GENERAL RELATIVITY
MIDTERM EXAM 1

April 16, 2016

Name: ______________________

Number: ______________________

ANSWER ALL THE QUESTIONS

Useful Information: $r_S = \frac{2GM}{c^2}$

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1. Consider two events A and B on the worldline of a particle whose coordinate separations are $c\,dt$, dx, dy, and dz in the inertial frame S in which the particle is observed. The particle is moving at a constant velocity v. In the frame S′ of a clock carried by the particle the events occur at the same place.

a) Write down the squared spacetime interval between the events in S′

b) Find the relation between the time interval dt' read by the particle's clock and the time interval dt read by a clock in S

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2. a) Write down the momentum four-vector for an object moving at constant velocity in terms of the total energy E and the components of the three-momentum vector $\vec{p}$.

b) For a particle moving in the x direction at constant velocity v write down the transformation law between $\underset{\sim}{p}'$ (in S') by using the Lorentz Transformation Matrix.

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3) When the angles θ and ϕ are used as parameters, points on a torus are given by

$$x = (a + b\cos\theta)\cos\phi \,, \quad y = (a + b\cos\theta)\sin\phi \,, \quad z = b\sin\theta \,,$$

with $-\pi < \theta \leq \pi$, $-\pi < \phi \leq \pi$ and a and b are constants.

a) Obtain the expressions for the coordinate basis vectors $\underset{\sim}{e}_\theta$ and $\underset{\sim}{e}_\phi$

b) Obtain the matrix for g_{ij}, where $i, j = \theta, \phi$.

4. In four dimensional curved space write down how the following objects transform under a coordinate transformation:
a) A^{α} , b) B_{α} , c) $\partial_{\alpha}\Phi$, where Φ is a scalar function
d) $T^{\sigma\gamma}{}_{\alpha\beta}$, e) $A_{\alpha}B^{\alpha}$

5. Consider the three dimensional space in spherical coordinates with

$$ds^2 = dr^2 + r^2 d^2\theta + r^2 \sin^2\theta \, d\phi^2$$

a) Write the matrix for the metric g_{ij}, $(i,j = r, \theta, \phi)$

b) Write down the matrix for g^{ij}

5c) Given $Y_i = \begin{bmatrix} 0 \\ -r^2 \\ \cos^2\theta \end{bmatrix}$. Obtain the contravariant three-vector Y^i.

5d) Given $X^i = \begin{bmatrix} 1 \\ r \\ 0 \end{bmatrix}$. Obtain the covariant three-vector X_i.

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6. By inserting $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ into the left side of

$$\partial^\alpha F^{\mu\nu} + \partial^\nu F^{\alpha\mu} + \partial^\mu F^{\nu\alpha} = \text{constant}$$

find the value of the constant.

b) The differential form of Gauss's law is

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0}$$

Show that the $\mu = t$ component of the equation

$$\partial_\nu F^{\mu\nu} = \frac{1}{c^2 \epsilon_0} J^\mu$$

gives the Gauss's law. Here J^μ is the current four-vector, and $F^{\mu\nu}$ is the electromagnetic field tensor.

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7. a) Calculate $\partial_\mu \partial_\nu F^{\mu\nu}$ by using the definition of $F^{\mu\nu}$ (in question 6) in terms of A^μ.

b) Show that $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ with $\mu = t$ and $\nu = x$ yields the x component of the equation $\vec{E} = -\frac{\partial \vec{A}}{c\partial t} - \vec{\nabla}\Phi$, where $\vec{A}$ and Φ are the vector and scalar potentials.

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8. a) Consider the following equations

i) $p^\mu = m u^\alpha \delta^\mu{}_\beta$, ii) $p^\mu \eta_{\mu\nu} p^\nu + m^2c^2 = 0$

iii) $\eta_{\alpha\beta} = \eta_{\mu\nu} \Lambda^\mu{}_\sigma \Lambda^\nu{}_\sigma$, iv) $\frac{dp^\alpha}{d\tau} = q F^{\mu\nu} u_\alpha$

v) $\eta_{\mu\nu} \eta^{\mu\alpha} = 4$

Circle the equations that violate the rules for indices.

b) Write down the numerical values of the following:

i) $\delta^\mu{}_\mu$, ii) $\eta_{\alpha\beta}\eta^{\alpha\beta}$, iii) $g^{\alpha\beta} g_{\alpha\beta}$, iv) $\eta_{\mu x} \delta^\mu{}_t$

v) $\delta^\mu{}_\nu \delta^\nu{}_\mu$

9. The square of the infinitesimal interval in Minkowski spacetime is

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

Obtain from this equation what the speed of light is.

