Lineer olmayan dif. denk.

Bağımlı - Bağımsız

$y' = t$ $\quad\quad y' = p$

$y'' = t'$ $\quad\quad y'' = p\dfrac{dp}{dy}$

$\vdots$ $\quad\quad \vdots$

20 Aralık Saat 18^{00} Yüzyüze kısa sınav

Derstpara web sayfası

Örnek $y' - 2xy = 0$ kuvvet serisi yardımı ile çözelim.

$$y = \sum_{n=0}^{\infty} a_n (x-x_0)^n \qquad y = \sum_{n=0}^{\infty} a_n x^n ; \quad y' = \sum_{\substack{n=1 \\ (n=0)}}^{\infty} n a_n x^{n-1}$$

$$\sum_{n=1}^{\infty} n a_n x^{n-1} - 2x \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} 2 a_n x^{n+1} = 0$$

$n+1 \to n-1$, $n \to n-2$

$$\sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=2}^{\infty} 2 a_{n-2} x^{n-1} = 0$$

$$1.a_1 x^0 + \sum_{n=2}^{\infty} n a_n x^{n-1} - \sum_{n=2}^{\infty} 2 a_{n-2} x^{n-1} = 0$$

$$a_1 + \sum_{n=2}^{\infty} \underbrace{(n a_n - 2 a_{n-2})} x^{n-1} = 0$$

$\boxed{a_1 = 0}$ ✓ $\quad n a_n - 2 a_{n-2} = 0 \quad (n \geq 2)$ ✓✓

Rekürens bağıntısı

$$\boxed{a_n = \frac{2}{n} a_{n-2}} \quad (n \geq 2)$$

$n=2 \quad a_2 = \frac{2}{2} a_0 = a_0$

$n=3 \quad a_3 = \frac{2}{3} a_1 = 0$

$n=4 \quad a_4 = \frac{\cancel{2}}{\cancel{4}_2} a_2 = \frac{1}{2} a_2 = \frac{1}{2} a_0$

$n=5 \quad a_5 = \frac{2}{5} a_3 = 0$

$n=6 \quad a_6 = \frac{\cancel{2}}{\cancel{6}_3} a_4 = \frac{1}{3} a_4 = \frac{1}{3.2} a_0$

$\vdots$

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

$$y = a_0 + a_0 x^2 + \frac{1}{2!} a_0 x^4 + \frac{1}{3!} a_0 x^6 + \frac{1}{4!} a_0 x^8 + \cdots$$

$$y = a_0 \left(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots \right)$$

$$y = a_0 \left(1 + x^2 + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \frac{(x^2)^4}{4!} + \cdots \right)$$

$$y = a_0 e^{x^2}$$

$\forall u \in \mathbb{R} \quad e^u = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \cdots$

$e^{x^2} = 1 + x^2 + \frac{(x^2)^2}{2!} + \cdots$

$e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \cdots$

$$y' - 2xy = 0$$

$$\frac{dy}{dx} = 2xy$$

$$\int \frac{dy}{y} = \int 2x\,dx \longrightarrow \ln y - \ln c = x^2$$

$$\frac{y}{c} = e^{x^2}$$

$$y = c e^{x^2}$$

örnek: $y'' - xy = 0$ kuvvet serisi çözümü nedir? $(x_0 = 0)$

$$y = \sum_{n=0}^{\infty} a_n x^n \qquad y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \qquad y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - x \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=0}^{\infty} \underset{a_{n-3}}{a_n} x^{\overset{n-3}{n+1}} = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{\overset{n-3}{n=0} \to n=3}^{\infty} a_{n-3} x^{n-2} = 0$$

$$2.1.a_2 x^0 + \sum_{n=3}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=3}^{\infty} a_{n-3} x^{n-2} = 0$$

