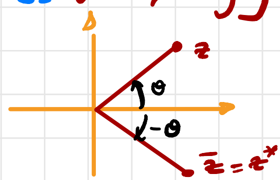
$e^{j\theta} = \cos\theta + j\sin\theta$ ⟶ Euler formülü

$$z = Re^{j\theta} = R\angle\theta$$

$z = x + jy$

Complex Conj. (Eşlenik)

$$\bar{z} = z^* = x - jy$$



$$z^* = R\cos\theta - jR\sin\theta$$

$$z^* = Re^{-j\theta}$$

Note:

$$z.z^* = R^2 = x^2 + y^2$$

$$z_1 = x_1 + jy_1 \qquad z_1 = R_1 e^{j\theta_1} \rightarrow \boxed{\begin{array}{l} R_1 = \sqrt{x_1^2 + y_1^2} \\ \theta_1 = \arctan\frac{y}{x} \end{array}}$$

$$z_2 = x_2 + jy_2 \qquad z_2 = R_2 e^{j\theta_2} \rightarrow \boxed{\begin{array}{l} R_2 = \sqrt{x^2 + y^2} \\ \theta_2 = \arctan\frac{y}{x} \end{array}}$$

$$\frac{z_1}{z_2} = \frac{R_1 \angle \theta_1}{R_2 \angle \theta_2} = \frac{R_1}{R_2} \angle \theta_1 - \theta_2$$

$$z_1 \cdot z_2 = R_1 \angle \theta_1 \, R_2 \angle \theta_2 = R_1 . R_2 \angle \theta_1 + \theta_2$$

$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j = \sqrt{-1}$$

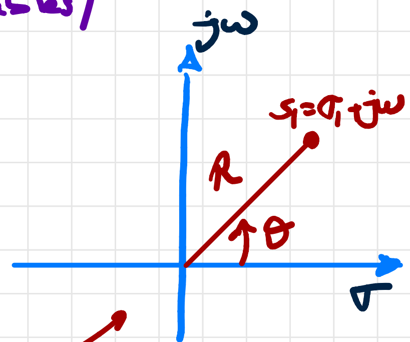
Karmaşık Değişkenler (Complex Variables)

$s \rightarrow$ Karmaşık değişken

$s = \sigma + j\omega$

$\sigma \rightarrow$ Gerçek eleman

$\omega \rightarrow$ İmajiner eleman



Karmaşık s-düzlemi (s-plain)

TRANSFER FONKSİYONU

Lineer Zamanla Değişmeyen   LZD = LTI

(Linear Time Invariant)

$$u(t) \longrightarrow \boxed{\begin{array}{c} LZD \\ g(t) \end{array}} \longrightarrow y(t)$$

$$G(s) = \mathcal{L}\{g(t)\}$$

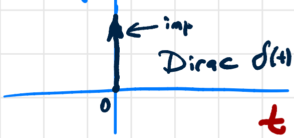
$$G(s) = \frac{Y(s)}{U(s)}$$

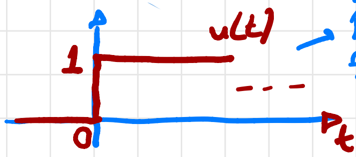
$Y(s)$: Girişin Laplace Transform

$U(s)$: Çıkışın L.T

$G(s)$: Transfer Fonk

Impulse func





→ Birim Basamak Fonk.

$$\frac{du(t)}{dt} = \delta(t)$$

$$G(s) = \frac{Y(s)}{U(s)}$$ Transfer Fonksiyonu

$Y(s)$ = Çıkışın Laplace Transform

$U(s)$ = Girişin Laplace Transform

LAPLACE TRANSFORM

* Adi Dif. Denklem çözümünde kullanılır.

$f(t)$ gerçek fonksiyon

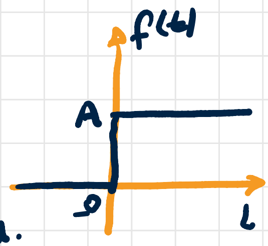
$$\int_0^\infty |f(t)e^{-\sigma t}|\, dt < \infty$$

$\sigma \rightarrow$ gerçek ve sonlu

$$F(s) = \int_{0^-}^{+\infty} f(t)e^{-st}\, dt = \mathcal{L}\{f(t)\}$$ = $f(t)$'nin Laplace Transform

Ex: $f(t)$ basamak fonk.

$$f(t) = \begin{cases} A \,; & t \geq 0 \\ 0 \,; & t < 0 \end{cases}$$



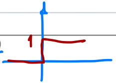
Buna göre $f(t)$ nin Laplace'ını bul.

