Homojen Dif. Denklem

$f(x,y)$ $\quad \left. \begin{array}{l} x \to \lambda x \\ y \to \lambda y \end{array} \right\}$ $\quad f(\lambda x,\lambda y) = \lambda^k . f(x,y)$

k. dereceden homojen fonk

$f(x,y) = x^2 + 3xy$ $\quad$ 2. dereceden homojen

$$f(\lambda x,\lambda y) = (\lambda x)^2 + 3\lambda x \lambda y = \lambda^2 x^2 + 3\lambda^2 xy = \lambda^2 (x^2 + 3xy)$$

$$= \lambda^2 f(x,y)$$

$$f(x,y) = x^2 + 3xy + 1 \Rightarrow \lambda^2 (x^2 + 3xy) + 1 \overset{?}{=} \lambda^k f(x,y)$$

$$\underbrace{x^2}_{\lambda^2} + \underbrace{3x^2y}_{\lambda^3}$$

$$g(x,y) = \tan \frac{x}{y} \to g(\lambda x,\lambda y) = \tan \frac{\lambda x}{\lambda y} = \tan \frac{x}{y}$$

$$= \lambda^0 g(x,y)$$

0. dereceden homojen fonk

$$h(x,y) = y e^{\frac{y}{x}} \to h(\lambda x,\lambda y) = (\lambda y).e^{\frac{\lambda y}{\lambda x}} = \lambda \, y e^{\frac{y}{x}}$$

1. dereceden homojen fonk $\quad h(x,y)$

$$\lambda \left(y e^{\frac{\lambda y}{\lambda x}} \right) = \lambda h(x,y)$$

$$f(x,y) = 3x + 5y + 1$$

$$f(\lambda x,\lambda y) = 3\lambda x + 5\lambda y + 1 = \lambda(3x + 5y) + 1$$

homojen olmayan f.

$* \quad y' = f(x,y)$ — k-derecede homojen fonk } Homojen dif

$*$

$M(x,y)\,dx + N(x,y)\,dy = 0$ } Homojen dif

Eğer aynı derecede homojen fonk ise

Not: Bir homojen dif dak. $\frac{y}{x}$'in (veya $\frac{x}{y}$'nin) bir fonk. olarak yazılabilir.

$* \quad y' = f\left(\frac{y}{x}\right)$ $\quad \boxed{\frac{y}{x} = u}$ değişken değişimi

$\frac{y}{x} = u \rightarrow y = u.x$

$y' = u'x + u$

$** \quad u'x + u = f(u)$

$\frac{du}{dx} \cdot x = f(u) - u \rightarrow \int \frac{dx}{x} = \int \frac{du}{f(u) - u}$

$\ln x = G(u) + C$

Genel çözüm $\boxed{\ln x = G\left(\frac{y}{x}\right) + C}$

Örnek

$$\underbrace{\left(y+\sqrt{x^2-y^2}\right)dx}\ \underbrace{-x}\,dy=0 \longrightarrow \text{Homojen dif.}$$

$\lambda y+\sqrt{\lambda^2x^2-\lambda^2y^2}$

$\lambda y+\sqrt{\lambda^2(x^2-y^2)}$

$\lambda y+\lambda\sqrt{x^2-y^2}$

$\lambda\left(y+\sqrt{x^2-y^2}\right)$

1. derecedn homojen

$N(x,y)=-x$ 1. derecedn homojen

$N(\lambda x,\lambda y)=-(\lambda x)=\lambda(\underbrace{-x}) =\lambda N(x,y)$

$$\left(y+\sqrt{x^2-y^2}\right)dx-x\,dy=0 \quad \text{Homojen dif.}$$

$$x\,dy=\left(y+\sqrt{x^2-y^2}\right)dx$$

$$\boxed{\frac{dy}{dx}=\frac{y+\sqrt{x^2-y^2}}{x}}$$

$$y=f\left(\frac{y}{x}\right)$$

$$y'=\frac{y}{x}+\frac{\sqrt{x^2-y^2}}{x}$$

$$y'=\frac{y}{x}+\sqrt{\frac{x^2-y^2}{x^2}}$$

$$y'=\frac{y}{x}+\sqrt{1-\left(\frac{y}{x}\right)^2}=f\left(\frac{y}{x}\right)$$

$$\boxed{*\ y'=\frac{y}{x}+\sqrt{1-\left(\frac{y}{x}\right)^2}} \quad \text{Homojen dif.}$$

$$\frac{y}{x}=u \rightarrow \begin{array}{l} y=ux \\ y'=u'x+u \end{array}$$

$$u'x+\cancel{u}=\cancel{u}+\sqrt{1-u^2}$$

$$\frac{du}{dx}x=\sqrt{1-u^2} \longrightarrow \int\frac{du}{\sqrt{1-u^2}}=\int\frac{dx}{x}$$

