Örnek $(\tan y - 3x^2)dx + x\sec^2 y\,dy = 0$

$$\frac{\partial M}{\partial y} = \sec^2 y = \frac{\partial N}{\partial x} \quad \text{Tam d.d.}$$

$$u(x,y) = C$$

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = (\tan y - 3x^2)dx + x\sec^2 y\,dy$$

$$\frac{\partial u}{\partial x} = \tan y - 3x^2 \quad ; \quad \frac{\partial u}{\partial y} = x\sec^2 y$$

$$u(x,y) = \int (\tan y - 3x^2)dx$$

$$u(x,y) = x\tan y - x^3 + \underbrace{h(y)}_{?}$$

$$\frac{\partial u}{\partial y} = x\sec^2 y + \frac{dh}{dy}$$

$$\cancel{x\sec^2 y} + \frac{dh}{dy} = \cancel{x\sec^2 y}$$

$$\frac{dh}{dy} = 0 \qquad h(y) = C_1$$

$$u(x,y) = C$$

$$x\tan y - x^3 + C_1 = C \qquad C - C_1 = C$$

$$\boxed{x\tan y - x^3 = C} \quad \text{Genel ç.}$$

$$\frac{\partial u}{\partial x} = \tan y - 3x^2$$

$$\cancel{\tan y} + \frac{dh}{dx} = \cancel{\tan y} - 3x^2$$

$$\frac{dh}{dx} = -3x^2$$

$$\frac{\partial u}{\partial y} = x\sec^2 y$$

$$u(x,y) = \int x\sec^2 y\,dy$$

$$\boxed{u(x,y) = x\tan y + \underbrace{h(x)}_{?}}$$

$$\int dh = \int -3x^2 dx$$

$$h(x) = -x^3 + C_1$$

$$\frac{\partial u}{\partial x} = \tan y + \frac{dh}{dx}$$

$$u(x,y) = C$$

$$x\tan y - x^3 + C_1 = C \longrightarrow C - C_1 = C$$

$$\boxed{x\tan y - x^3 = C}$$

Tam Dif. Denkleme Dönüştürme (integrasyon çarpanı)

$$M(x,y)dx + N(x,y)dy = 0 \qquad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \text{ Tam dif. değil}$$

$\lambda(x,y)$

$$\underbrace{\lambda(x,y).M(x,y)}_{M_1(x,y)}dx + \underbrace{\lambda(x,y)\,N(x,y)}_{N_1(x,y)}dy = 0$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x} \quad \text{Tam dif.} \checkmark$$

$\lambda(x,y) \rightarrow$ integresyon çarpanı

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$$\frac{\partial}{\partial y}(\lambda.M) = \frac{\partial}{\partial x}(\lambda.N)$$

$$\frac{\partial \lambda}{\partial y}M + \lambda\frac{\partial M}{\partial y} = \frac{\partial \lambda}{\partial x}N + \lambda\frac{\partial N}{\partial x}$$

$$* \quad \lambda\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{\partial \lambda}{\partial x}N - \frac{\partial \lambda}{\partial y}.M$$

1°) $\lambda = \lambda(x)$ olması halinde

$$\frac{d\lambda}{dx} N = \lambda \cdot \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\int \frac{d\lambda}{\lambda} = \int \underbrace{\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}}_{x \text{ değişkenli bir fonk.}} dx$$

$$\boxed{\ln \lambda = \int \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \right) dx} \quad \lambda = \lambda(x)$$

2°) $\lambda = \lambda(y)$

$$-\frac{d\lambda}{dy} M = \lambda \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\int \frac{d\lambda}{\lambda} = \int \underbrace{\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}}_{y \text{ değişkenli bir fonk}} dy$$

$$\boxed{\ln \lambda = \int \left(\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} \right) dy} \quad \lambda = \lambda(y)$$

M.A. çarpanı

Örnek

$$\overbrace{(xy+1) \cdot y}^{xy^2+y} dx + \underbrace{(2y - x)} dy = 0$$

$$\frac{\partial M}{\partial y} = 2xy + 1 \neq \frac{\partial N}{\partial x} = -1 \quad \text{Tam dif. değil}$$