Sol: $$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\, dt$$

$$F(s)=\int_0^{\infty} f(t)e^{-st}\,dt=\int_0^{\infty} A\,e^{-st}\,dt$$

$$F(s)=-\frac{A}{s}e^{-st}\Big|_0^{\infty}=\left(-\frac{A}{s}\cancel{e^{-s\cdot\infty}}^{\,0}\right)-\left(-\frac{A}{s}\cancel{e^{-s\cdot 0}}^{\,1}\right)\Rightarrow F(s)=\frac{A}{s}$$

Table 2.3 Important Laplace Transform Pairs



$f(t)$	$F(s)$
Step function, $u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
$f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-) - \ldots - f^{(k-1)}(0^-)$
$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(s)}{s} + \frac{1}{s} \int_{-\infty}^{0} f(t)\, dt$
Impulse function $\delta(t)$	1
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{1}{\omega}\left[(\alpha-a)^2+\omega^2\right]^{1/2} e^{-at} \sin(\omega t + \phi)$, $\phi = \tan^{-1} \frac{\omega}{\alpha - a}$	$\frac{s+\alpha}{(s+a)^2+\omega^2}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$, $\zeta < 1$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
$\frac{1}{a^2+\omega^2} + \frac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at} \sin(\omega t - \phi)$, $\phi = \tan^{-1} \frac{\omega}{-a}$	$\frac{1}{s\left[(s+a)^2+\omega^2\right]}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t + \phi\right)$, $\phi = \cos^{-1}\zeta$, $\zeta < 1$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
$\frac{\alpha}{a^2+\omega^2} + \frac{1}{\omega}\left[\frac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2} e^{-at} \sin(\omega t + \phi)$. $\phi = \tan^{-1}\frac{\omega}{\alpha - a} - \tan^{-1}\frac{\omega}{-a}$	$\frac{s+\alpha}{s\left[(s+a)^2+\omega^2\right]}$

BAZI ÖNEMLİ LAPLACE DÖNÜŞÜMLERİ

$$s \equiv \frac{d}{dt}$$

$$\frac{1}{s} \equiv \int_0^t dt$$

*** $$\mathcal{L}\left\{ \frac{df(t)}{dt} = f'(t) = \dot{f} \right\} = sF(s) - f(0)$$

$$\mathcal{L}\left\{ \frac{d^2 f(t)}{dt^2} \right\} = s^2 F(s) - sf(0) - f'(0)$$

$$G(s) = \frac{p(s) \rightarrow \text{sıfır (zeros)}}{q(s) \rightarrow \text{kutup (poles)}}$$

(Transfer Fonksiyonu)

$q(s) \rightarrow$ Karakteristik Denklem dir.

TERS LAPLACE → Heaviside → Ele.. Eş.

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - jw}^{\sigma + jw} F(s) e^{st} ds = \mathcal{L}^{-1}\{F(s)\}$$

1. Dereceden lineer sistem Dif. Denklemi

$$\frac{dy(t)}{dt} + a_0 y(t) = f(t)$$

2. Dereceden lineer sistem Dif. Denk.

$$\frac{d^2y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = f(t)$$

Çözümleri Laplace ile yapılacak.

Ex:

$$\frac{d^2x(t)}{dt^2} - 2\frac{dx(t)}{dt} + x(t) = e^t \qquad x(t) = ?$$

$$x(0) = -2 \qquad \frac{dx(t)}{dt} = -3 \Big\} \text{ Başlangıç koşulları}$$

$$\mathcal{L}\left\{ \ddot{x}(t) - 2\dot{x}(t) + x(t) \right\} = \left\{ \left(s^2X(s) - s\,x(0) - x'(0)\right) \right.$$

$$\left. - 2\left(sX(s) - x(0)\right) + X(s) \right\}$$

$$\mathcal{L}\left\{ e^t \right\} = \frac{1}{s-1}$$

$$\left(s^2X(s) - s\,\overset{-2}{x(0)} - \overset{-3}{x'(0)} - 2\left(sX(s) - \overset{-2}{x(0)}\right) + X(s) = \frac{1}{s-1}$$

