$$ay'' + by' + cy = 0$$

$$ar^2 + br + c = 0 \qquad r_{1,2} = \frac{-b \mp \sqrt{\Delta}}{2a}$$

$$r_1 = r_2 = r = \frac{-b}{2a}$$

$$y \overset{?}{=} k(x)e^{rx}$$

$$y' = k'e^{rx} + k r e^{rx}$$

$$y'' = k''e^{rx} + 2k'r e^{rx} + k r^2 e^{rx}$$

$$\cancel{e^{rx}}\left(ak'' + 2ak'r + akr^2 + bk' + bkr + ck\right) = 0$$

$$ak'' + (\underbrace{2ar + b}_{0})k' + (\underbrace{ar^2 + br + c}_{0})k = 0 \checkmark$$

$$\cancel{a}k'' = 0$$

$$\frac{d^2k}{dx^2} = 0 \checkmark \rightarrow \frac{d}{dx}\left(\frac{dk}{dx}\right) = 0$$

$$\frac{dk}{dx} = C_1$$

$$\int dk = \int C_1 dx$$

2 kat

$$r = r_1 = r_2 = \frac{-b}{2a} \qquad k(x) = C_1 x + C_2$$

$$y = k(x)e^{rx} = (C_1 x + C_2)e^{rx}$$

$$y = C_1 \underbrace{xe^{rx}}_{y_1} + C_2 \underbrace{e^{rx}}_{y_2} \checkmark$$

lineer bağımsız

$r = r_1 = r_2 = r_3$ (üç katlı)

$$y = C_1 x^2 e^{rx} + C_2 x e^{rx} + C_3 e^{rx}$$

$$(C_1 x^2 + C_2 x + C_3) e^{rx}$$

SONUÇ 2. Karakteristik polinomun köklerinin eşit (katlı) kök olması durumunda d.d ait genel çözüm; e^{rx} çarpanı ile beraber katlı kök sayısından bir derece düşük polinomun genel şeklidir.

$r = r_1 = r_2$ (2 kat) $\quad y = e^{rx}(C_1 x + C_2)$ $\quad x e^{rx}\ \ e^{rx}$

$r = r_1 = r_2 = r_3$ (3 kat) $\quad y = e^{rx}(C_1 x^2 + C_2 x + C_3)$ $\quad x^2 e^{rx}\ \ x e^{rx}\ \ e^{rx}$

örnek $\quad y'' - 2y' + y = 0$

$r^2 - 2r + 1 = 0 \qquad (r-1)^2 = 0$

$r = 1$ (2 kat) $\qquad (r_1 = r_2 = 1)$

$$y = e^x (C_1 x + C_2) = C_1 x e^x + C_2 e^x$$

örnek $\quad y''' + 3y'' + 3y' + y = 0$

$r^3 + 3r^2 + 3r + 1 = 0$

$(r+1)^3 = 0 \qquad r_1 = r_2 = r_3 = -1$ (üç kat)

$$y = e^{-x}(C_1 x^2 + C_2 x + C_3) = C_1 x^2 e^{-x} + C_2 x e^{-x} + C_3 e^{-x}$$

örnek $y''' - 3y'' - 4y' + 12y = 0$

$$r^3 - 3r^2 - 4r + 12 = 0$$

$$r^2(r-3) - 4(r-3) = 0$$

$$(r-3)(r^2-4) = 0$$

$r_1 = 3$, $r_2 = 2$, $r_3 = -2$

$$y = c_1 e^{3x} + c_2 e^{2x} + c_3 e^{-2x}$$

örnek $4y'' + 4y' + y = 0$

$$4r^2 + 4r + 1 = 0$$

$$(2r+1)^2 = 0$$

$r = -\frac{1}{2}$ (2 kat)

$$y = e^{-\frac{x}{2}}(c_1 x + c_2)$$

$$y = c_1 x e^{-\frac{x}{2}} + c_2 e^{-\frac{x}{2}}$$

örnek $y''' - 5y'' + 8y' - 4y = 0$

* $r^3 - 5r^2 + 8r - 4 = 0$

$(r-r_1)(r-r_2)(r-r_3) = 0$

$r_1 = 1$ ✓

$(r-1)(\ldots\ldots) = 0$

$$r^3 - 5r^2 + 8r - 4 \;\Big|\; \frac{r-1}{r^2 - 4r + 4}$$

	1	-5	8	-4
1		1	-4	4
	1	-4	4	0

$r^2 - 4r + 4$

	r^3	r^2	r	sabit
	1	-5	8	-4
kök 1		1	-4	4
	1	-4	4	0
	r^2	r	sabit	

$$r^2 - 4r + 4 = 0$$

$$r^3-5r^2+8r-4=(r-1)(r^2-4r+4)=0$$

$$(r-1)\,(r-2)^2=0$$

$$r_1=1 \qquad r_2=r_3=2 \quad (2\text{ kat})$$

$$y=C_1e^{x}+e^{2x}(C_2x+C_3)$$

$$\underbrace{r_1=r_2=-3}_{2\text{ kat}} \qquad \underbrace{r_3=2}_{\checkmark} \qquad \underbrace{r_4=r_5=r_6=\frac{1}{2}}_{3\text{ kat}}$$

$$y=\underbrace{e^{-3x}(C_1x+C_2)}+\underbrace{C_3e^{2x}}+\underbrace{e^{\frac{x}{2}}(C_4x^2+C_5x+C_6)}$$

