Fredholm int. Denklemleri

$$u(x) = f(x) + \lambda \int_a^b K(x,t)\, u(t)\, dt$$

$a \leq x \leq b$

$f(x) \quad K(x,t) \quad a \leq t \leq b$

Sabit çekirdekli int. Denklem

$$u(x) = f(x) + \lambda \int_a^b c\, u(t)\, dt \qquad c \in \mathbb{R}$$

$$* \quad u(x) = f(x) + \lambda c \int_a^b u(t)\, dt \qquad A = \int_a^b u(t)\, dt$$

$$\boxed{u(x) = f(x) + \lambda c A} \quad \text{çözüm modeli}$$

$$\cancel{f(x)} + \cancel{\lambda} c \cdot A = \cancel{f(x)} + \cancel{\lambda} c \cdot \int_a^b \left[f(t) + \lambda c A \right] dt$$

$$A = \int_a^b \left[f(t) + \lambda c A \right] dt$$

$$A = \int_a^b f(t)\, dt + \lambda c A \underbrace{\int_a^b dt}_{t}$$

$$A = \int_a^b f(t)\, dt + \lambda c A (b-a)$$

$$A \left(1 - \lambda c (b-a)\right) = \int_a^b f(t)\, dt$$

$$A = \frac{\int_a^b f(t)\,dt}{1-\lambda c(b-a)} \qquad 1-\lambda c(b-a) \neq 0$$

$$u(x) = f(x) + \lambda c A$$

$$u(x) = f(x) + \lambda c \frac{\int_a^b f(t)\,dt}{1-\lambda c(b-a)}$$

örnek

$$u(x) = x^2 + \lambda \int_0^{10} 3u(t)\,dt$$

$$u(x) = x^2 + 3\lambda \int_0^{10} u(t)\,dt \qquad A = \int_0^{10} u(t)\,dt = ?$$

$$u(x) = x^2 + 3\lambda A \rightarrow \text{çözüm model}$$

$$\cancel{x^2} + \cancel{3\lambda} A = \cancel{x^2} + \cancel{3\lambda} \int_0^{10} (t^2 + 3\lambda A)\,dt$$

$$A = \int_0^{10} (t^2 + 3\lambda A)\,dt = \int_0^{10} t^2\,dt + 3\lambda A \int_0^{10} dt$$

$$A = \frac{1000}{3} + 30\lambda A$$

$$A(1-30\lambda) = \frac{1000}{3} \Rightarrow A = \frac{1000}{3(1-30\lambda)}$$

$$\lambda \neq \frac{1}{30}$$

$$u(x) = x^2 + 3\lambda A$$

$$u(x) = x^2 + \frac{3000\lambda}{3(1-30\lambda)}$$

Dejenere Çekirdekli İnt. denk.

$$u(x) = f(x) + \lambda \int_a^b K(x,t)\, u(t)\, dt$$

$$K(x,t) = r(x)\, s(t)$$

$$K(x,t) = x\sqrt{t}$$

$$K(x,t) = e^{x-t} = e^x e^{-t}$$

$$u(x) = f(x) + \lambda \int_a^b r(x)\, s(t)\, u(t)\, dt$$

$$u(x) = f(x) + \lambda r(x) \int_a^b s(t)\, u(t)\, dt \qquad A = \int_a^b s(t)\, u(t)\, dt$$

$$u(x) = f(x) + \lambda r(x) A \longrightarrow \text{Çözüm modeli}$$

$$\cancel{f(x)} + \cancel{\lambda r(x)} A = \cancel{f(x)} + \cancel{\lambda r(x)} \int_a^b s(t) \left[f(t) + \lambda r(t) A \right] dt$$

$$A = \int_a^b s(t) f(t)\, dt + \lambda A \int_a^b s(t)\, r(t)\, dt$$

$$A \left(1 - \lambda \int_a^b s(t)\, r(t)\, dt \right) = \int_a^b s(t)\, f(t)\, dt$$

$$A = \frac{\int_a^b s(t) f(t)\, dt}{1 - \lambda \int_a^b s(t)\, r(t)\, dt} \qquad \left(1 - \lambda \int_a^b s(t)\, r(t)\, dt \neq 0 \right)$$

$$u(x) = f(x) + \lambda r(x) \frac{\int_a^b s(t) f(t)\, dt}{1 - \lambda \int_a^b s(t)\, r(t)\, dt}$$

