Yüksek Mertebeden Lineer Olmayan Dif. Denk.

* $F(x, y, y', y'', y''', ---, y^{(n)}) = 0 \quad (n \geq 2)$

a) Bağımlı değişkeni içermeyen dif. denklemler

$$F(x, y', y'', y''', ---, y^{(n)}) = 0$$

$$\left.\begin{array}{l} y' = t \\ y'' = t' \\ y''' = t'' \\ \vdots \\ y^{(n)} = t^{(n-1)} \end{array}\right\} \quad \underbrace{F(x, t, t', t'', ---, t^{(n-1)})}_{} = 0$$

$$t = \underbrace{\varphi(x, c_1, c_2, ---, c_{n-1})}_{} = \frac{dy}{dx}$$

$$\int dy = \int \varphi(x, c_1, ---, c_n)\, dx$$

$$y = \phi(x, c_1, c_2, ---, c_{n-1}, c_n)$$

Örnek $y'' - (y')^2 = 0$ dif. denklemini çözünüz.

$$\left.\begin{array}{l} y' = t \\ y'' = t' \end{array}\right\} \quad t' - t^2 = 0$$

$$\frac{dt}{dx} = t^2 \Rightarrow \int \frac{dt}{t^2} = \int dx$$

$$-\frac{1}{t} = x + C_1$$

$$t = -\frac{1}{x + C_1}$$

$$\underbrace{\frac{d^2y}{dx^2}} = -\frac{1}{x+C_1}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = -\frac{1}{x+C_1} \Rightarrow \int d\left(\frac{dy}{dx}\right) = \int -\frac{1}{x+C_1}\,dx$$

$$\frac{dy}{dx} = -\ln(x+C_1) + C_2$$

$$\int \ln x\,dx = x\ln x - x$$

$$\int dy = \int \left[-\ln(x+C_1) + C_2\right] dx$$

$$\boxed{y = -(x+C_1)\ln(x+C_1) + x + C_1 + C_2x + C_3}$$

Örnek $\quad xy'' - 4y' = 0$

1.Yol $\quad x^2y'' - 4xy' = 0$

(x ile çarparak ...)

Cauchy-Euler $\qquad D = \frac{d}{dt}$

$x = e^t \quad y' = e^{-t}Dy$

$y'' = e^{-2t}D(D-1)y$

⋮

2.Yol $\quad y' = t$

$y'' = t'$

$$xt' - 4t = 0$$

$$x\frac{dt}{dx} = 4t$$

$$\int \frac{dt}{t} = \int 4\frac{dx}{x}$$

$$\ln t = 4\ln x + \ln C_1$$

$$t = C_1x^4$$

$$t = \frac{dy}{dx} = C_1 x^4$$

$$\int dy = \int C_1 x^4 \, dx$$

$$y = C_1 \frac{x^5}{5} + C_2$$

Örnek $\quad x y'' - y' = \sqrt{x^2 + (y')^2}$

$$\left.\begin{array}{l} y' = t \\ y'' = t' \end{array}\right\} \quad x t' - t = \sqrt{x^2 + t^2}$$

$$x t' = t + \sqrt{x^2 + t^2}$$

$$t' = \frac{t}{x} + \frac{\sqrt{x^2 + t^2}}{x}$$

$$t' = \frac{t}{x} + \sqrt{\frac{x^2 + t^2}{x^2}}$$

$$t' = \frac{t}{x} + \sqrt{1 + \left(\frac{t}{x}\right)^2}$$

$$\frac{t}{x} = u \rightarrow t = ux$$

$$t' = u'x + u$$

$$u'x + \cancel{u} = \cancel{u} + \sqrt{(1+u)^2}$$

$$\frac{du}{dx} x = \sqrt{1+u^2} \rightarrow \int \frac{du}{\sqrt{1+u^2}} = \int \frac{dx}{x}$$

$$\ln\left(u + \sqrt{1+u^2}\right) = \ln x + \ln C_1$$

$$u + \sqrt{1+u^2} = C_1 x$$

$$\left(\sqrt{1+u^2}\right)^2 = (C_1 x - u)^2$$

$$1 + \cancel{u^2} = C_1^2 x^2 - 2C_1 x u + \cancel{u^2}$$

$$\boxed{u = \frac{C_1^2 x^2 - 1}{2C_1 x}} = \frac{t}{x}$$

$$t = \boxed{\frac{C_1^2 x^2 - 1}{2C_1} = \frac{dy}{dx}}$$

$$\int dy = \int \frac{C_1^2 x^2 - 1}{2C_1} dx = \int \left(\frac{C_1^{\cancel{2}} x^2}{\cancel{2C_1}} - \frac{1}{2C_1}\right) dx$$

$$y = \frac{C_1}{6} x^3 - \frac{x}{2C_1} + C_2$$

b) Bağımsız değişkeni içermeyen dif. denk.