$$X(s)\left(s^2 - 2s + 1\right) + 2s - 1 = \frac{1}{s-1}$$

$$X(s)\,(s-1)^2 = \frac{1}{s-1} - 2s + 1$$

$$\mathcal{L}\left\{ \frac{n!}{(s-a)^{n+1}} \right\} = e^{at} \cdot t^n$$

$$X(s) = \frac{1}{(s-1)^3} + \frac{-2s+1}{(s-1)^2}$$

$$\mathcal{L}^{-1}\left\{ \frac{1}{(s-1)^3} \right\} = \mathcal{L}^{-1}\left\{ \frac{1}{2}\,\frac{2}{(s-1)^3} \right\} = \frac{1}{2}e^t t^2$$

$$\frac{-2s+1}{(s-1)^2} = \frac{A}{s-1} + \frac{B}{(s-1)^2}$$

$(s-1)$

$$-2s+1 = A(s-1) + B$$

$s=1 \rightarrow -1 = B$

$s=0 \rightarrow 1 = -A+B \longrightarrow A=-2$

$$\frac{-2s+1}{(s-1)^2} = \frac{-2}{s-1} + \frac{-1}{(s-1)^2}$$

Tablodan !

$$\mathcal{L}^{-1}\left\{\frac{-2}{s-1} - \frac{1}{(s-1)^2}\right\} = -2e^t - te^t$$

$$\boxed{x(t) = \frac{1}{2}e^t t^2 - 2e^t - te^t}$$

Ex: $\dfrac{d^2y(t)}{dt^2} + 5\dfrac{dy(t)}{dt} + 6y(t) = 0$ $\quad \left.\begin{aligned} y(0)&=2 \\ y'(0)&=3 \end{aligned}\right\} y(t)\ ?$

20.10.2023

3. HAFTA

Dinamik Sistemlerin

modellenmesi

Dinamik Sistemlerin Modellenmesi

(LTI)

Lineer Sistemleri iki genel yöntemle

↳ Transfer Fonksiyonu ile → Sadece Lineer Zamanla Değişmeyen Sist.

↳ Durum Değişkenleri Metodu ile. → LTI nonlineer

Mekanik Sistemlerin Modellenmesi:

- ↳ Ötelemeli (Translational)
- ↳ Dönel (Rotational)
- ↳ İkisi birlikte

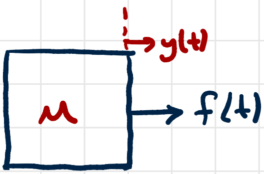
Ötelemeli Hareketi (Translational Motion)

- * ivme (acceleration)
- * hız (velocity)
- * yer değiştirme (displacement)

Newton Kanunu

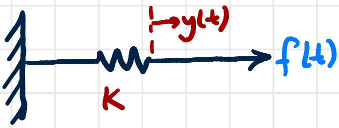
$$\Sigma F = m \cdot a$$

$N \quad kg \quad \frac{m}{s^2}$



$$f(t) = M \cdot a(t) = M \cdot \frac{d^2 y(t)}{dt^2} = M \frac{d\upsilon(t)}{dt}$$

Lineer Yay (Linear Spring)



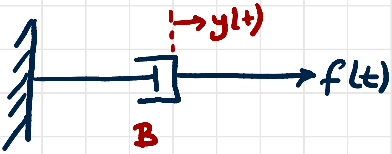
$$f(t) = K \cdot y(t)$$

f(t): Kuvvet; K: yay sabiti; y(t): yer değiştirme

Ötelemeli Hareket Sürtünme Çeşitleri

- * Viskoz Sürtünme (Viscous Friction)
- * Statik Sürtünme (Static Friction)
- * Kinetik Sürtünme (Kinetic Friction)

Viskoz Sürtünme

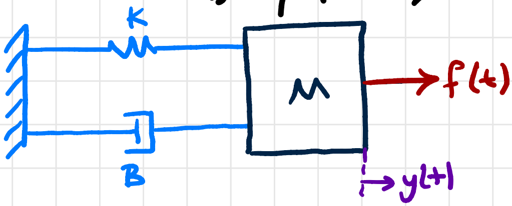


$$f(t) = B \cdot \frac{dy(t)}{dt} = B \cdot \dot{y}$$

B: sürtünme sabiti

$\frac{dy(t)}{dt}$: hız

Örnek: Sistemin transfer fonksiyonunu bulunuz. Başlangıç koşulları sıfır.