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10. a) Write down the squared infinitesimal interval ds^2 describing the spacetime outside a static spherical distribution of mass M.

b) What is the infinitesimal radial distance along a radial line?

c) Calculate the radial distance between two events A and B with coordinates r_A and r_B along such a line in the limit $\frac{r_s}{r} \ll 1$.

d) What is the proper time $\Delta\tau$ measured by a clock between two events at its location?

11. Consider two light signals emitted at a radial coordinate r_E. The signals are received at the radial coordinate r_R.

a) What is the ratio $\frac{\Delta \tau_R}{\Delta \tau_E}$, where $\Delta \tau_R$ and $\Delta \tau_E$ are the proper times measured by clocks at rest at the radial coordinates r_R and r_E?

b) What is the ratio $\frac{\lambda_R}{\lambda_E}$ between the wavelengths of the received and emitted light signals?

c) Calculate $\frac{\lambda_R}{\lambda_E}$ in the limit $\frac{r_s}{r_R} \ll 1$ and $\frac{r_s}{r_E} \ll 1$.

d) Express $\frac{\lambda_R}{\lambda_E}$ in terms of the radial difference $h = r_R - r_E$ and the gravitational acceleration g measured at the emission radius r_E.

11 e) What is the effect used to observe this gravitational redshift?

12. The acceleration of an object in Schwarzschild spacetime is given by

$$\frac{d^2 r}{d\tau^2} = -\frac{GM}{r^2} + \frac{l^2}{r^3} - \frac{3GMl^2}{c^2 r^4},$$

where $l = r^2 \frac{d\phi}{d\tau}$

a) What is the acceleration of an object moving radially?

b) What is the corresponding Newtonian equation in this case?

13. The two dimensional curved surface

$$ds^2 = \frac{dr^2}{(1 - r_s/r)} + r^2 d\phi^2 \; ; (t = \text{const.}, \theta = \pi/2)$$

can be embedded in a three dimensional Euclidean space described by $ds^2 = dr^2 + r^2 d\phi^2 + dz^2$

In this three dimensional space we can describe a two dimensional surface by specifying its height z as a function of the coordinate r.

Do the calculation and find the expression for $z(r)$.

14. The four-velocity of a particle in the Schwarzschild spacetime is given by

$$\underset{\sim}{U} = \begin{bmatrix} dt/d\tau \\ dr/d\tau \\ d\theta/d\tau \\ d\phi/d\tau \end{bmatrix}$$

Consider the vectors $\underset{\sim}{\xi} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\underset{\sim}{\eta} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Calculate the quantities

$$e = -\underset{\sim}{\xi} \cdot \underset{\sim}{U} \quad \text{and} \quad \ell = \underset{\sim}{\eta} \cdot \underset{\sim}{U}$$

in the Schwarzschild spacetime

15. a) What are the pathologies associated with the Schwarzschild metric.

b) When does a collapsing star (namely a star shrinking in size due to its own gravity) become a black hole?

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EXAM 1

19.11.2016

Name : ______________________

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1 a) Explain the Equivalence Principle by giving an example.

b) Give an example for proof that non-uniform gravitational fields curve the space-time.

2 a) Show that in flat space-time light travels at the speed of light c.

b) The 4-force, or the covariant force is defined by

$$K^\mu \equiv \frac{dp^\mu}{d\tau}, \quad \text{where } p^\mu \text{ is the 4-momentum}$$

i) What are the components of K^μ?

ii) How does K^μ transform under a Lorentz transformation for a system moving in the y direction?

c) Under a coordinate change $S \to S'$, how is $\eta_{\mu\nu} A^\mu B^\nu$ related to $\eta_{\mu\nu} A^{\mu\prime} B^{\nu\prime}$?

