1. Mertebeden Yüksek Dereceden Dif. Denklemler

1) Clairaut Dif. Denklemi:

$$y = xy' + \underbrace{\varphi(y')}_{}$$ lineerliği bozar

$\boxed{y' = P}$

$$\boxed{y = P.x + \varphi(P)} \quad *$$

$$\cancel{P} = y' = \frac{dP}{dx}x + \cancel{P} + \frac{dP}{dx}\varphi'(P)$$

$A.B = 0$

$A = 0 \vee B = 0$

$$\frac{dP}{dx}.(x + \varphi'(P)) = 0$$

1°) $\frac{dP}{dx} = 0 \rightarrow P = C$

$$\boxed{y = Cx + \varphi(C)}$$ Genel Çözüm

2°) $x + \varphi'(P) = 0 \rightarrow \begin{cases} x = -\varphi'(P) \\ y = P(-\varphi'(P)) + \varphi(P) \end{cases}$

Tekil Çözümün parametrik denk.

$$\boxed{F(x, y) = 0 \text{ Tekil Çözüm}}$$

örnek $y = xy' + y' - (y')^2$ dif. denkleminin tüm çözümlerini araştırınız.

$$y' = P \quad * \quad \boxed{y = P.x + P - P^2}$$

$$\cancel{P} = y' = \frac{dP}{dx}x + \cancel{P} + \frac{dP}{dx} - 2P\frac{dP}{dx}$$

$$\frac{dP}{dx}(x + 1 - 2P) = 0$$

1) $\frac{dP}{dx} = 0 \rightarrow P = C \rightarrow \boxed{y = Cx + C - C^2}$

2) $x + 1 - 2P = 0 \qquad \underline{x = 2P - 1}$

$$y = P(2P-1) + P - P^2 = P^2$$
$$2P^2 - \cancel{P} + \cancel{P} - P^2$$

$\left.\begin{array}{l} x = 2P - 1 \\ y = P^2 \end{array}\right\}$ Tekil çözümün parametrik denklemleri

$$P = \frac{x+1}{2} \qquad \boxed{y = \left(\frac{x+1}{2}\right)^2} \text{ Tekil çözüm}$$

örnek

$$y = xy' + \sqrt{y'}$$

$$y' = P \qquad \boxed{y = Px + \sqrt{P}} \quad *$$

$$\cancel{P} = y' = \frac{dP}{dx}x + \cancel{P} + \frac{dP}{dx}\frac{1}{2\sqrt{P}}$$

$$\frac{dP}{dx}\left(x + \frac{1}{2\sqrt{P}}\right) = 0$$

1) $\frac{dp}{dx}=0 \rightarrow p=C$ $\quad y=Cx+\sqrt{C}$
Genel Ç.

2) $x+\frac{1}{2\sqrt{p}}=0$ $\quad \underbrace{\left(x=-\frac{1}{2\sqrt{p}}\right)}$ T.Ç. P.D.

$y=\cancel{p}\left(-\frac{1}{2\cancel{\sqrt{p}}}\right)+\sqrt{p}$

$\left.\begin{array}{l} x=-\frac{1}{2\sqrt{p}} \\ y=\frac{\sqrt{p}}{2} \end{array}\right\}$

$y=-\frac{\sqrt{p}}{2}+\sqrt{p}=\frac{\sqrt{p}}{2}$

$x=-\frac{1}{2\sqrt{p}} \Rightarrow \sqrt{p}=-\frac{1}{2x}$

$y=\frac{\sqrt{p}}{2} \Rightarrow y=\frac{-1/2x}{2}$

$$\boxed{y=\frac{-1}{4x}}$$

2) Lagrange Dif. Denklemi

$y=x\,f(y')+\varphi(y')$

$y'=p$ $\quad \underbrace{y=x.f(p)+\varphi(p)}$ *

$p=y'=f(p)+x.\frac{dp}{dx}f'(p)+\frac{dp}{dx}.\varphi'(p)$

$\frac{dp}{dx}\left(x f'(p)+\varphi'(p)\right)+f(p)-p=0$

$$\frac{dx}{dp}\left(f(p)-p\right)+x f'(p)+\varphi'(p)=0$$

Lineer dif.