$$ay''+by'+cy=0 \qquad ar^2+br+c=0$$

$$r_{1,2}=\frac{-b\mp\sqrt{\Delta}}{2a}=\boxed{\alpha\mp i\beta} \checkmark$$

Kompleks (karmaşık) kök

$$\Delta<0 \qquad \sqrt{-1}=i$$

$$r_1=\alpha+i\beta \qquad r_2=\alpha-i\beta$$

$$y_1=e^{(\alpha+i\beta)x} \checkmark \qquad y_2=e^{(\alpha-i\beta)x} \checkmark \qquad a^{m+n}=a^m\cdot a^n$$

$$y=K_1e^{(\alpha+i\beta)x}+K_2e^{(\alpha-i\beta)x} \checkmark$$

$$y=K_1e^{\alpha x}e^{i\beta x}+K_2e^{\alpha x}e^{-i\beta x}$$

$$y=e^{\alpha x}\left(K_1e^{i\beta x}+K_2e^{-i\beta x}\right)$$

$$e^{\mp i\beta x} = \cos\beta x \mp i\sin\beta x$$

$$y = e^{\alpha x}\left(K_1(\cos\beta x + i\sin\beta x) + K_2(\cos\beta x - i\sin\beta x)\right)$$

$$y = e^{\alpha x}\left(\underbrace{(K_1+K_2)}_{C_1}\cos\beta x + \underbrace{i(K_1-K_2)}_{C_2}\sin\beta x\right)$$

$$y = e^{\alpha x}\left(C_1\cos\beta x + C_2\sin\beta x\right)$$

$$ay'' + by' + cy = 0 \qquad ar^2 + br + c = 0$$

$$r_{1,2} = \underbrace{\alpha \mp i\beta}$$

$$y = e^{\alpha x}\left(C_1\cos\beta x + C_2\sin\beta x\right)$$

$$r_{1,2} = 2 \mp 3i \qquad y = e^{2x}\left(C_1\cos 3x + C_2\sin 3x\right)$$

örnek

$$\underbrace{y'' + 9y = 0} \qquad r^2 + 9 = 0$$

$$r^2 = -9 \qquad r_{1,2} = \mp 3i$$

$$y = \underbrace{e^{0x}}\left(C_1\cos 3x + C_2\sin 3x\right)$$

$$y = C_1\underline{\cos 3x} + C_2\underline{\sin 3x}$$

örnek

$$y'' - 2y' + 3y = 0$$

$$r^2 - 2r + 3 = 0$$

$$r_{1,2} = \frac{-(-2) \mp \sqrt{(-2)^2 - 4 \cdot 3 \cdot 1}}{2} = \frac{2 \mp \sqrt{4-12}}{2}$$

$$= \frac{2 \mp \sqrt{-8}}{2} = \frac{2 \mp \sqrt{8(-1)}}{2} = \frac{2 \mp i\sqrt{8}}{2}$$

$$= \frac{2 \mp i\, 2\sqrt{2}}{2} = 1 \mp \sqrt{2}\, i$$

$$r_{1,2} = 1 \mp \sqrt{2}\, i$$

$$y = e^x \left(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x\right)$$

$$y = C_1 e^x \cos\sqrt{2}x + C_2 e^x \sin\sqrt{2}x$$

Örnek 2 $\quad y'' + 2y' + 10y = 0$

$$r^2 + 2r + 10 = 0$$

$$r_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \mp \sqrt{4 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$$

$$= \frac{-2 \mp \sqrt{-36}}{2} = \frac{-2 \mp \sqrt{36(-1)}}{2} = \frac{-2 \mp 6i}{2}$$

$$r_{1,2} = -1 \mp 3i$$

$$y = e^{-x}\left(C_1 \sin 3x + C_2 \cos 3x\right)$$

örnek

$$y''' - 5y'' + 12y' - 8y = 0$$

$$r^3 - 5r^2 + 12r - 8 = 0 \qquad r_1 = 1$$

	1	-5	12	-8
		1	-4	8
1	1	-4	8	0

$$r^2 - 4r + 8 = 0$$

$$r_{2,3} = 2 \mp 2i$$

$$y = C_1 e^{x} + e^{2x}\left(C_2 \cos 2x + C_3 \sin 2x\right)$$

örnek

$$y^{IV} + 8y'' + 16y = 0$$

$$r^4 + 8r^2 + 16 = 0 \qquad \underbrace{(r^2+4)}_{\mp 2i}\underbrace{(r^2+4)}_{\mp 2i} = 0$$

$$\underbrace{(r^2+4)}^{2} = 0$$

$$r_{1,2} = \mp 2i \quad (2 \text{ kat})$$

$$y = (C_1 x + C_2)\cos 2x + (C_3 x + C_4)\sin 2x$$

$$r_{1,2} = \sqrt{3} \mp 4i \quad (3 \text{ kat})$$

$$y = e^{\sqrt{3}x}\left(\left(C_1 x^2 + C_2 x + C_3\right)\cos 4x + \left(C_4 x^2 + C_5 x + C_6\right)\sin 4x\right)$$

$y_1 = e^{\sqrt{3}x} x^2 \cos 4x \qquad y_2 = e^{\sqrt{3}x} x \cos 4x \qquad y_3 = e^{\sqrt{3}x} \cos 4x$

$y_4 = e^{\sqrt{3}x} x^2 \sin 4x \qquad y_5 = e^{\sqrt{3}x} x \sin 4x \qquad y_6 = e^{\sqrt{3}x} \sin 4x$