3 a) Write down how the following tensors transform under a coordinate transformation $S \to S'$:

i) $C_{\mu\nu}^{\ \ \alpha}$

ii) $A^{\mu\nu}_{\ \ \alpha} + B^{\mu\nu}_{\ \ \alpha}$

iii) $\delta^{\mu}_{\ \nu}$

iv) $g_{\mu\nu} A^{\mu} B^{\nu}$

v) $R^{\alpha\beta}_{\ \ \mu\nu}$

4 a) The metric equation for the cartesian coordinates x, y is

$$ds^2 = dx^2 + dy^2$$

The parabolic coordinates p, q are given by $p(x,y) = x$ and $q(x,y) = y - cx^2$. Find the metric $[g_{\mu'\nu'}]$ in this coordinate system.

4b) The vector $\underset{\sim}{B}$ has components $B^p = 1$ and $B^q = 0$.
Find this vector's components in the x, y coordinate system.

4c) Show that $B^2 = \underset{\sim}{B} \cdot \underset{\sim}{B}$ of this vector has the same value in both systems.

4d) Find the basis vectors $\underset{\sim}{e}_p$ and $\underset{\sim}{e}_q$.

4e) Calculate the matrix $\begin{bmatrix} \underset{\sim}{e}_p \cdot \underset{\sim}{e}_p & \underset{\sim}{e}_p \cdot \underset{\sim}{e}_q \\ \underset{\sim}{e}_q \cdot \underset{\sim}{e}_p & \underset{\sim}{e}_q \cdot \underset{\sim}{e}_q \end{bmatrix}$. What is the meaning of this matrix?

5. In the isotropic coordinates ρ, θ, ϕ, the metric equation is

$$ds^2 = -\left(1-\frac{GM}{2c^2\rho}\right)^2\left(1+\frac{GM}{2c^2\rho}\right)^{-2}c^2dt^2 + \left(1+\frac{GM}{2c^2\rho}\right)^4\left(d\rho^2+\rho^2d\theta^2+\rho^2\sin^2\theta\, d\phi^2\right)$$

a) Write down the matrix $[g_{\mu\nu}]$.

b) Write down the inverse matrix $[g^{\mu\nu}]$.

c) Write down the relation between $d\tau$, the infinitesimal proper time interval, and dt, the coordinate time interval.

d) Write down the relation between the infinitesimal radial distance dR and $d\rho$.

e) Does the coordinate ρ of a point equal to the distance of that point from the origin? Why?

6 a) Write down the Schwarzschild metric equation ds^2 in spherical polar coordinates r, θ, ϕ.

b) What is the physical meaning of this equation?

c) What is the condition on a compact spherical object to be a black hole?

d) What happens to ds^2 as $r \to \infty$?

e) What happens to $d\tau$ as $r \to \infty$?

7. In Schwarzschild coordinates the vector X^μ and the covector Y_μ are given as $X^\mu = \begin{bmatrix} 0 \\ 1 \\ r \\ 0 \end{bmatrix}$ and $Y_\mu = \begin{bmatrix} 1 \\ 0 \\ -r \\ 1 \end{bmatrix}$

a) Calculate X_μ.

b) Calculate Y^μ.

8. The electromagnetic field tensor $F^{\mu\nu}$ is given by

$$[F^{\mu\nu}] = \begin{bmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & B_z & -B_y \\ -\frac{E_y}{c} & -B_z & 0 & B_x \\ -\frac{E_z}{c} & B_y & -B_x & 0 \end{bmatrix}$$

a) Calculate $\partial_\mu \partial_\nu F^{\mu\nu}$.

b) In terms of the electromagnetic four potential A^μ, $F^{\mu\nu}$ is given as $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

Calculate $\partial^\alpha F^{\mu\nu} + \partial^\nu F^{\alpha\mu} + \partial^\mu F^{\nu\alpha}$.

c) Show from $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ that $F^{tx} = \frac{E_x}{c}$

d) Show that $F^{yz} = B_x$

e) Calculate from $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ $F^{tt} + F^{xx} + F^{yy} + F^{zz} = ?$

9. a) What is the Gravitational Red Shift ?

b) If two light signals are emitted at the location r_E at different times and received at r_R, find the expression $\frac{\lambda_R - \lambda_E}{\lambda_E}$ in leading order, where λ is the wavelength of the light signal.

c) How can this extremely small red shift be measured experimentally ?

