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**INTRODUCTION TO TORQUE**

Welcome to the presentation on torque. So, if you watched the presentation on the center of mass, which you should have, you might have gotten a little bit of a glancing view of what torque is. And now we'll do some more in detail. So in general, from the center of mass video, we learned, if this is a ruler and this is the ruler's center of mass. And if I were to apply force at the center of mass, I would accelerate the whole ruler in the direction of the force. If I have the force applying at the center of mass there, the whole ruler would accelerate in that direction. And we'd figure it out by taking the force we're applying to it and dividing by the mass of the ruler. And in that center of mass video, I imply-- well, what happens if the force is applied here? Away from the center of mass? Well, in this situation, the object, assuming it's a free floating object on the Space Shuttle or something, it will rotate around the center of mass. And that's also true, if we didn't use the center of mass, but instead we fixed the point. Let's say we had another ruler. Although it has less height than the previous one. Instead of worrying about its center of mass, let's say that it's just fixed at a point here. Let's say it's fixed here. So if this could be the hand of a clock, and it's nailed down to the back of the clock right there. So if we were trying to rotate it, it would always rotate around this point. And the same thing would happen. If I were to apply a force at this point, maybe I could break the nail off the back of the clock, or something, but I won't rotate this needle or this ruler, or whatever you want to call it. But if I would apply a force here, I would rotate the ruler around the pivot point. And this force that's applied a distance away from the pivot point, or we could say from the axis of rotation, or the center of mass. That's called torque. And torque, the letter for torque is this Greek, I think that's tau, it's a curvy T. And torque is defined as force times distance. And what force and what distance is it? It's the force that's perpendicular to the object. I guess you could say to the distance vector. If this is the distance vector-- let me do it in a different color. If this is the distance vector, the component of the force is perpendicular to this distance vector. And this is torque. And so what are its units? Well, force is newtons, and distance is meters, so this is newton meters. And you're saying, hey Sal, newtons times meters, force times distance, that looks an awful lot like work. And it's very important to realize that this isn't work, and that's why we won't call this joules. Because in work, what are we doing? We are translating an object. If this is an object, and I'm applying a force, I'm taking the force over the distance in the same direction as the force. Here the distance and the force are parallel to each other. You could say the distance vector and the force vector are in the same direction. Of course, that's translational. The whole object is just moving. It's not rotating or anything. In the situation of torque, let me switch colors. The distance vector, this is the distance from the fulcrum or the pivot point of the center of mass, to where I'm applying the force. This distance vector is perpendicular to the force that's being applied. So torque and work are fundamentally two different things, even though their units are the same. And this is a little bit of notational. This distance is often called the moment arm distance. And I don't know where that came from. Maybe one of you all can write me a message saying where it did come from. And often in some of your physics classes they'll often call torque as a moment. But we'll deal with the term torque. And that's more fun, because eventually we can understand concepts like torque horsepower in cars. So let's do a little bit of math, hopefully I've given you a little bit of intuition. So let's say I had this ruler. And let's say that this is its pivot point right here. So it would rotate around that point. It's nailed to the wall or something. And let's say that I apply a force-- Let's say the moment arm distance. So let's say this distance, let me do it in different color. Let's say that this distance right here is 10 meters. And I were to apply a force of 5 newtons perpendicular to the distance vector, or to dimension of the moment arm, you could view it either way. So torque is pretty easy in this situation. Torque is going to be equal to the force, 5 newtons, times the distance, 10. So it would be 50 newton meters. And you're probably saying, well, Sal, how do I know if this torque is going to be positive or negative? And this is where there's just a general arbitrary convention in physics. And it's good to know. If you're rotating clockwise torque is negative. Let me go the other way. If you were rotating counterclockwise, like we were in this example, rotating counterclockwise, the opposite direction of which a clock would move in. Torque is positive. And if you rotate clockwise the other way, torque is negative. So clockwise is negative. And I'm not going to go into the whole cross product and the linear algebra of torque right now, because I think that's a little bit beyond the scope. But we'll do that once we do more mathematically intensive physics. But, so, good enough. There's a torque of 50 newton meters. And that's all of the torque that is acting on this object . So it's going to rotate in this direction. And we don't have the tools yet to figure out how quickly it will rotate. But we know it will rotate. And that's vaguely useful. But what if I said that the object is not rotating? And that I have another force acting here? And let's say that that force is-- I don't know, let me make up something, that's 5 meters to the left of the pivot point. If I were tell you that this object does not rotate. So if I tell you that the object is not rotating, that means the net torque on this ruler must be 0, because it's not-- its rate of change of rotation is not changing. I should be a little exact. If I'm applying some force here, and still not rotating, then we know that the net torque on this object is 0. So what is the force being applied here? Well, what is the net torque? Well, it's this torque, which we already figured out. It's going in the clockwise direction. So it's 5-- Let me do it in a brighter color. 5 times 10. And then the net torque. The sum of all the torques have to be equal to 0. So what's this torque? So let's call this f. This is the force. So, plus-- Well, this force is acting in what direction? Clockwise or counterclockwise? Well, it's acting in the clockwise direction. This force wants to make the ruler rotate this way. So this is actually going to be a negative torque. So let's say, put a negative number here times f, times its moment arm distance, times 5, and all of this has to equal 0. The net torque is 0, because the object's rate of change of rotation isn't changing, or if it started off not rotating, it's still not rotating. So here we get 50 minus 5 f is equal to 0. That's 50 is equal to 5 f. f is equal to 10. If we follow the units all the way through, we would get that f is equal to 10 newtons. So that's interesting. I applied double the force at half the distance. And it offsetted half the force at twice the distance. And that should all connect, or start to connect, with what we talked about with mechanical advantage. You could view it the other way. Let's say these are people applying these forces. Say this guy over here is applying 10 newtons. He's much stronger. He's twice as strong as this guy over here. But because this guy is twice as far away from the pivot point, he balances the other guy. So you can kind of view it as this guy having some mechanical advantage or having a mechanical advantage of 2. And watch the mechanical advantage videos if that confuses you a little bit. But this is where to torque is useful. Because if an object's rate of rotation is not changing, you know that the net torque on that object is 0. And you can solve for the forces or the distances. I'm about to run out of time, so I will see you in the next video.