Örnek $u(x) = x^2 + \lambda \int_0^1 \underbrace{x^2 t^2}\, u(t)\,dt \qquad K(x,t) = x^2 \cdot t^2$

Dejenere çek.

$$u(x) = x^2 + \lambda x^2 \underbrace{\int_0^1 t^2 u(t)\,dt} \qquad A = \int_0^1 t^2 u(t)\,dt$$

$$u(x) = x^2 + \lambda x^2 A \longrightarrow \text{Çözüm modeli}$$

$$\cancel{x^2} + \cancel{\lambda x^2} A = \cancel{x^2} + \cancel{\lambda x^2} \int_0^1 t^2 \left[t^2 + \lambda t^2 A \right] dt$$

$$A = \int_0^1 t^4\,dt + \lambda A \int_0^1 t^4\,dt = \frac{1}{5} + \frac{\lambda A}{5}$$

$$5A = 1 + \lambda A \longrightarrow A(5-\lambda) = 1$$

$$A = \frac{1}{5-\lambda}$$

$$(\lambda \neq 5)$$

$$\boxed{u(x) = x^2 + \frac{\lambda x^2}{5-\lambda}}$$

Soru- $u(x) = x e^{-x} + \lambda \int_0^1 e^{x-t} u(t)\,dt$

Cevap: $u(x) = x e^{-x} + \lambda e^{x} \cdot \dfrac{1-3e^{-2}}{4(1-\lambda)}$

$(\lambda \neq 1)$

Dejenere Çekirdeğin Genel Hali:

$$K(x,t) = \sum_{i=1}^{n} r_i(x)\, s_i(t) = r_1(x) s_1(t) + r_2(x) s_2(t) + \dots + r_n(x) s_n(t)$$

$$K(x,t) = \sin(x+t) \dots$$

$$u(x) = f(x) + \lambda \int_a^b \underbrace{K(x,t)}\, u(t)\, dt$$

$$u(x) = f(x) + \lambda \int_a^b \left[r_1(x)\, s_1(t) + \dots + r_n(t)\, s_n(t) \right] u(t)\, dt$$

$$* \; u(x) = f(x) + \lambda r_1(x) \underbrace{\int_a^b s_1(t) u(t)\, dt}_{A_1} + \dots + \lambda r_n(x) \underbrace{\int_a^b s_n(t) u(t)\, dt}_{A_n}$$

$$u(x) = f(x) + \lambda r_1(x) A_1 + \lambda r_2(x) A_2 + \dots + \lambda r_n(x) A_n$$

çözüm modeli

$$\cancel{f(x)} + \lambda r_1(x) A_1 + \lambda r_2(x) A_2 + \dots + \lambda r_n(x) A_n$$

$$= \cancel{f(x)} + \lambda r_1(x) \int_a^b s_1(t) \left[f(t) + \lambda r_1(t) A_1 + \dots + \lambda r_n(t) A_n \right] dt$$

$$+ \dots + \lambda r_n(x) \int_a^b s_n(t) \left[f(t) + \lambda r_1(t) A_1 + \dots + \lambda r_n(t) A_n \right] dt$$

$$A_1 = \int_a^b s_1(t) \left[f(t) + \lambda r_1(t) A_1 + \dots + \lambda r_n(t) A_n \right] dt$$

$$A_2 = \int_a^b s_2(t) \left[f(t) + \lambda r_1(t) A_1 + \dots + \lambda r_n(t) A_n \right] dt$$

$$\vdots$$

$$A_n = \int_a^b s_n(t) \left[f(t) + \lambda r_1(t) A_1 + \dots + \lambda r_n(t) A_n \right] dt$$

$$A_1 = \int_a^b s_1(t) f(t)\,dt + \lambda A_1 \int_a^b s_1(t) r_1(t)\,dt + \cdots + \lambda A_n \int_a^b s_1(t) r_n(t)\,dt$$

$$A_2 = \int_a^b s_2(t) f(t)\,dt + \lambda A_1 \int_a^b s_2(t) r_1(t)\,dt + \cdots + \lambda A_n \int_a^b s_2(t) r_n(t)\,dt$$