$$2a_2 + \sum_{n=3}^{\infty} \left(n(n-1) a_n - a_{n-3} \right) x^{n-2} = 0$$

$$a_2 = 0 \qquad n(n-1)a_n - a_{n-3} = 0 \quad (n \geq 3)$$

$$a_n = \frac{1}{n(n-1)} a_{n-3} \qquad (n \geq 3)$$

$n=3 \qquad a_3 = \frac{1}{3.2} a_0$

$n=4 \qquad a_4 = \frac{1}{4.3} a_1$

$n=5 \qquad a_5 = \frac{1}{5.4} a_2 = 0$ ✓

$n=6 \qquad a_6 = \frac{1}{6.5} a_3 = \frac{1}{6.5.3.2} a_0$

$n=7 \qquad a_7 = \frac{1}{7.6} a_4 = \frac{1}{\underline{7.6.4.3}} a_1$

$n=8 \qquad a_8 = \frac{1}{8.7} a_5 = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots$$

$$y = a_0 + a_1 x + \frac{1}{6} a_0 x^3 + \frac{1}{12} a_1 x^4 + \frac{1}{180} a_0 x^6 + \frac{1}{12.42} a_1 x^7 + \dots$$

$$y = a_0\left(1 + \frac{x^3}{6} + \frac{x^6}{180} + \dots\right) + a_1\left(x + \frac{x^4}{12} + \frac{x^7}{12.42} + \dots\right)$$

örnek $y'' - (x-2)y' + 2y = 0$ $\quad x = \underset{x_0}{2}$ noktası civarında kuvvet serisi çözümü?

$$y=\sum_{n=0}^{\infty} a_n (x-2)^n \qquad y'=\sum_{n=1}^{\infty} n a_n (x-2)^{n-1} \qquad y''=\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - (x-2)\sum_{n=1}^{\infty} n a_n (x-2)^{n-1} + 2\sum_{n=0}^{\infty} a_n (x-2)^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=0}^{\infty} n a_n \boxed{(x-2)^n} + \sum_{n=0}^{\infty} 2a_n \boxed{(x-2)^n} = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=0}^{\infty} (n-2) a_n (x-2)^n = 0$$

$n-2$ $\quad n-2$ $\quad n-2$ $\quad n-2$

$$\sum_{\boxed{n=2}}^{\infty} n(n-1) a_n x^{n-2} - \sum_{\boxed{n=2}}^{\infty} (n-4) a_{n-2} (x-2)^{n-2} = 0$$

$$\sum_{n=2}^{\infty} \left(n(n-1)a_n - (n-4)a_{n-2}\right) x^{n-2} = 0$$

$$n(n-1)a_n - (n-4)a_{n-2} = 0 \qquad (n \geq 2)$$

Rekürans bağ.

$$a_n = \frac{n-4}{n(n-1)} a_{n-2} \qquad (n \geq 2)$$

$n=2 \qquad a_2 = \frac{-2}{2\cdot 1} a_0 = -a_0$

$n=3 \qquad a_3 = \frac{-1}{3\cdot 2} a_1 = -\frac{1}{6} a_1$

$n=4 \qquad a_4 = 0$

$n=5 \qquad a_5 = \frac{1}{5.4} a_3 = \frac{1}{5.4}\left(-\frac{1}{6} a_1\right) = \frac{-1}{120} a_1$

$n=6 \qquad a_6 = \frac{2}{6.5} a_4 = 0$

$\vdots$

$$y = \sum_{n=0}^{\infty} a_n (x-2)^n = a_0 + a_1(x-2) + a_2(x-2)^2 + a_3(x-2)^3 + \dots$$

$$y = a_0 + a_1(x-2) - a_0(x-2)^2 - \frac{1}{6} a_1 (x-2)^3 - \frac{1}{120} a_1 (x-2)^5 - \dots$$

$$y = a_0\left(1 - (x-2)^2\right) + a_1\left[(x-2) - \frac{(x-2)^3}{6} - \frac{(x-2)^5}{120} + \dots\right]$$

örnek $\quad y'' - xy' + 3y = 0 \qquad x=2$

$$y = \sum_{n=0}^{\infty} a_n (x-2)^n$$

$$(x-2+2)\sum_{n=1}^{\infty} n a_n (x-2)^{n-1} \qquad -2+2$$

$$(x-2)\sum \dots + 2\sum \qquad a+0=a$$

$$y'' - (x-2+2)y' + 3y = 0$$

$$y'' - (x-2)y' - 2y' + 3y = 0$$

! NOKTA