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ROLLING WITHOUT SLIPPING PROBLEMS

[Instructor] So we saw last time that there's two types of kinetic energy, translational and rotational, but these kinetic energies aren't necessarily proportional to each other. In other words, the amount of translational kinetic energy isn't necessarily related to the amount of rotational kinetic energy. However, there's a whole class of problems. A really common type of problem where these are proportional. So that's what we're gonna talk about today and that comes up in this case. So, imagine this. Imagine we, instead of pitching this baseball, we roll the baseball across the concrete. So, say we take this baseball and we just roll it across the concrete. What's it gonna do? It's gonna rotate as it moves forward, and so, it's gonna do something that we call, rolling without slipping. At least that's what this baseball's most likely gonna do. I mean, unless you really chucked this baseball hard or the ground was really icy, it's probably not gonna skid across the ground or even if it did, that would stop really quick because it would start rolling and that rolling motion would just keep up with the motion forward. So when you have a surface like leather against concrete, it's gonna be grippy enough, grippy enough that as this ball moves forward, it rolls, and that rolling motion just keeps up so that the surfaces never skid across each other. In other words, this ball's gonna be moving forward, but it's not gonna be slipping across the ground. There's gonna be no sliding motion at this bottom surface here, which means, at any given moment, this is a little weird to think about, at any given moment, this baseball rolling across the ground, has zero velocity at the very bottom. This bottom surface right here isn't actually moving with respect to the ground because otherwise, it'd be slipping or sliding across the ground, but this point right here, that's in contact with the ground, isn't actually skidding across the ground and that means this point right here on the baseball has zero velocity. So this is weird, zero velocity, and what's weirder, that's means when you're driving down the freeway, at a high speed, no matter how fast you're driving, the bottom of your tire has a velocity of zero. It's not actually moving with respect to the ground. It's just, the rest of the tire that rotates around that point. So that point kinda sticks there for just a brief, split second. That makes it so that the tire can push itself around that point, and then a new point becomes the point that doesn't move, and then, it gets rotated around that point, and then, a new point is the point that doesn't move. So, they all take turns, it's very nice of them. Other points are moving. This point up here is going crazy fast on your tire, relative to the ground, but the point that's touching the ground, unless you're driving a little unsafely, you shouldn't be skidding here, if all is working as it should, under normal operating conditions, the bottom part of your tire should not be skidding across the ground and that means that bottom point on your tire isn't actually moving with respect to the ground, which means it's stuck for just a split second. It has no velocity. So that's what we mean by rolling without slipping. Why is this a big deal? I'll show you why it's a big deal. This implies that these two kinetic energies right here, are proportional, and moreover, it implies that these two velocities, this center mass velocity and this angular velocity are also proportional. So that's what I wanna show you here. So, how do we prove that? How do we prove that the center mass velocity is proportional to the angular velocity? Well imagine this, imagine we coat the outside of our baseball with paint. So I'm about to roll it on the ground, right? Roll it without slipping. Let's say I just coat this outside with paint, so there's a bunch of paint here. Now let's say, I give that baseball a roll forward, well what are we gonna see on the ground? We're gonna see that it just traces out a distance that's equal to however far it rolled. So if it rolled to this point, in other words, if this baseball rotates that far, it's gonna have moved forward exactly that much arc length forward, right? 'Cause if this baseball's rolling without slipping, then, as this baseball rotates forward, it will have moved forward exactly this much arc length forward. So in other words, if you unwind this purple shape, or if you look at the path that traces out on the ground, it would trace out exactly that arc length forward, and why do we care? Why do we care that it travels an arc length forward? 'Cause that means the center of mass of this baseball has traveled the arc length forward. So the center of mass of this baseball has moved that far forward. That's the distance the center of mass has moved and we know that's equal to the arc length. What's the arc length? Remember we got a formula for that. If something rotates through a certain angle. So if we consider the angle from there to there and we imagine the radius of the baseball, the arc length is gonna equal r times the change in theta, how much theta this thing has rotated through, but note that this is not true for every point on the baseball. Consider this point at the top, it was both rotating around the center of mass, while the center of mass was moving forward, so this took some complicated curved path through space. It might've looked like that. This distance here is not necessarily equal to the arc length, but the center of mass was not rotating around the center of mass, 'cause it's the center of mass. The center of mass here at this baseball was just going in a straight line and that's why we can say the center mass of the baseball's distance traveled was just equal to the amount of arc length this baseball rotated through. In other words it's equal to the length painted on the ground, so to speak, and so, why do we care? Why do we care that the distance the center of mass moves is equal to the arc length? Here's why we care, check this out. We can just divide both sides by the time that that took, and look at what we get, we get the distance, the center of mass moved, over the time that that took. That's just the speed of the center of mass, and we get that that equals the radius times delta theta over deltaT, but that's just the angular speed. So this shows that the speed of the center of mass, for something that's rotating without slipping, is equal to