10. a) Lower the indices of $F^{\alpha\beta}$ to obtain $F_{\mu\nu}$ in flat space-time

b) Write down the matrix $[F_{\mu\nu}]$

c) What does the equation $\frac{dp^{\mu}}{d\tau} = q F^{\mu\nu} U_{\nu}$ correspond to in the non-relativistic limit? Here p^{μ}, q, and U_{ν} are the 4-momentum, charge, and 4-velocity of a particle.

GENERAL RELATIVITY

EXAM 1

November 18, 2017

Instructor: Dr. Murat Özer

Student's Name:

1.a) A rocket in deep space is accelerating upward at $a = 9.8\, m/s^2$. The period of a simple pendulum on the floor is found to be 1 sec. How much does a similar pendulum on Earth takes for 60 oscillations? Explain why?

b) Two objects 1 m apart are dropped from the top of a 500-m-high tower on Earth. Draw the paths of the objects as they fall.

c) Again two objects 1 m apart are dropped from the ceiling of a 500-m-high rocket in deep space accelerating upward at $a = 9.8\, m/s^2$. Draw the paths of the objects as they fall.

2. In a coordinate transformation, the four coordinates x^α, with $\alpha = 0, 1, 2,$ and 3, that label events in spacetime are changed to new labels x'^α that are functions of the old ones, $x'^\alpha = x'^\alpha(x)$

Write down the relation between the quantity in the new coordinates and in the old coordinates for the following

(i) A'_μ

(ii) A'^μ

(iii) dx'^μ

(iv) $A'_\mu B'^\mu$

(v) $T'^\mu{}_\nu$

(vi) $R'_{\mu\nu}$

(vii) $R'^{\mu\nu}$

(viii) ϕ'

(ix) $\partial'_\mu \phi'$

(x) $ds'^2 = g'_{\mu\nu} dx'^\mu dx'^\nu$

3. The infinitesimal spacetime distance $d\underset{\sim}{s}$ between two neighboring points can be written as

$$d\underset{\sim}{s} = c\,dt\,\underset{\sim}{e}_t + dr\,\underset{\sim}{e}_r + d\theta\,\underset{\sim}{e}_\theta + d\phi\,\underset{\sim}{e}_\phi\,,$$

where the $\underset{\sim}{e}_\mu$s are the coordinate basis vectors.

a) What are the basis vectors (in column vector form) for the Schwarzschild geometry?

b) How is a four-vector $\underset{\sim}{A}$ is written in terms of $\underset{\sim}{e}_\mu$?

c) Write down the expression for A^2 for $\underset{\sim}{A}$ in the Schwarzschild spacetime.

4) Evaluate the following quantities

(i) $\delta^{\alpha}_{0}\,\eta_{\alpha\beta}\,\eta^{\beta\sigma}\,\delta^{0}_{\sigma}$

(ii) $\delta^{\alpha}_{\mu}\,\eta_{\alpha\beta}\,\eta^{\beta\sigma}\,\delta^{\mu}_{\sigma}$

(iii) $g^{\alpha\beta}\,g_{\beta\sigma}\,g^{\sigma\rho}\,g_{\rho\alpha}$

(iv) $g^{\alpha\beta}\,\eta_{\beta\sigma}\,\eta^{\sigma\rho}\,g_{\rho\alpha}$

(v) $g^{\alpha\beta}\,\delta^{\mu}_{\beta}\,\delta^{\nu}_{\alpha}\,g_{\mu\nu}$

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5. In the hypothetical spacetime described by

$$ds^2 = -\left(1-\frac{r_s}{r}\right)c^2dt + 2\sqrt{\frac{r_s}{r}}\,c\,dt\,dr + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2,$$

where $r_s = 2GM/c^2$.

a) Write down the matrix $[g_{\mu\nu}]$

5b) Given the contravariant vector $Y^\mu = \begin{bmatrix} -\frac{r}{r_s} \\ 0 \\ \frac{1}{r^2} \\ \frac{1}{r^2 \sin^2\theta} \end{bmatrix}$,

obtain the covariant vector Y_μ.