$$\vdots$$

$$A_n = \int_a^b s_n(t) f(t)\,dt + \lambda A_1 \int_a^b s_n(t) r_1(t)\,dt + \cdots + \lambda A_n \int_a^b s_n(t) r_n(t)\,dt$$

$$A_1 - \lambda A_1 \int_a^b s_1(t) r_1(t)\,dt - \cdots - \lambda A_n \int_a^b s_1(t) r_n(t)\,dt = \int_a^b s_1(t) f(t)\,dt$$

$$-\lambda A_1 \int_a^b s_2(t) r_1(t)\,dt + A_2 - \lambda A_2 \int_a^b s_2(t) r_2(t)\,dt - \cdots - \lambda A_n \int_a^b s_2(t) r_n(t)\,dt = \int_a^b s_2(t) f(t)\,dt$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$-\lambda A_1 \int_a^b s_n(t) r_1(t)\,dt - \cdots + A_n - \lambda A_n \int_a^b s_n(t) r_n(t)\,dt = \int_a^b s_n(t) f(t)\,dt$$

$$\int_a^b s_i(t) r_j(t)\,dt = C_{ij} \quad (i, j = 1, \ldots, n)$$

$$\int_a^b s_i(t) f(t)\,dt = B_i \quad (i = 1, \ldots, n)$$

$$(1-\lambda C_{11})A_1 - \lambda C_{12}A_2 - \cdots - \lambda C_{1n}A_n = B_1$$

$$-\lambda C_{21}A_1 + (1-\lambda C_{22})A_2 - \cdots - \lambda C_{2n}A_n = B_2$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$-\lambda C_{n1}A_1 - \lambda C_{n2}A_2 - \cdots + (1-\lambda C_{nn})A_n = B_n$$

$A_1, A_2, \ldots, A_n$ Cramer Sistemi

Örnek

$$u(x) = \cos x + \lambda \int_0^{\pi} (x \sin t + t \sin x)\, u(t)\, dt$$

$$u(x) = \cos x + \lambda x \underbrace{\int_0^{\pi} \sin t\, u(t)\, dt}_{A_1} + \lambda \sin x \underbrace{\int_0^{\pi} t\, u(t)\, dt}_{A_2}$$

$u(x) = \cos x + \lambda x A_1 + \lambda \sin x A_2$ çözüm modeli

$$\cancel{\cos x} + \lambda x A_1 + \lambda \sin x A_2 = \cancel{\cos x} + \lambda x \int_0^{\pi} \sin t \left[\cos t + \lambda t A_1 + \lambda \sin t A_2\right] dt$$
$$+ \lambda \sin x \int_0^{\pi} t \left[\cos t + \lambda t A_1 + \lambda \sin t A_2\right] dt$$

$$A_1 = \int_0^{\pi} \sin t \cos t\, dt + \lambda A_1 \int_0^{\pi} t \sin t\, dt + \lambda A_2 \int_0^{\pi} \sin^2 t\, dt$$

$$A_2 = \int_0^{\pi} t \cos t\, dt + \lambda A_1 \int_0^{\pi} t^2\, dt + \lambda A_2 \int_0^{\pi} t \sin t\, dt$$

$$\left.\begin{aligned}(1-\lambda\pi)A_1-\lambda\frac{\pi}{2}&=0\\-\lambda\frac{\pi^3}{3}A_1+(1-\lambda\pi)A_2&=-2\end{aligned}\right\}$$

$$A_1=-\frac{\lambda\pi}{\Delta}\qquad A_2=-\frac{2(1-\lambda\pi)}{\Delta}$$

$$u(x)=\cos x+\lambda x A_1+\lambda\sin x A_2$$

$$\boxed{u(x)=\cos x-\frac{\lambda}{(1-\lambda\pi)^2-\frac{\lambda^2\pi^4}{6}}\left(\lambda\pi x+2(1-\lambda\pi)\sin x\right)}$$

SORU

$$u(x)=\cos x+\lambda\int_0^{\pi}\sin(x-t)u(t)\,dt$$

Cevap: $u(x)=\cos x+\lambda\sin x\dfrac{2\pi}{4+\lambda^2\pi^2}-\lambda^2\cos x\dfrac{\pi^2}{4+\lambda^2\pi^2}$

Dejenere Çekirdekli Homojen İnt. denklem