the radius of that object times the angular speed about the center of mass. So the speed of the center of mass is equal to r times the angular speed about that center of mass, and this is important. This you wanna commit to memory because when a problem says something's rotating or rolling without slipping, that's basically code for V equals r omega, where V is the center of mass speed and omega is the angular speed about that center of mass. Now, you might not be impressed. You might be like, "Wait a minute. "Didn't we already know this? "Didn't we already know that V equals r omega?" We did, but this is different. This V we showed down here is the V of the center of mass, the speed of the center of mass. This V up here was talking about the speed at some point on the object, a distance r away from the center, and it was relative to the center of mass. So, in other words, say we've got some baseball that's rotating, if we wanted to know, okay at some distance r away from the center, how fast is this point moving, V, compared to the angular speed? Well if this thing's rotating like this, that's gonna have some speed, V, but that's the speed, V, relative to the center of mass. What we found in this equation's different. This is the speed of the center of mass. This tells us how fast is that center of mass going, not just how fast is a point on the baseball moving, relative to the center of mass. This gives us a way to determine, what was the speed of the center of mass? And it turns out that is really useful and a whole bunch of problems that I'm gonna show you right now. Let's do some examples. Let's get rid of all this. So let's do this one right here. Let's say you took a cylinder, a solid cylinder of five kilograms that had a radius of two meters and you wind a bunch of string around it and then you tie the loose end to the ceiling and you let go and you let this cylinder unwind downward. As it rolls, it's gonna be moving downward. Let's say you drop it from a height of four meters, and you wanna know, how fast is this cylinder gonna be moving? How fast is this center of mass gonna be moving right before it hits the ground? That's what we wanna know. We're calling this a yo-yo, but it's not really a yo-yo. A yo-yo has a cavity inside and maybe the string is wound around a tiny axle that's only about that big. We're winding our string around the outside edge and that's gonna be important because this is basically a case of rolling without slipping. You might be like, "this thing's not even rolling at all", but it's still the same idea, just imagine this string is the ground. It's as if you have a wheel or a ball that's rolling on the ground and not slipping with respect to the ground, except this time the ground is the string. This cylinder is not slipping with respect to the string, so that's something we have to assume. We're gonna assume this yo-yo's unwinding, but the string is not sliding across the surface of the cylinder and that means we can use our previous derivation, that the speed of the center of mass of this cylinder, is gonna have to equal the radius of the cylinder times the angular speed of the cylinder, since the center of mass of this cylinder is gonna be moving down a distance equal to the arc length traced out by the outside edge of the cylinder, but this doesn't let us solve, 'cause look, I don't know the speed of the center of mass and I don't know the angular velocity, so we need another equation, another idea in here, and that idea is gonna be conservation of energy. This problem's crying out to be solved with conservation of energy, so let's do it. So we're gonna put everything in our system. We're gonna say energy's conserved. Starts off at a height of four meters. That means it starts off with potential energy. So I'm gonna say that this starts off with mgh, and what does that turn into? Well this cylinder, when it gets down to the ground, no longer has potential energy, as long as we're considering the lowest most point, as h equals zero, but it will be moving, so it's gonna have kinetic energy and it won't just have translational kinetic energy. So, it will have translational kinetic energy, 'cause the center of mass of this cylinder is going to be moving. The center of mass of the cylinder is gonna have a speed, but it's also gonna have rotational kinetic energy because the cylinder's gonna be rotating about the center of mass, at the same time that the center of mass is moving downward, so we have to add 1/2, I omega, squared and it still seems like we can't solve, 'cause look, we don't know V and we don't know omega, but this is the key. This is why you needed to know this formula and we spent like five or six minutes deriving it. This is the link between V and omega. So, we can put this whole formula here, in terms of one variable, by substituting in for either V or for omega. Now, I'm gonna substitute in for omega, because we wanna solve for V. So, I'm just gonna say that omega, you could flip this equation around and just say that, "Omega equals the speed "of the center of mass divided by the radius." So I'm gonna use it that way, I'm gonna plug in, I just solve this for omega, I'm gonna plug that in for omega over here. Let's just see what happens when you get V of the center of mass, divided by the radius, and you can't forget to square it, so we square that. So after we square this out, we're gonna get the same thing over again, so I'm just gonna copy that, paste it again, but this whole term's gonna be squared. So I'm gonna have a V of the center of mass, squared, over radius, squared, and so, now it's looking much better. We just have one variable in here that we don't know, V of the center of mass. This I might be freaking you out, this is the moment of inertia, what do we do with that? With a moment of inertia of a cylinder, you often just have to look these up. The moment of inertia of a cylinder turns out to be 1/2 m, the mass of the cylinder, times the radius of the cylinder squared. So we can take this, plug that in for I, and what are we gonna get? If I just copy this, paste that again. If we substitute in for our I, our moment of inertia, and I'm gonna scoot this over just a little bit, our moment of inertia was 1/2 mr squared. So I'm gonna have 1/2, and this is in addition to this 1/2, so this 1/2 was already here. There's another 1/2, from the moment of inertia term, 1/2mr squared, but this r is the same as that r, so look it, I've got a, I've got a r squared and a one over r squared, these end up canceling