Değişkenlerine ayrılabilen

$$\arcsin u = \ln x + \ln C$$

$$\arcsin u = \ln Cx$$

$$u = \sin(\ln Cx) \rightarrow \frac{y}{x} = \sin(\ln Cx)$$

$$\boxed{y = x \sin(\ln Cx)}$$

Genel Ç.

$$y' = \frac{y + \sqrt{x^2 - y^2}}{x} \quad \text{Homojen dif.}$$

$$\frac{y}{x} = u \rightarrow y = ux$$

$$y' = u'x + u$$

$$u'x + u = \frac{ux + \sqrt{x^2 - u^2x^2}}{x} = \frac{u\cancel{x} + \cancel{x}\sqrt{1-u^2}}{\cancel{x}}$$

$$u'x + u = u + \sqrt{1-u^2}$$

---

2. Yol $\quad (y + \sqrt{x^2 - y^2})dx - x\,dy = 0 \quad$ Homojen dif

$$\frac{y}{x} = u \rightarrow y = u.x$$

$$dy = x\,du + u\,dx \quad \left(\frac{dy}{dx} = x\frac{du}{dx} + u\right)$$

$$(ux + \sqrt{x^2 - u^2x^2})dx - x(x\,du + u\,dx) = 0$$

$$(u\cancel{x} + x\sqrt{1-u^2} - \cancel{x}u)dx - x^2du = 0$$

$$x\sqrt{1-u^2}\,dx - x^2du = 0 \quad \text{Değişkenlerine ayr.}$$

$$x\sqrt{1-u^2}\,dx = x^2du \rightarrow \frac{du}{\sqrt{1-u^2}} = \frac{\cancel{x}\,dx}{x^{\cancel{2}}} \rightarrow \ldots$$

Devamı yukarıda var!

Örnek

$$\underbrace{(x^3+y^3)}\,dx+\underbrace{3xy^2}\,dy=0$$

3. dereceden homojen

$$\frac{y}{x}=u \rightarrow \quad y=ux$$

$$dy=u\,dx+x\,du$$

$$(x^3+u^3x^3)dx+3x\,u^2x^2(u\,dx+x\,du)=0$$

$$(\underbrace{x^3+u^3x^3+3u^3x^3}_{4u^3x^3})dx+3x^4u^2du=0$$

$$x^3.(1+4u^3)dx+3x^4.u^2\,du=0$$

$$\underbrace{\int\frac{\cancel{x^3}}{x^{\cancel{4}}}dx}+\int\frac{3u^2}{1+4u^3}du=\int 0$$

$$\int\frac{3u^2}{1+4u^3}du \qquad 1+4u^3=t$$

$$12u^2du=dt$$

$$3u^2du=\frac{dt}{4}$$

$$=\frac{1}{4}\int\frac{dt}{t}=\frac{1}{4}\ln(1+4u^3)$$

$$\ln x+\frac{1}{4}\ln(1+4u^3)=\ln C$$

$$4\ln x+\ln(1+4u^3)=4\ln C$$

$$\ln x^4+\ln(1+4u^3)=\ln C^4$$

$$\ln(x^4(1+4u^3))=\ln C^4$$

$$x^4(1+4u^3)=C^4$$

$$x^4\left(1+4\frac{y^3}{x^3}\right)=C \rightarrow \boxed{x^4+4xy^3=C}$$

SORU

$$\left(x\cos^2\left(\frac{y}{x}\right)-y\right)dx+x\,dy=0$$

cos x

$\lambda^0\cos(\lambda x)$

$$\boxed{\ln x=-\tan\left(\frac{y}{x}\right)+1} \longleftarrow y(1)=\frac{\pi}{4}$$