Örnek $y'' - 2y' + 2y = xe^x$

$$r^2 - 2r + 2 = 0 \qquad r_{1,2} = 1 \mp i$$

$$y_h = e^x(c_1 \sin x + c_2 \cos x)$$

$$y_p = (ax+b)e^x = a(xe^x) + be^x$$

$$y_p' = ae^x + a(xe^x) + be^x$$

$$y_p'' = 2ae^x + axe^x + be^x$$

$$\cancel{e^x}(\cancel{2a} + ax + b - \cancel{2a} - \cancel{2ax} - \cancel{2b} + \cancel{2ax} + \cancel{2b}) = x\cancel{e^x}$$

$$ax + b = x$$

$$a = 1 \qquad b = 0$$

$$y_p = xe^x$$

$$\boxed{y = y_h + y_p = e^x(c_1 \sin x + c_2 \cos x) + xe^x}$$

Örnek $y'' + y = e^{-x} \cdot \cos x$

$-1 \mp i$

$$r^2 + 1 = 0 \Rightarrow r_{1,2} = \mp i$$

$$y_h = c_1 \sin x + c_2 \cos x$$

$$y_p = e^{-x}(A\cos x + B\sin x)$$ Rezonans yok!

$$= A(e^{-x}\cos x) + Be^{-x}\sin x$$

$$y_p' = -A(e^{-x}\cos x) - A(e^{-x}\sin x) - B(e^{-x}\sin x) + B(e^{-x}\cos x)$$

$$y_p'' = \cancel{Ae^{-x}\cos x} + \underbrace{Ae^{-x}\sin x + Ae^{-x}\sin x} - \cancel{Ae^{-x}\cos x}$$
$$+ \cancel{Be^{-x}\sin x} - \underbrace{Be^{-x}\cos x - Be^{-x}\cos x} - \cancel{Be^{-x}\sin x}$$

$$y_p'' = 2Ae^{-x}\sin x - 2Be^{-x}\cos x$$

$$y'' + y = e^{-x}\cos x$$

$$2A\cancel{e^{-x}}\sin x - 2B\cancel{e^{-x}}\cos x + A\cancel{e^{-x}}\cos x + B\cancel{e^{-x}}\sin x = \cancel{e^{-x}}\cos x$$

$$(2A+B)\sin x + (A-2B)\cos x = \cos x - 4\sin x$$

$$2A + B = 0 \qquad A = 1/5$$
$$A - 2B = 1 \qquad B = -2/5$$

$$y_p = e^{-x}\left(\frac{1}{5}\cos x - \frac{2}{5}\sin x\right)$$

$$y = y_h + y_p$$

f	değil	y_p

örnek $\quad y'' + 4y = x\sin x$ $\quad$ ($\sin 2x$)

$$r^2 + 4 = 0 \qquad r_{1,2} = \mp 2i \qquad y_h = C_1\sin 2x + C_2\cos 2x$$

$$y_p = (ax+b)\cos x + (cx+d)\sin x$$

$$y_p' = a\cos x - (ax+b)\sin x + c\sin x + (cx+d)\cos x$$

$$y_p'' = -2a\sin x - (ax+b)\cos x + 2c\cos x - (cx+d)\sin x$$

$$-2a\sin x - (ax+b)\cos x + 2c\cos x - (cx+d)\sin x + 4(ax+b)\cos x$$
$$+ 4(cx+d)\sin x = x\sin x$$

$$(3cx+3d-2a)\sin x + (3ax+3b+2c)\cos x = x\sin x$$

1) $3cx + 3d - 2a = x \rightarrow$

$3c = 1 \rightarrow \boxed{c = \frac{1}{3}}$

$3d - 2a = 0 \rightarrow d = 0$

2) $3ax + 3b + 2c = 0$

$3a = 0 \rightarrow a = 0$

$3b + 2c = 0$

$\boxed{b = -\frac{2}{9}}$

$$y_p = (ax+b)\cos x + (cx+d)\sin x$$

$$y_p = -\frac{2}{9}\cos x + \frac{x}{3}\sin x$$

$$y = y_h + y_p$$

Örnek $y'' - 3y' + 2y = x^2 + 1 + e^x - \sin 2x$

$r^2 - 3r + 2 = 0$ $\quad r_1 = 1 \quad r_2 = 2$

$$y_h = c_1 e^x + c_2 e^{2x}$$

$$y_{p_1} = ax^2 + bx + c$$
$$y_{p_1}' = 2ax + b$$
$$y_{p_1}'' = 2a$$

$$y'' - 3y' + y = x^2 + 1$$

$$2a - 6ax - 3b + 2ax^2 + 2bx + 2c = x^2 + 1$$

$$2a = 1 \rightarrow a = \frac{1}{2}$$
$$2b - 6a = 0 \rightarrow b = \frac{3}{2}$$
$$2a - 3b + 2c = 1 \rightarrow c = \frac{9}{4}$$

$$y_{p_1} = \frac{x^2}{2} + \frac{3x}{2} + \frac{9}{4}$$

$\alpha = 1 = r_1$ (tek kat)

$$y'' - 3y' + 2y = e^x$$

$y_{p_2} = Axe^x$ (rezonans)

$$A = -1$$
$$y_{p_2} = -xe^x$$

$$y_{p_3} = A\sin 2x + B\cos 2x$$

$$y_{p_3}' = 2A\cos 2x - 2B\sin 2x$$

$$y_{p_3}'' = -4A\sin 2x - 4B\cos 2x$$

$$y'' - 3y' + 2y = -\sin 2x$$

$$-4A\sin 2x - 4B\cos 2x - 6A\cos 2x + 6B\sin 2x$$
$$+ 2A\sin 2x + 2B\cos x = -\sin 2x$$

$$(-2A + 6B)\sin 2x + (-6A - 2B)\cos 2x = -\sin 2x$$

$$\left.\begin{aligned} -2A + 6B &= -1 \\ -6A - 2B &= 0 \end{aligned}\right\} \quad A = 1/20 \quad B = -3/20$$

$$y_{p_3} = \frac{1}{20}\sin 2x - \frac{3}{20}\cos 2x$$

$$y_p = y_{p_1} + y_{p_2} + y_{p_3}$$

$$y = y_h + y_p$$

$$y = C_1 e^{x} + C_2 e^{2x} + \frac{x^2}{2} + \frac{3x}{2} + \frac{9}{4} - x e^{x} + \frac{1}{20}\sin 2x - \frac{3}{20}\cos 2x$$