$\lambda = \lambda(x)$

$$\ln\lambda = \int \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \right) dx = \int \frac{2xy+1-(-1)}{2y-x} dx = \underbrace{\int \frac{2(xy+1)}{2y-x} dx}$$

$\lambda = \lambda(y)$

$$\ln\lambda = \int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} dy = \int \frac{\overbrace{-1-(2xy+1)}^{-2(1+xy)}}{(xy+1)\cdot y} dy$$

$$= \int \frac{-2\cancel{(1+xy)}}{\cancel{(xy+1)}\cdot y} dy = \int -\frac{2}{y} dy = -2\ln y$$

$$\ln\lambda = -2\ln y \Rightarrow \lambda = \frac{1}{y^2}$$

$$\frac{1}{y^2}(xy^2+y)dx + \frac{1}{y^2}(2y-x)dy = 0$$

$$\underbrace{\left(x+\frac{1}{y}\right)}dx + \underbrace{\left(\frac{2}{y}-\frac{x}{y^2}\right)}dy = 0$$

$$\frac{\partial M_1}{\partial y} = -\frac{1}{y^2} = \frac{\partial N_1}{\partial x}$$ Tam dif.

$$u(x,y) = C$$

$$\frac{\partial u}{\partial x} = x + \frac{1}{y}$$

$$u(x,y) = \int \left(x+\frac{1}{y}\right) dx$$

$$u(x,y) = \frac{x^2}{2} + \frac{x}{y} + h(y)$$

?

$$\frac{\partial u}{\partial y} = \frac{-x}{y^2} + \frac{dh}{dy}$$

$$\frac{\partial u}{\partial y} = \frac{2}{y} - \frac{x}{y^2}$$

$$\cancel{-\frac{x}{y^2}} + \frac{dh}{dy} = \underbrace{\frac{2}{y}} - \cancel{\frac{x}{y^2}}$$

$$\frac{dh}{dy} = \frac{2}{y}$$

$$\int dh = \int \frac{2}{y} dy \rightarrow h(y) = 2\ln y + c_1$$

$u(x,y)=C$

$M\,dx + N\,dy = 0$

$$\boxed{\frac{x^2}{2}+\frac{x}{y}+2\ln y = C}$$

Örnek $2y\,dx + (1-\ln y-2x)\,dy = 0$

$\frac{\partial M}{\partial y}=2 \qquad \frac{\partial N}{\partial x}=-2$ Tam dif. değil

$\lambda=\lambda(y) \qquad \ln\lambda=\int\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}\,dy=\int\frac{-2-2}{2y}\,dy$

$=\int\frac{-4}{2y}\,dy=-\int\frac{2}{y}\,dy=-2\ln y$

$\ln\lambda=\ln y^{-2} \Rightarrow \lambda=\frac{1}{y^2} \qquad \int\frac{\ln y}{y^2}\,dy$

cevap: $\boxed{\frac{2x}{y}+\frac{\ln y}{y}=C}$

Soru $(x-3y)dx - x\,dy = 0$ dif. denklemini $\boxed{\lambda=x^n}$ olacak şekilde bir int. çarpanı bularak tam dif. hale dönüştürüp

Çözünüz:

$-x^3 \quad -3x^2$

$-x^4 \quad -4x^3$

$x^n(x-3y)dx - x^n x\,dy = 0$

$(\underbrace{x^{n+1}-3x^n y}_{})dx - \underbrace{x^{n+1}}_{}dy = 0$

$\frac{\partial M_1}{\partial y}=\frac{\partial N_1}{\partial x} \rightarrow \cancel{+}3\cancel{x^n}=\cancel{+}(n+1)\cancel{x^n}$

$n=2$

$\boxed{\lambda=x^2}$

$\boxed{\lambda=x^\alpha . y^\beta \qquad \lambda=x^\alpha+y^\beta}$