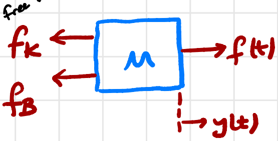


$$F(s) \longrightarrow \boxed{T(s)} \longrightarrow Y(s)$$

$T(s) = ?$

$$T(s) = \frac{Y(s)}{F(s)}$$

Serbest Cisim Diyagramı
free body diag



$$Ma = M \cdot \frac{d^2y(t)}{dt^2} = M\ddot{y}(t)$$

$$f_K = K \cdot y(t)$$

$$f_B = B \cdot \frac{dy(t)}{dt} = B \cdot \dot{y}(t)$$

$$\Sigma F = 0$$

$$f(t) - f_K - f_B - Ma = 0$$

$$f(t) = K \cdot y(t) + B\dot{y}(t) + M\ddot{y}(t)$$

$$f(t) = Ky(t) + B\frac{dy(t)}{dt} + M\frac{d^2y(t)}{dt^2}$$

Başlangıç durumları

$$\mathcal{L}: \quad F(s) = K\,Y(s) + B\left(sY(s) - \cancel{y(0)}^{0}\right) + M\left[s^2Y(s) - s\cancel{y(0)}^{0} - \cancel{\dot{y}(0)}^{0}\right]$$

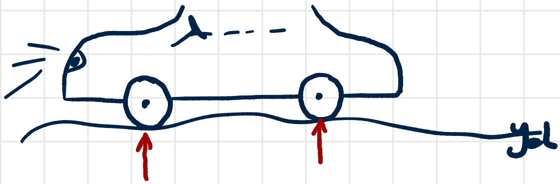
$$F(s) = KY(s) + BsY(s) + Ms^2Y(s)$$

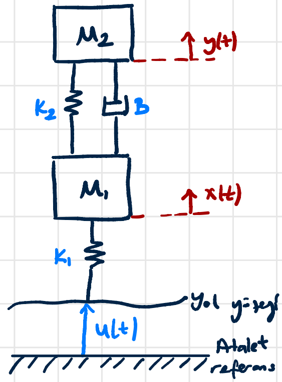
$$F(s) = Y(s)\left(K + Bs + Ms^2\right) \qquad T(s) = \frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

$$\dot{T}(s) = \frac{s}{Ms^2 + Bs + K}$$

$$\frac{V(s)}{F(s)} = \frac{sY(s)}{F(s)}$$

Örnek: İki kütleli Süspansiyon modeli

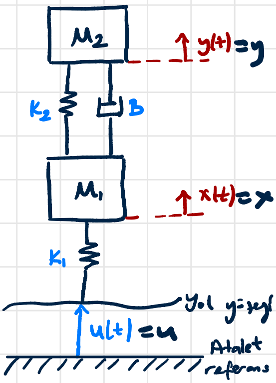


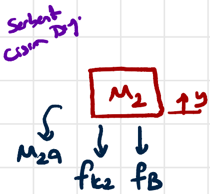


(Başlangıç koşulları sıfır alınacak!)

$$U(s) \rightarrow \boxed{T_1(s)} \rightarrow Y(s) \qquad X(s) \rightarrow \boxed{T_2(s)} \rightarrow Y(s)$$

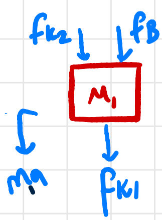
$T_1(s) = ?$ $\qquad$ $T_2(s) = ?$





$$f_{k2} = K_2(y-x)$$
$$f_B = B(\dot{y}-\dot{x})$$
$$M_2 a = M_2 \cdot \ddot{y}$$

$$M_2\ddot{y} + f_{k2} + f_B = 0$$
$$M_2\ddot{y} + K_2(y-x) + B(\dot{y}-\dot{x}) = 0$$
$$M_2\ddot{y} + B\dot{y} + K_2 \cdot y = B\dot{x} + K_2 x \quad (1)$$



$$f_{k2} = K_2 \cdot (x-y)$$
$$f_B = B(\dot{x}-\dot{y})$$
$$f_{k1} = K_1 \cdot (x-u)$$

$$M_1\ddot{x} + f_{k2} + f_B + f_{k1} = 0$$
$$M_1\ddot{x} + K_2(x-y) + B(\dot{x}-\dot{y}) + K_1(x-u) = 0$$
$$M_1\ddot{x} + B\dot{x} + (K_1+K_2)x = B\dot{y} + K_2 y + K_1 u \quad (2)$$