, and this is really strange, it doesn't matter what the radius of the cylinder was, and here's something else that's weird, not only does the radius cancel, all these terms have mass in it. So no matter what the mass of the cylinder was, they will all get to the ground with the same center of mass speed. In other words, all yo-yo's of the same shape are gonna tie when they get to the ground as long as all else is equal when we're ignoring air resistance. No matter how big the yo-yo, or have massive or what the radius is, they should all tie at the ground with the same speed, which is kinda weird. So now, finally we can solve for the center of mass. We've got this right hand side. The left hand side is just gh, that's gonna equal, so we end up with 1/2, V of the center of mass squared, plus 1/4, V of the center of mass squared. That's just equal to 3/4 speed of the center of mass squared. If you take a half plus a fourth, you get 3/4. So if I solve this for the speed of the center of mass, I'm gonna get, if I multiply gh by four over three, and we take a square root, we're gonna get the square root of 4gh over 3, and so now, I can just plug in numbers. If I wanted to, I could just say that this is gonna equal the square root of four times 9.8 meters per second squared, times four meters, that's where we started from, that was our height, divided by three, is gonna give us a speed of the center of mass of 7.23 meters per second. Now, here's something to keep in mind, other problems might look different from this, but the way you solve them might be identical. For instance, we could just take this whole solution here, I'm gonna copy that. Let's try a new problem, it's gonna be easy. It's not gonna take long. Let's say we take the same cylinder and we release it from rest at the top of an incline that's four meters tall and we let it roll without slipping to the bottom of the incline, and again, we ask the question, "How fast is the center of mass of this cylinder "gonna be going when it reaches the bottom of the incline?" Well, it's the same problem. It looks different from the other problem, but conceptually and mathematically, it's the same calculation. This thing started off with potential energy, mgh, and it turned into conservation of energy says that that had to turn into rotational kinetic energy and translational kinetic energy. Again, if it's a cylinder, the moment of inertia's 1/2mr squared, and if it's rolling without slipping, again, we can replace omega with V over r, since that relationship holds for something that's rotating without slipping, the m's cancel as well, and we get the same calculation. This cylinder again is gonna be going 7.23 meters per second. The center of mass is gonna be traveling that fast when it rolls down a ramp that was four meters tall. So recapping, even though the speed of the center of mass of an object, is not necessarily proportional to the angular velocity of that object, if the object is rotating or rolling without slipping, this relationship is true and it allows you to turn equations that would've had two unknowns in them, into equations that have only one unknown, which then, let's you solve for the speed of the center of mass of the object.

