Dejenere Çekirdekli Denklemler
(Direkt çözüm yöntemi)

$$K(x,t) = r(x).s(t)$$

$x^2 t^3$

$\sin x . \cos t$

$$u(x) = f(x) + \lambda \int_a^b K(x,t)\,u(t)\,dt$$

$$u(x) = f(x) + \lambda \int_a^b \underbrace{r(x)s(t)}\,u(t)\,dt$$

$$u(x) = f(x) + \lambda r(x) \underbrace{\int_a^b s(t)\,u(t)\,dt}_{A}$$

$$A = \int_a^b s(t)u(t)\,dt$$

$$\boxed{u(x) = f(x) + \lambda r(x).A}$$ çözüm modeli

$$\cancel{f(x)} + \cancel{\lambda} r(\cancel{x}).A = \cancel{f(x)} + \cancel{\lambda} r(\cancel{x}) \int_a^b s(t)\left[f(t) + \lambda r(t)A\right]dt$$

$$A = \int_a^b s(t)f(t)\,dt + \lambda A \int_a^b s(t)r(t)\,dt$$

$$\left[1 - \lambda \int_a^b s(t)\,r(t)\,dt\right] A = \int_a^b s(t)\,f(t)\,dt$$

$$A = \frac{\int_a^b s(t)f(t)\,dt}{1 - \lambda \int_a^b s(t)f(t)\,dt}$$

$$u(x) = f(x) + \lambda r(x).A$$

$$1 - \lambda \int_a^b s(t)f(t)\,dt \neq 0$$

Örnek

$$u(x) = x^2 + \lambda \int_0^1 x^2 t^2 u(t)\,dt$$

$$u(x) = x^2 + \lambda x^2 \underbrace{\int_0^1 t^2 u(t)\,dt}_{A} \rightarrow \boxed{u(x) = x^2 + \lambda x^2 A}$$?

$$\cancel{x^2} + \cancel{\lambda x^2} A = \cancel{x^2} + \cancel{\lambda x^2} \int_0^1 t^2 \left[t^2 + \lambda t^2 A\right] dt$$

$$A = \underbrace{\int_0^1 t^4\,dt}_{\frac{1}{5}} + \lambda A \underbrace{\int_0^1 t^4\,dt}_{\frac{1}{5}}$$

$$A\left(1 - \frac{\lambda}{5}\right) = \frac{1}{5} \rightarrow A\left(\frac{5-\lambda}{\cancel{5}}\right) = \frac{1}{\cancel{5}}$$

$$A = \frac{1}{5-\lambda} \qquad (\lambda \neq 5)$$

$$u(x) = x^2 + \lambda x^2 A$$

$$\boxed{u(x) = x^2 + \frac{\lambda x^2}{5-\lambda}} \quad (\lambda \neq 5)$$

$$\underbrace{e^x . e^{-t}}$$

SORU

$$u(x) = x e^{-x} + \lambda \int_0^1 e^{x-t} u(t)\,dt$$

Cevap: $u(x) = x e^{-x} + \lambda e^x \left(\dfrac{1 - 3e^{-2}}{4(1-\lambda)}\right)$

$(\lambda \neq 1)$

Dejenere Çekirdeğin Genel Hali:

$$K(x,t) = \sum_{i=1}^{n} r_i(x) s_i(t) = r_1(x)s_1(t) + r_2(x)s_2(t) + \dots + r_n(x)s_n(t)$$

$$u(x) = f(x) + \lambda \int_a^b K(x,t)\,u(t)\,dt$$

$$u(x) = f(x) + \lambda \int_a^b \left[r_1(x)s_1(t) + r_2(x)s_2(t) + \dots + r_n(x)s_n(t)\right] u(t)\,dt$$

$$\Rightarrow u(x) = f(x) + \lambda r_1(x) \underbrace{\int_a^b s_1(t)u(t)\,dt}_{A_1} + \lambda r_2(x) \underbrace{\int_a^b s_2(t)u(t)\,dt}_{A_2} + \dots + \lambda r_n(x) \underbrace{\int_a^b s_n(t)u(t)\,dt}_{A_n}$$

$$\boxed{u(x) = f(x) + \lambda r_1(x)A_1 + \lambda r_2(x)A_2 + \dots + \lambda r_n(x)A_n}$$

Çözüm modeli

$$\cancel{f(x)} + \lambda r_1(x)A_1 + \lambda r_2(x)A_2 + \dots + \lambda r_n(x)A_n$$

$$= \cancel{f(x)} + \lambda r_1(x) \int_a^b s_1(t)\left[f(t) + \lambda r_1(t)A_1 + \lambda r_2(t)A_2 + \dots + \lambda_n r_n(t)A_n\right]dt$$

$$+ \dots + \lambda r_n(x) \int_a^b s_n(t)\left[f(t) + \lambda r_1(t)A_1 + \lambda r_2(t)A_2 + \dots + \lambda_n r_n(t)A_n\right]dt$$

$$A_1 = \underbrace{\int_a^b s_1(t)f(t)\,dt}_{B_1} + \lambda A_1 \underbrace{\int_a^b s_1(t)r_1(t)\,dt}_{C_{11}} + \lambda A_2 \int_a^b s_1(t)r_2(t)\,dt + \dots + \lambda A_n \int_a^b s_1(t)r_n(t)\,dt$$