$$F(y, y', y'', y''', \dots, y^{(n)}) = 0 \qquad y' = P \checkmark$$

$$y'' = \frac{dP}{dx}$$

$$\left.\begin{aligned} y' &= P \\ y'' &= P\frac{dP}{dy} \\ y''' &= P\frac{d}{dy}\left(\frac{dP}{dy}\right) \\ &\vdots \end{aligned}\right\} \quad F\left(y, P, P\frac{dP}{dy}, \dots\right) = 0$$

Bağımlı

Bağımsız

$$P = \varphi(y, C_1, C_2, \dots, C_{n-1})$$

$$\frac{dy}{dx} = \varphi(y, c_1, \dots, c_{n-1})$$

$$\int \frac{dy}{\varphi(y, c_1, \dots, c_n)} = \int dx \Rightarrow \boxed{x = \phi(y, c_1, c_2, \dots, c_{n-1}, c_n)}$$

Örnek $y'' - (y')^3 = 0$

$$\left.\begin{aligned} y' &= p \\ y'' &= p\frac{dp}{dy} \end{aligned}\right\} \quad p\frac{dp}{dy} - p^3 = 0$$

$$p \cdot \left(\frac{dp}{dy} - p^2\right) = 0$$

1) $p = 0 \rightarrow \frac{dy}{dx} = 0 \rightarrow y = C$ Sahteker çözüm

2) $\frac{dp}{dy} - p^2 = 0 \rightarrow \frac{dp}{dy} = p^2$

$$\int \frac{dp}{p^2} = \int dy$$

$$-\frac{1}{p} = y + C_1$$

$$p = \boxed{-\frac{1}{y + C_1} = \frac{dy}{dx}}$$

$$\int dx = \int -(y + C_1)\, dy$$

$$\boxed{x = -\frac{y^2}{2} - C_1 y + C_2}$$

Örnek

$$yy'' = (y')^2$$

$$\left.\begin{aligned} y' &= P \\ y'' &= P\frac{dP}{dy} \end{aligned}\right\} \quad yP\frac{dP}{dy} = P^2$$

$$P\left(y\frac{dP}{dy} - P\right) = 0$$

1) $P = 0 \rightarrow \frac{dy}{dx} = 0 \rightarrow y = C$

2) $\underbrace{y\frac{dP}{dy} - P = 0}$

$$y\frac{dP}{dy} = P$$

$$\int \frac{dP}{P} = \int \frac{dy}{y}$$

$$\ln P = \ln y + \ln C_1$$

$$P = C_1 y = \frac{dy}{dx} \rightarrow \int \frac{dy}{y} = \int C_1\, dx$$

$$\ln y - \ln C_2 = C_1 x$$

$$\ln \frac{y}{C_2} = C_1 x$$

$$\frac{y}{C_2} = e^{C_1 x} \rightarrow \boxed{y = C_2 e^{C_1 x}}$$

Örnek $yy'' = (y')^2 + y(y')^3$

$$y' = P, \quad y'' = P\frac{dP}{dy} \quad \Bigg\} \quad yP\frac{dP}{dy} = P^2 + yP^3$$

$$P\left(y\frac{dP}{dy} - P - yP^2\right) = 0$$

1) $P = 0 \rightarrow y = C$

2) $y\frac{dP}{dy} - P - yP^2 = 0$

$$\frac{dP}{dy} - \frac{P}{y} - P^2 = 0$$

$$\frac{dP}{dy} - \frac{1}{y}P = P^2 \longrightarrow \text{Bernoulli}$$

$$\frac{1}{P^2}\frac{dP}{dy} - \frac{1}{y}\frac{1}{P} = 1 \qquad z = \frac{1}{P}$$

$$\frac{dz}{dy} = -\frac{1}{P^2}\frac{dP}{dy}$$

$$-\frac{dz}{dy} - \frac{1}{y}z = 1$$

$$\frac{dz}{dy} + \frac{1}{y}z = -1 \quad \text{Lineer dif.}$$

Size ait ... ~