Lineer Olmayan Volterra int. Denklemleri

$$u(x) = f(x) + \int_0^x K(x,t)\,F(u(t))\,dt$$

2. tür lineer olmayan Volterra int. denklemi

$$f(x) = \int_0^x K(x,t)\,F(u(t))\,dt$$

1. tür lin. olm. Volterra int. denk.

$F(u(x)) \to$ $u(x)$'in lineer olmayan fonk.

$u^2(x)$ $e^{u(x)}$

$\sin(u(x))$ $\ldots$

2. tür lineer olmayan Volterra int. Denklemi

$$u(x) = f(x) + \int_0^x K(x,t)\,F(u(t))\,dt$$

Ardışık Yaklaştırma Yönt:

$u_0(x)$ $0,\ 1,\ x$

$$u_{n+1}(x) = f(x) + \int_0^x K(x,t)\,F(u_n(t))\,dt \quad (n \geq 0)$$

$$u_1(x) = f(x) + \int_0^x K(x,t)F\big(u_0(t)\big)dt$$

$$u_2(x) = f(x) + \int_0^x K(x,t)F\big(u_1(t)\big)dt$$

$$\vdots$$

$$u_{n+1}(x) = f(x) + \int_0^x K(x,t)F\big(u_n(t)\big)dt$$

Sonuç olarak $u(x)$ çözümü

$$u(x) = \lim_{n\to\infty} u_{n+1}(x) \quad \text{ile elde edilir.}$$

Örnek

$$u(x) = e^x + \frac{1}{3}x\left(1 - e^{3x}\right) + \int_0^x x\,u^3(t)\,dt \qquad F(u(t)) = [u(t)]^3$$

$u_0(x) = 1$ seçelim;

$$u_{n+1}(x) = e^x + \frac{1}{3}x\left(1 - e^{3x}\right) + \int_0^x x\,u_n^3(t)\,dt$$

$$u_0(x) = 1$$

$$u_1(x) = e^x + \frac{1}{3}x\left(1 - e^{3x}\right) + \int_0^x x\,u_0^3(t)\,dt$$

$$u_1(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x}{3} - \frac{x}{3}\left(1 + 3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \dots\right) + x^2$$

$$u_1(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \cancel{\frac{x}{3}} - \cancel{\frac{x}{3}} - \cancel{x^2} - \frac{9x^3}{3!} - \dots + \cancel{x^2}$$

$$u_1(x) = 1 + x + \frac{x^2}{2!} - \frac{4}{3}x^3 - \dots$$

$$u_2(x) = e^x + \frac{x}{3}\left(1 - e^{3x}\right) + \int_0^x x\, u_1^3(t)\, dt$$

$$\vdots$$

$$u_2(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{67}{60}x^5 - \cdots$$

$$\vdots$$

$$u_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \cdots$$

$$\vdots$$

$$u(x) = \lim_{n \to \infty} u_{n+1}(x) = e^x$$

Soru:

$$u(x) = \cos x + \frac{1}{8}\cos 2x - \frac{1}{4}x^2 - \frac{1}{8} + \int_0^x (x-t)\, u^2(t)\, dt$$

$$u_0(x) = 1$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

Cevap: $u(x) = \cos x$

Adomian Ayrıştırma Yöntemi

$n \geq 0$ $\quad A_n$ Adomian polinomları

$$u(x) = f(x) + \int_0^x K(x,t)\, F(u(t))\, dt$$

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \qquad F(u(t)) \qquad A_n$$

$$\sum_{n=0}^{\infty} u_n(x) = f(x) + \int_0^x K(x,t) \sum_{n=0}^{\infty} A_n(t)\, dt$$

$$u_0(x)+u_1(x)+u_2(x)+\dots = f(x) + \int_0^x K(x,t)\left[A_0(t)+A_1(t)+\dots\right]dt$$

$$u_0(x) = f(x)$$

$$u_1(x) = \int_0^x K(x,t)A_0(t)\,dt$$

$$u_2(x) = \int_0^x K(x,t)A_1(t)\,dt$$

$$\vdots$$

$$u_{k+1}(x) = \int_0^x K(x,t)A_k(t)\,dt \qquad (k \geq 0)$$

Adomian Polinomlarının Hesaplanması

$$F(u(x))$$

$$A_0 = F(u_0)$$

$$A_1 = u_1 F'(u_0)$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2!}u_1^2 F''(u_0)$$

$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!}u_1^3 F'''(u_0)$$

$$\vdots$$

$$F(u) = A_0 + A_1 + A_2 + A_3 + \dots$$

$$= F(u_0) + (u_1+u_2+u_3+\dots)F'(u_0) + \frac{1}{2!}\left(u_1^2+2u_1u_2+2u_1u_3+u_2^2+\dots\right)F''(u_0)+\dots$$

$$= F(u_0) + (u-u_0)F'(u_0) + \frac{1}{2!}(u-u_0)^2 F''(u_0) + \dots$$

Sonuçta A_n bir polinom serisi şeklindedir.

örnek

$$u(x) = x + \int_0^x u^2(t)\,dt \qquad F(u(x)) = [u(x)]^2$$

$$\sum_{n=0}^{\infty} u_n(x) = x + \int_0^x \sum_{n=0}^{\infty} A_n(t)\,dt$$

$$\begin{cases} u_0(x) = x \\ u_{k+1}(x) = \int_0^x A_k(t)\,dt & k \geq 0 \end{cases}$$

$$F(u_0(t)) = u_0^2(t) = t^2, \quad F'(u_0(t)) = 2t, \quad F''(u_0(t)) = 2$$

$$u_0(x) = x$$

$$u_1(x) = \int_0^x A_0(t)\,dt = \int_0^x F(u_0(t))\,dt = \int_0^x t^2\,dt = \frac{x^3}{3}$$

$$u_2(x) = \int_0^x A_1(t)\,dt = \int_0^x u_1(t)F'(u_0(t))\,dt = \int_0^x \frac{t^3}{3}2t\,dt = \frac{2}{15}x^5$$

$$u_3(x) = \int_0^x A_2(t)\,dt = \int_0^x \left[u_2(t)F'(u_0(t)) + \frac{1}{2!}u_1^2(t)F''(u_0(t))\right]dt$$

$$= \int_0^x \left(\frac{2}{15}t^5 \cdot 2t + \frac{1}{2}\frac{t^6}{9} \cdot 2\right)dt = \frac{17}{315}x^7$$

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \dots$$

$$u(x) = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \dots \approx \tan x$$

SORU

$$u(x) = \sin x + \frac{1}{4}\sin^2 x - \frac{1}{4}x^2 + \int_0^x (x-t)\,u^2(t)\,dt$$

$$u_0(x) = \sin x + \frac{1}{4}\sin^2 x - \frac{1}{4}x^2.$$

$$u_{k+1}(x) = \int_0^x (x-t)\,A_k(t)\,dt$$

$$u_1(x) = -\frac{1}{4}\sin^2 x + \cdots$$

$$u(x) = \sin x$$