$\vdots$

$$A_n = \underbrace{\int_a^b s_n(t)f(t)\,dt} + \lambda A_1 \int_a^b s_n(t)r_1(t)\,dt + \lambda A_2 \int_a^b s_n(t)r_2(t)\,dt + \dots + \lambda A_n \int_a^b s_n(t)r_n(t)\,dt$$

$$\int_a^b s_i(t) r_j(t)\,dt = C_{ij} \quad \begin{pmatrix} i=1,\dots,n \\ j=1,\dots,n \end{pmatrix} \qquad B_i, C_{ij} \in \mathbb{R}$$

$$\int_a^b s_i(t) f(t)\,dt = B_i \quad (i=1,\dots,n)$$

$$\begin{cases} (1-\lambda C_{11})A_1 - \lambda C_{12}A_2 - \dots - \lambda C_{1n}A_n = B_1 \\ -\lambda C_{21}A_1 + (1-\lambda C_{22})A_2 - \dots - \lambda C_{2n}A_n = B_2 \\ \quad\vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \\ -\lambda C_{n1}A_1 - \lambda C_{n2}A_2 - \dots + (1-\lambda C_{nn})A_n = B_n \end{cases}$$

Bu sistem bir lineer denklem sistemidir. Bu sistemden $A_1, A_2, \dots, A_n$ çözülür.

$$u(x) = f(x) + \lambda\left[A_1 r_1(x) + A_2 r_2(x) + \dots + A_n r_n(x)\right]$$

SORU

$$u(x) = 11x + 10x^2 + x^3 - \int_0^1 (30\underbrace{xt^2} + 20\underbrace{x^2 t})\,dt$$

$$u(x) = 11x + 10x^2 + x^3 - 30x\underbrace{\int_0^1 t^2 u(t)\,dt} - 20x^2 \underbrace{\int_0^1 t u(t)\,dt}$$

$$u(x) = 11x + 10x^2 + x^3 - 30xA_1 - 20x^2 A_2$$

çözüm modeli

$$11x + 10x^2 + x^3 - 30xA_1 - 20x^2A_2$$

$$= \cancel{11x} + \cancel{10x^2} + \cancel{x^3} - 30x\int_0^1 t^2\left[11t + 10t^2 + t^3 - 30tA_1 - 20t^2A_2\right]dt$$

$$-20x^2\int_0^1 t\left[\qquad // \qquad \right]dt$$

$$A_1 = 11\int_0^1 t^3 dt + 10\int_0^1 t^4 dt + \int_0^1 t^5 dt - 30A_1\int_0^1 t^3 dt - 20A_2\int_0^1 t^4 dt$$

$$A_2 = 11\int_0^1 t^2 dt + 10\int_0^1 t^3 dt + \int_0^1 t^4 dt - 30A_1\int_0^1 t^2 dt - 20A_2\int_0^1 t^3 dt$$

$$A_1 = \left(\frac{11}{4} + 2 + \frac{1}{6}\right) - \frac{30}{4}A_1 - 4A_2$$

$$A_2 = \left(\frac{11}{3} + \frac{10}{4} + \frac{1}{5}\right) - 10A_1 - 5A_2$$

$$\left(1 + \frac{15}{2}\right)A_1 + 4A_2 = \frac{59}{12}$$

$$10A_1 + 6A_2 = \frac{191}{30}$$

$$17A_1 + 8A_2 = \frac{59}{6}$$

$$10A_1 + 6A_2 = \frac{191}{30}$$

$$A_1 = \frac{11}{30} \qquad A_2 = \frac{9}{20}$$

$$u(x) = 11x + 10x^2 + x^3 - 30x\underbrace{A_1}_{\downarrow \frac{11}{30}} - 20x^2\underbrace{A_2}_{\downarrow \frac{9}{20}}$$

$$u(x) = \cancel{11x} + 10x^2 + x^3 - \cancel{11x} - 9x^2$$

$$\boxed{u(x) = x^2 + x^3}$$

SORULAR

1) $u(x) = \cos x + \lambda \int_0^{\pi} (x\sin t + t\sin x)\, u(t)\, dt$

Cevap: $u(x) = \cos x - \dfrac{\lambda}{(1-\lambda\pi)^2 - \frac{\lambda^2\pi^2}{6}} \left(\lambda\pi x + 2(1-\lambda\pi)\sin x\right)$

2) $u(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, u(t)\, dt$

Cevap: $u(x) = \cos x + \lambda \sin x \dfrac{2\pi}{4+\lambda^2\pi^2} - \lambda^2 \cos x \dfrac{\pi^2}{4+\lambda^2\pi^2}$