Laplace

$$(1) \quad (M_2 s^2 + Bs + K_2)\, Y(s) = Bs\, X(s) + K_2\, X(s)$$

$$(2) \quad [M_1 s^2 + Bs + (K_1+K_2)]\, X(s) = (Bs + K_2)\, Y(s) + K_1\, u(s)$$

$$\frac{Y(s)}{X(s)} = \frac{Bs + K_2}{M_2 s^2 + Bs + K_2}$$

(2) denklemine (1)'i yerleştir. Çöz.

① $(M_2 s^2 + Bs + K_2) Y(s) = Bs X(s) + K_2 X(s)$

② $[M_1 s^2 + Bs + (K_1 + K_2)] X(s) = (Bs + K_2) Y(s) + K_1 u(s)$

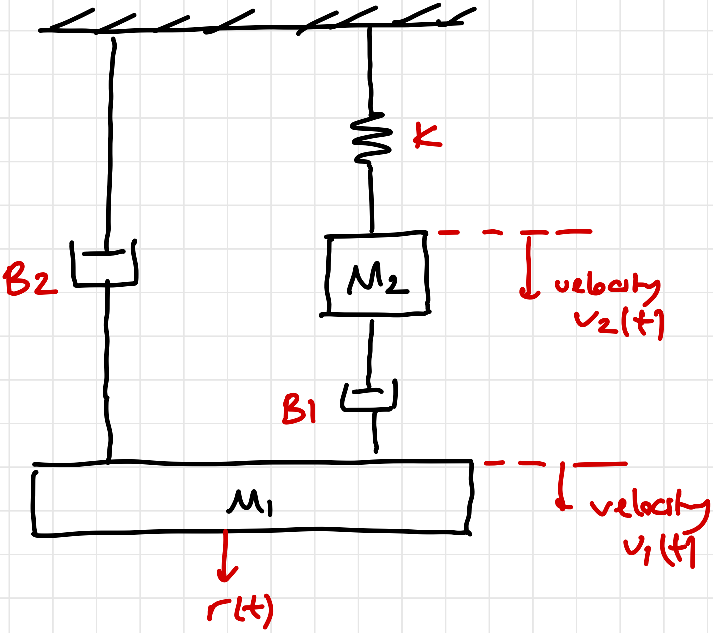
$$X(s) = \frac{M_2 s^2 + Bs + K_2}{Bs + K_2} \cdot Y(s)$$

$$[M_1 s^2 + Bs + (K_1 + K_2)] \frac{M_2 s^2 + Bs + K_2}{Bs + K_2} \cdot Y(s) - (Bs + K_2) Y(s) = K_1 u(s)$$

$$\left(\frac{(M_1 s^2 + Bs + (K_1 + K_2)) M_2 s^2 + Bs + K_2 - (Bs + K_2)^2}{Bs + K_2}\right) Y(s) = K_1 u(s)$$

$$\frac{Y(s)}{u(s)} = \frac{K_1 (Bs + K_2)}{M_1 M_2 s^4 + (M_1 + M_2) Bs^3 + (K_1 M_2 + (M_1 + M_2) K_2] s^2 + K_1 Bs + K_1 K_2}$$

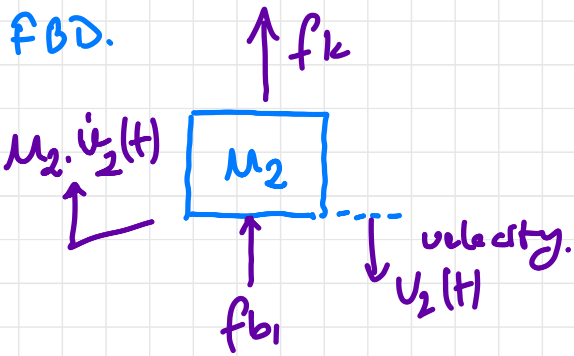
Ex:



$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow V_1(s) \qquad \frac{V_1(s)}{R(s)} = ?$$

$$R(s) \longrightarrow \boxed{G_2(s)} \longrightarrow X_1(s) \qquad \frac{X_1(s)}{R(s)} = ?$$

FBD.