**VOCABULARY:**

presentation: sunum

torque: tork

to watch: seyretmek

to glance: göz atmak

glancıng: tesadüfi, önemsiz

shuttle: mekik

shuttle bus

To float: (havada suda) yüzmek, asılı kalmak

the hand of a clock

short hand = hour hand:akrep (saat)

hand = minute hand: yelkovan

pivot: eksen, dönme noktası,

axis: eksen

looks an awful lot like: felaket derecede .....benziyor

switch: elektrik düğmesi,şalter

to switch: değiştirmek

notation: notasyon

to deal with: uğraşmak, alakadar olmak

fun: eğlence

horsepower: beygir gücü

pretty: sevimli, oldukça

arbitrary: gelişigüzel, tesadüfi

convention: convensiyon

clockwise: saat yönünde

counterclockwise: saat yönünün aksinde

beyond: ötesi, ötede

scope: faaliyet alanı, kapsam

intensive: yoğun,aşırı,şiddetli

tool: alet

vague: belirsiz, müphem,hayal meyal

vaguely: belli belirsiz

to make up: uydurmak,telafi etmek,makyaj yapmak

make up exam: telafi imtihanı

make up: makyaj

bright: parlak

to offsett: dengelemek

to connect: bağlamak, bitiştirmek, ulamak

advantage: avantaj

guy: adam, herif

twice as far away: iki misli daha uzak

balance: denge,(banka hesabı)bakiye, terazi

to balance: dengelemek, tartmak

kind of: gibi, az çok

I'm about to run out of time: vaktim bitmek üzere

to roll: yuvarlanmak

roll: rulo,tomar,dürüm

to slip: kaymak,sürçmek

necessarily: mutlaka

proportional: orantılı

directly proportional: doğru orantılı

indirectly proportional: ters orantılı

in other words: yani

to pitch: (top) atmak

concrete: beton

cement: çimento

to chuck: atmak,fırlatmak,bırakmak

to skid: patinaj yapmak

Grippy

leather: deri

surface: satıh, yüzey

to contact: temas etmek

weird: tuhaf, acayip

freeway: otoban, expres yol

kinda = kind of

brief: kısa, hulasa,özet

to brief: özetlemek, talimat veya bilgi vermek

split second: saniyeden az bir zaman,göz açıp kapayıncaya kadar

crazy: çılgın,deli

a big deal: önemli bir şey

coat: palto, katman,

to coat: kaplamak

coating:kaplama

coated: kaplı, kaplanmış

trace: iz

to trace: iz bıramak

however far it rolled: nekadar uzağa yuvarlandıysa

to wind /waynd/: (saat vs)kurmak, sarmak

to unwind: çözmek

'Cause=because

care: bakım, umursama

to care: bakmak, ihtimam göstermek,umursamak

wanna= want to

to commit to memory: ezberlemek

impressed: etkilenmiş

cavity: oyuk, kovuk

wound: kurulmuş, sarılmış

to cry out: bağırmak

to spend: harcamak, geçirmek

to substitute: (bir formulde) yerine koymak

to flip: tersine çevirmek

to plug in: yerine koymak

to freak out: korkutmak

to scoot: çabucak hareket etmek, kaymak,kaydırmak

to cancel: iptal etmek

to ignore: aldırmamak, umursamamak, önemsememek

to keep in mind: akılda tutmak

incline: eğik düzlem

to recapitulate: özetlemek, tekrarlamak

to recap = recapitulate