$$\frac{dx}{dp}+\frac{f'(p)}{f(p)-p}x=-\frac{\varphi'(p)}{f(p)-p}$$ Lineer dif

Lagrange d.d. Genel Çözümü p.d.

$$\begin{cases} x=F(p,c) \\ y=F(p,c)f(p)+\varphi(p) \end{cases}$$

Örnek

$$y=x(\underbrace{1+y'}_{f(y')})+\underbrace{(y')^2}_{\varphi(y')} \qquad y'=p$$

$$y=x.(1+p)+p^2$$

$$\cancel{p}=y'=1+\cancel{p}+x.\frac{dp}{dx}+2p\frac{dp}{dx}$$

$y'+\underbrace{p(x)}.y=q(x)$

$\ln\lambda=\int p(x)\,dx$

$$\frac{dp}{dx}(x+2p)+1=0$$

$$\frac{dx}{dp}+x+2p=0 \longrightarrow \frac{dx}{dp}+x=-2p$$

Lineer dif.

$$\ln\lambda=\int 1.\,dp=p$$

$$\lambda=e^p$$

$$e^{p}\frac{dx}{dp}+e^{p}\cdot x=-2pe^{p}$$

$$\frac{d}{dp}(e^{p}\cdot x)=-2pe^{p}\Rightarrow\int d(e^{p}x)=\int-2pe^{p}dp$$

$$e^{p}\cdot x=-2(p-1)e^{p}+C$$

$$\boxed{x=-2(p-1)+Ce^{-p}}$$

$$\int pe^{p}dp=pe^{p}-\int e^{p}dp=pe^{p}-e^{p}=(p-1)e^{p}$$

$p=u$ $\quad e^{p}dp=dv$

$dp=du$ $\quad e^{p}=v$

$$\begin{cases}x=-2(p-1)+Ce^{-p}\\ y=(1+p)\left[-2(p-1)+Ce^{-p}\right]+p^{2}\end{cases}$$

Lagrange dif. denk. Genel Çöz. Par. Denk.

Örnek

$$y=2xy'-(y')^{2}-1$$

$y'=p$

$$\boxed{y=2xp-p^{2}-1}$$

$$\cancel{p}=y'=2\cancel{p}+2x\frac{dp}{dx}-2p\frac{dp}{dx}$$

$$\frac{dp}{dx}(2x-2p)+p=0\ *$$

$\frac{p}{x}=u \quad p=ux$

$$\underbrace{p\,dx+(2x-2p)dp=0}_{\text{Homojen}}$$

$$p\frac{dx}{dp}+2x-2p=0$$

$$\frac{dx}{dp}+\frac{2x}{p}=2 \quad \text{Lineer dif.}$$

$$\ln \lambda = \int \frac{2}{p}\, dp = 2\ln p \Rightarrow \lambda = p^2$$

$$\underbrace{p^2 \frac{dx}{dp} + 2xp}_{} = 2p^2$$

$$\frac{d}{dp}\left(p^2 \cdot x\right) = 2p^2$$

$$\int d\left(p^2 x\right) = \int 2p^2\, dp$$

$$p^2 x = \frac{2p^3}{3} + C$$

$$\boxed{x = \frac{2}{3}p + \frac{C}{p^2}}$$

$$\begin{cases} x = \frac{2}{3}p + \frac{C}{p^2} \\ y = 2\left(\frac{2}{3}p + \frac{C}{p^2}\right)p - p^2 - 1 \end{cases}$$

Lagrange d.d.
Genel çöz. P.D.

1. mertebeden dif. denklemlerin sonu!