5c. Calculate $Y_\mu Y^\mu$

6. The electromagnetic field tensor $F^{\mu\nu}$ is defined as

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu ,$$

where A^μ is the electromagnetic four-potential.

a) Show that $F^{\mu\nu}$ is antisymmetric

b) Calculate $\partial_\mu \partial_\nu F^{\mu\nu}$

c) Calculate $\partial^\alpha F^{\mu\nu} + \partial^\nu F^{\alpha\mu} + \partial^\mu F^{\nu\alpha}$

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7. The four-velocity of a particle in the Schwarzschild spacetime is given by

$$\underset{\sim}{u} = \begin{bmatrix} dt/d\tau \\ dr/d\tau \\ d\theta/d\tau \\ d\phi/d\tau \end{bmatrix}$$

Consider the vectors

$$\underset{\sim}{\xi} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \underset{\sim}{\zeta} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

7 a) calculate $e = -\underset{\sim}{\xi} \cdot \underset{\sim}{U}$

b) calculate $\ell = \underset{\sim}{\zeta} \cdot \underset{\sim}{U}$

c) calculate $\underset{\sim}{U} \cdot \underset{\sim}{U}$

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8. Consider the hypothetical spacetime metric

$$ds^2 = -c^2 dt^2 + \frac{dr^2}{1 - \frac{r^2}{R^2}} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

a) Is this spacetime flat or curved? Why?

8 b) At a given time, what is the spatial metric for $\theta = \frac{\pi}{2}$?

c) Embed the surface with the metric in b) in the three dimensional Euclidean space described by cylindrical coordinates (r, ϕ, z) whose metric is

$$ds^2 = dr^2 + r^2 d\phi^2 + dz^2$$

9. Sinusoidal coordinates p and q on a flat 2D plane are defined by the relations $p = x$ and $q = y - a\sin(bx)$ where a and b are constants.

a) Find the metric of the sinusoidal coordinate system.

b) Find the coordinate basis vectors of this coordinate system

c) Find the metric of this coordinate system from the basis vectors found in part b).

10. a) Find the value of the square $U^2 = \underset{\sim}{U} \cdot \underset{\sim}{U}$ of the four-velocity $\underset{\sim}{U}$ of an object.

b) Evaluate the t-component of the four-velocity $\underset{\sim}{U}$ for an object at rest in the Schwarzschild spacetime.

c) The geodesic equation for any spacetime is

$$g_{\alpha\beta} \frac{d^2 x^\beta}{d\tau^2} = -\left(\partial_\nu g_{\alpha\mu} - \frac{1}{2} \partial_\alpha g_{\mu\nu}\right) U^\mu U^\nu .$$

Obtain the geodesic equation for r in the Schwarzschild spacetime.

GENERAL RELATIVITY
EXAM 1

01.12.2018

Name:

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1. a) Give an example for proof that nonuniform gravitational fields curve the spacetime.

b) Write down the square of the line element ds in flat spacetime.

c) Write down the matrix $[\eta_{\mu\nu}]$ for the metric $\eta_{\mu\nu}$ for this spacetime.

d) Write down the matrix $[\eta^{\mu\nu}]$.

2. a) A small enough cabin is accelerated upward in deep space at an acceleration of $a = 1.62\ m/s^2$. Describe a cabin equivalent to this one so that an experiment done in deep space will give the same result when done in this equivalent cabin.

b) A small enough cabin is in free fall in the gravitational field of Planet X. Describe a cabin equivalent to this one.

3. Four-vectors X^α and Y_α in the Schwarzschild space time are given as

$$X^\alpha = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} , \quad Y_\alpha = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

a) Find the covariant vector X_α

b) Find the contravariant vector Y^α

4. a) Given two tensors $B_{\alpha\beta}$ and $C^{\alpha\beta}$ with the property that $B_{\alpha\beta} = B_{\beta\alpha}$, $C^{\alpha\beta} = -C^{\beta\alpha}$.