$$f_k = k \cdot \int^t v_2(t)\,dt$$

$$F_k = \frac{k}{s} \overset{0}{\cdot} V_2(s)$$

$$f_{b_1} = b_1\left(v_2(t) - v_1(t)\right)$$

$$F_{b_1} = b_1\left(V_2(s) - V_1(s)\right)$$

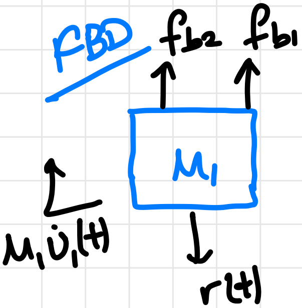
$$\sum F = m \cdot a$$

$$-f_k - f_{b_1} = M_2 \cdot \dot{v}_2(t) \Rightarrow f_k + f_{b_1} + M_2 \dot{V}_2(t) = 0$$

$$\frac{k}{s} V_2(s) + b_1 \left(V_2(s) - V_1(s) \right) + M_2 . s \, V_2(s) = 0$$

$$① \left(M_2 s + b_1 + \frac{k}{s} \right) V_2(s) = b_1 V_1(s)$$

$$V_2(s) = \frac{b_1}{M_2 s + b_1 + \frac{k}{s}} V_1(s)$$



$$f_{b_1} = b_1 \cdot \left(v_1(t) - v_2(t) \right)$$

$$f_{b_2} = b_2 . \, v_1(t)$$

$$\sum F = m \cdot a$$

$$M_1 \dot{v}_1(t) + f_{b_1} + f_{b_2} - r(t) = 0$$

$$M_1 s V_1(s) + b_1 \left(V_1(s) - V_2(s) \right) + b_2 . V_1(s) - R(s) = 0$$

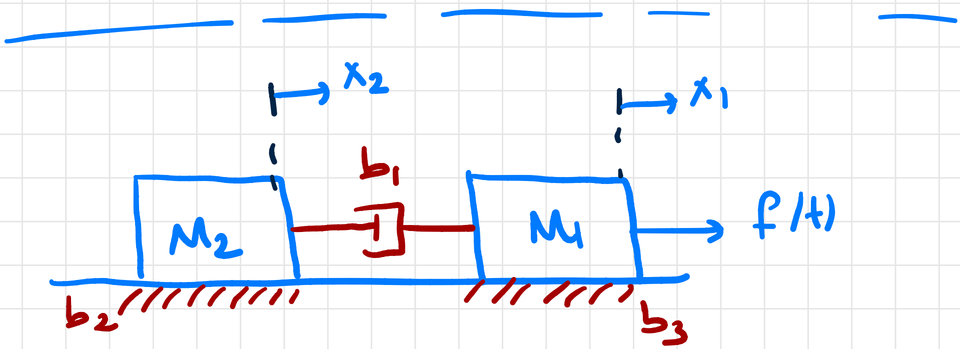
$$\left(M_1 s + b_1 + b_2 \right) V_1(s) - b_1 V_2(s) = R(s)$$

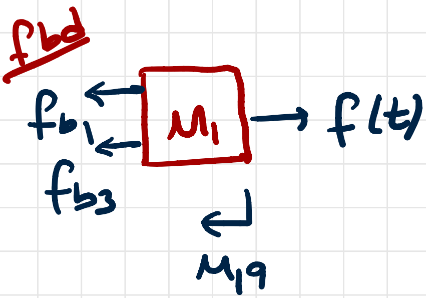
$$\left(M_1 s + b_1 + b_2 \right) V_1(s) - \frac{b_1}{M_2 s + b_1 + \frac{k}{s}} V_1(s) = R(s)$$

$$\left[(M_1 s + b_1 + b_2)(M_2 s^2 + b_1 s + k) - b_1^2 s\right] V_1(s) = (M_2 s^2 + b_1 s + k) R(s)$$

$$G(s) = \frac{V_1(s)}{R(s)} = \frac{(M_2 s^2 + b_1 s + k)}{\left[(M_1 s + b_1 + b_2)(M_2 s^2 + b_1 s + k) - b_1^2 s\right]}$$