Find the numerical value of $B_{\alpha\beta} C^{\alpha\beta}$.

b) Any tensor $T^{\alpha\beta}$ can be written as $T^{\alpha\beta} = S^{\alpha\beta} + A^{\alpha\beta}$, where $S^{\alpha\beta} = S^{\beta\alpha}$ and $A^{\alpha\beta} = -A^{\beta\alpha}$. Find the tensors $S^{\alpha\beta}$ and $A^{\alpha\beta}$ in terms of $T^{\alpha\beta}$ and $T^{\beta\alpha}$.

5. A coordinate transformation from the old coordinates (x^0, x^1, x^2, x^3) to the new coordinates $(x^{0'}, x^{1'}, x^{2'}, x^{3'})$ is made. Write down the expressions in the new coordinates for the following tensors: in terms of

a) A_α, b) A^α, c) $B^{\alpha\beta}$, d) $B_{\alpha\beta}$, e) $C^\alpha{}_\beta$

f) ϕ (a scalar), g) $D_\alpha E^\alpha$, h) $D^\alpha E_\alpha$, i) $g_{\alpha\beta}\,dx^\alpha dx^\beta$

6. From the roof of a building of height h two signals are sent down to the ground. Let the wavelengths of the emitted and received signals be λ_E and λ_R. Find the ratio $\frac{\lambda_R}{\lambda_E}$ from the Schwarzschild metric.

b) Find $\frac{\lambda_R - \lambda_E}{\lambda_E}$. What kind of a shift is this?

7. The metric of a hypothetical spacetime is given by

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \left(1 + \frac{2GM}{c^2 r}\right)dr^2$$

$$+ r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

a) What is the metric for the equatorial plane?

b) Embed the space in a) in the 3-dimensional Euclidean space $ds^2 = dr^2 + r^2 d\phi^2 + dz^2$ and find the expression for $z(r)$.

8. The Schwarzschild metric in coordinates (v, r, θ, ϕ) is given by

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dv^2 + 2\,dv\,dr + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

a) Write down the matrix $[g_{\mu\nu}]$

b) Obtain the equation satisfied by photons moving radially in the equatorial plane.

9. The electromagnetic field tensor $F_{\mu\nu}$ is given in matrix form as

$$[F_{\mu\nu}] = \begin{bmatrix} 0, & -\frac{E_x}{c}, & -\frac{E_y}{c}, & -\frac{E_z}{c} \\ \frac{E_x}{c}, & 0, & B_z, & -B_y \\ \frac{E_y}{c}, & -B_z, & 0, & B_x \\ \frac{E_z}{c}, & B_y, & -B_x, & 0 \end{bmatrix}$$

a) Raise the indices of $F_{\mu\nu}$ to obtain $F^{\alpha\beta}$ in flat spacetime.

b) Write down the matrix $[F^{\mu\nu}]$

c) The nonrelativistic force equation on a charged particle of mass m and charge q is: $m\frac{d\vec{v}}{dt} = q\vec{E} + q\vec{v}\times\vec{B}$.
Write down the special relativistic equation for such a particle.

10. a) Consider the following equations

i) $A^2 = \eta_{\alpha\beta} A^\alpha A^\beta$, ii) $\frac{dp^\mu}{dt} = q F^{\mu\nu} U_\nu$

iii) $T_{\alpha\beta\gamma} = g_{\alpha\mu} g_{\beta\nu} g_{\gamma\sigma} T^{\mu\nu\sigma}$

iv) $m \frac{dU^\alpha}{d\tau} = q F^{\alpha\beta} U^\beta$, v) $T^{\alpha\beta\gamma} = g^{\alpha\mu} g^{\beta\nu} g^{\gamma\gamma} T_{\mu\nu\gamma}$

Circle the equations that are wrong.

b) Write down what the following operations give:

i) $g^{\mu\alpha} R_{\alpha\gamma} =$

ii) $g_{\mu\alpha} T^{\alpha\nu} =$

iii) $g^{\mu\alpha} g_{\alpha\nu} =$

iv) $g^{\mu\alpha} g_{\mu\alpha} =$

v) $\eta_{\mu 0} \delta^\mu_0 =$