$$\frac{X_1(s)}{R(s)} = \frac{V_1(s)}{s R(s)} = \frac{G(s)}{s}$$





$$f(t) = M_1 \ddot{x}_1$$

$$f_{b_1} = b_1 \cdot (\dot{x}_1 - \dot{x}_2)$$

$$f_{b_3} = b_3 \cdot \dot{x}_1$$

DÖNEL HAREKET

$$\Sigma F = m.a$$

$$\Sigma \text{Torques} = J.\alpha$$

J: inertia

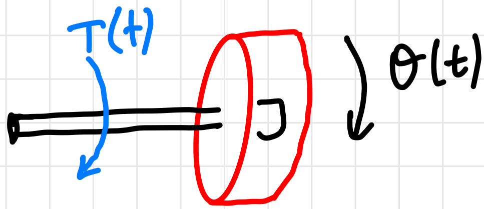
α: angular acceleration

- Torque (T)
- Angular velocity (ω)
- Angular displacement (θ)

r : radius

M : Kütle

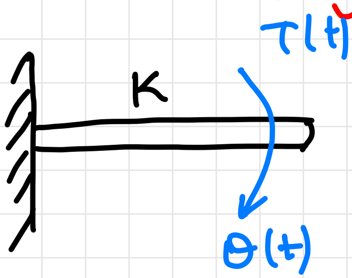
$$\text{Inertia}, J = \frac{1}{2} M r^2$$



Tork-Eylemsizlik Sistemi (Torque-inertia system)

$$J\alpha(t) = J\frac{d\omega(t)}{dt} = J\frac{d^2\theta(t)}{dt^2}$$

Torsional Spring



Torque-spring system.

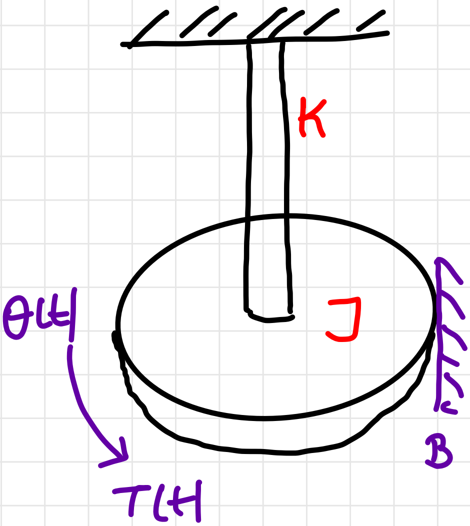
$$T(t) = K.\theta(t)$$

Dönel Sistem Sürtünmesi

1) Viscous friction $\quad T(t) = B \cdot \dfrac{d\theta(t)}{dt}$

2) Static friction $\quad T(t) = \pm f_s \Big|_{\dot{\theta}=0}$

3) Coulomb friction $\quad T(t) = f_c \dfrac{\frac{d\theta(t)}{dt}}{\left|\frac{d\theta(t)}{dt}\right|}$



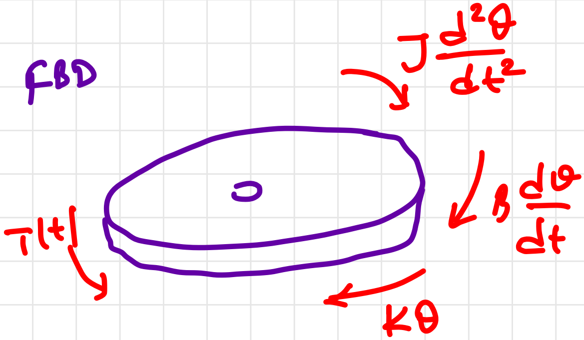
$$T(t) = B \cdot \frac{d\theta(t)}{dt}$$

$\theta(t) \rightarrow$ yerdeği

$w(t) \rightarrow$ Hız

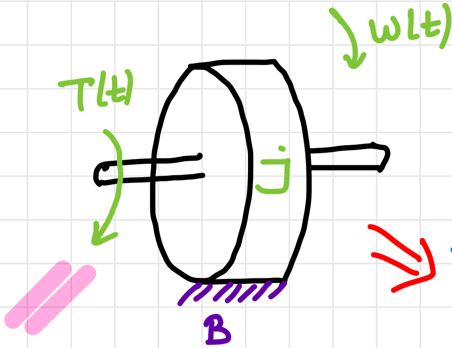
$\alpha(t) \rightarrow$ ivme

FBD

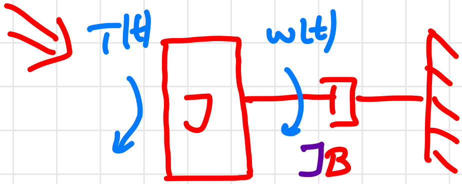


$$T(t) = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K.\theta(t)$$

Ex:

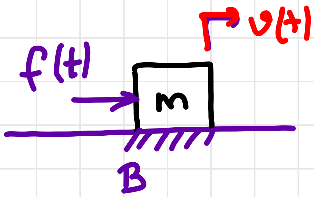


$\Omega(s) \rightarrow \boxed{G(s)} \rightarrow T(s)$



$t \rightarrow F$

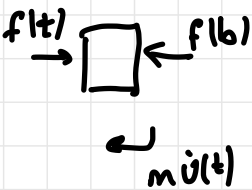
$\omega \rightarrow \Omega$



$\sum J = J \cdot \alpha$

$J_b = B.\dot{\theta} = B.\omega$

$\sum F = m.a$



$T(t) - J_b - J.\dot{\omega} = 0$

$T(t) = J_b + J\dot{\omega}$

$T(s) = B.\Omega(s) + Js\,\Omega(s) = 0$

$\frac{T(s)}{\Omega(s)} = \frac{1}{Js+B}$

$f(t) = f_b + m\dot{v}(t)$

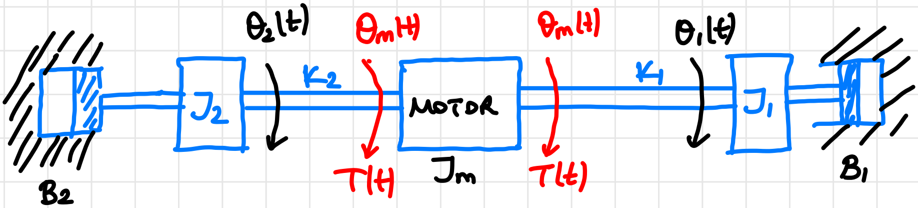
$f(t) = B.v(t) + m\dot{v}(t)$

$F(s) = B\,V(s) + Ms\,V(s)$

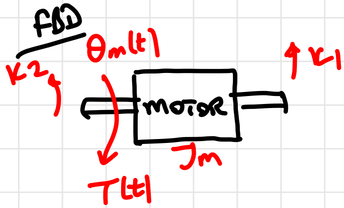
$\frac{F(s)}{V(s)} = \frac{1}{Ms+B}$

Örnek: Dönel sistemin Tork denklemlerini yazınız. Ardından

$\frac{\theta_1(s)}{\theta_m(s)} = ?$ $\quad \frac{\theta_2(s)}{\theta_m(s)} = ?$ transfer fonksiyonlarını bulunuz.



FBD

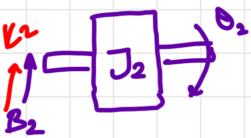


$\Sigma J = J.\alpha$

$f_{K1} = K_1(\theta_m - \theta_1)$

$f_{K2} = K_2(\theta_m - \theta_2)$

$$J_m \frac{d^2\theta_m(t)}{dt} + K_1(\theta_m(t) - \theta_1(t)) + K_2(\theta_m(t) - \theta_2(t)) = T(t)$$



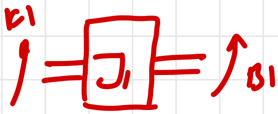
$$J_2 \frac{d^2\theta_2(t)}{dt} + K_2(\theta_2(t) - \theta_m(t)) + B_2 \frac{d\theta_2(t)}{dt} = 0$$

$\mathcal{L}$

$$J_2 s^2 \theta_2(s) + K_2\theta_2(s) - K_2\theta_m(s) + B_2 s\,\theta_2(s) = 0$$

$$\theta_2(s)(J_2 s^2 + B_2 s + K_2) = K_2\theta_m(s)$$

$$\frac{\theta_2(s)}{\theta_m(s)} = \frac{K_2}{J_2 s^2 + B_2 s + K_2}$$



$J_1 \alpha$

$$J_1 \frac{d^2\theta_1(t)}{dt} + K_1(\theta_1(t) - \theta_m(t)) + B_1 \frac{d\theta_1(t)}{dt} = 0$$

$$J_1 s^2 \theta_1(s) + K_1\theta_1(s) - K_1\theta_m(s) + B_1 s\,\theta_1(s) = 0$$

$$\boxed{\frac{\theta_1(s)}{\theta_m(s)} = \frac{K_1}{J_1 s^2 + B_1 s + K_1}}$$