Dif. Denklem Oluşturulması

$$F(x,y,y',y'',\dots,y^{(n)})=0 \rightarrow G(x,y,\underbrace{c_1,c_2,\dots,c_n}_{n \text{ tane}})=0$$

$$G(x,y,\overbrace{c_1,c_2,\dots,c_n}^{n \text{ tane}})=0$$

Genel çözüm

n. mertebeden dif.

n kere türetiyoruz.

n+1 tane denklemdeki $c_1,c_2,\dots,c_n$ keyfi sabitleri yok ederek

$$F(x,y,y',y'',\dots,y^{(n)})=0$$

örnek

$y = Cx^2$ eğri ailesine ait (genel çözümüne) dif. denklemi kurunuz.

$$\left\{\begin{array}{l} y = \underset{\text{1 tane}}{C}x^2 \\ y' = 2Cx \end{array}\right\} \text{1. mertebe}$$

$$C = \frac{y}{x^2}$$

$$y' = 2\frac{y}{x^{\cancel{2}}}\cancel{x}$$

$$\boxed{xy' = 2y} \; *$$

$$\frac{y = \cancel{C}x^{\cancel{2}}}{y' = 2\cancel{C}\cancel{x}}$$

$$\frac{y}{y'} = \frac{x}{2}$$

$$xy' = 2y$$

örnek

3 tane

$$y = \overbrace{A \sin 3x + B}^{2.\ \text{mertebe}} \cos 3x$$

$$y' = 3A\cos 3x - 3B\sin 3x$$

$$y'' = -9A\sin 3x - 9B\cos 3x$$

$$-9\underbrace{(A\sin 3x + B\cos 3x)}_{y}$$

$$y'' = -9y$$

$$y'' + 9y = 0$$

2 mertebe sabit katsayılı lineer dif. denklem

örnek

$$y = ax^3 + bx^2 + cx + d$$

$$y' = 3ax^2 + 2bx + c$$

$$y'' = 6ax + 2b$$

$$y''' = 6a$$

$$y^{IV} = 0$$

4. mertebe

örnek

$$y = A^x$$

$$y' = \underbrace{A^x}_{y} \ln A \rightarrow y' = y \frac{\ln y}{x}$$

$$x y' = y \ln y$$

1. mertebe 1. dereceden

$$* \quad \ln y = \ln A^x$$

$$\ln y = x \ln A \rightarrow \ln A = \frac{\ln y}{x}$$

$$x y' = y^2$$

$$x y' = \ln y$$

1. MERTEBEDEN DİF. DENKLEMLER

$F(x,y,y')=0 \longrightarrow G(x,y,C)=0$

Genel çözüm

$\boxed{y'=f(x,y)}$ **

$\frac{dy}{dx}=f(x,y) \Rightarrow f(x,y)dx-dy=0$

$\boxed{M(x,y)dx+N(x,y)dy=0}$ *

$N(x,y)dy=-M(x,y)dx$

$\frac{dy}{dx}=-\frac{M(x,y)}{N(x,y)}$ $f(x,y)$

$y'=2xy$

$\frac{dy}{dx}=2xy \Rightarrow 2xy\,dx-dy=0$

$\boxed{xy'=2y}$

$x\frac{dy}{dx}=2y$

$x\,dy=2y\,dx$

$\boxed{2y\,dx-x\,dy=0}$

Değişkenlerine Ayrılabilen Dif. Denklemler

$$\underbrace{M(x,y)}dx + \underbrace{N(x,y)}dy = 0$$

$$f_1(x)g_1(y)dx + f_2(x)g_2(y)dy = 0$$

$$f_1(x)g_1(y)dx = -f_2(x)g_2(y)dy$$

$$\frac{f_1(x)}{f_2(x)}dx = -\frac{g_2(y)}{g_1(y)}dy$$

$$\int \frac{f_1(x)}{f_2(x)}dx = \int -\frac{g_2(y)}{g_1(y)}dy$$

$$\boxed{F_1(x) + C = F_2(y)}$$

Genel çözüm

Örnek $\quad y' = 2x \leftrightarrow 2x\,dx - dy = 0$

$$\frac{dy}{dx} = 2x$$

$$\int dy = \int 2x\,dx$$

$$y = x^2 + C$$

örnek

$$y' + y\cos x = 0$$

$$\frac{dy}{dx} = -y\cos x$$

$$\int \frac{1}{y}dy = \int \cos x \, dx$$

$$\ln y = -\sin x + C$$

$$y = e^{-\sin x + C} = e^{-\sin x} \cdot e^{C}$$

$$\boxed{y = C e^{-\sin x}}$$

$$\ln y - \ln C = -\sin x$$

$$\ln \frac{y}{C} = -\sin x$$

$$\frac{y}{C} = e^{-\sin x}$$

$$\boxed{y = C e^{-\sin x}}$$

örnek

$$x\cdot(1-y^2)dx + y\cdot(1-x^2)dy = 0$$

$$\int\left(\underbrace{\frac{x}{1-x^2}}\,dx + \underbrace{\frac{y}{1-y^2}}\,dy\right) = \int 0$$

$$-\frac{1}{2}\ln(1-x^2) - \frac{1}{2}\ln(1-y^2) = -\frac{1}{2}\ln C$$

$$-\frac{1}{2}\left[\ln(1-x^2) + \ln(1-y^2)\right] = -\frac{1}{2}\ln C$$

$$\ln(1-x^2)(1-y^2) = \ln C$$

$$\boxed{(1-x^2)(1-y^2) = C}$$

$$\int\frac{x}{1-x^2}dx = -\frac{1}{2}\int\frac{du}{u} = -\frac{1}{2}\ln u = -\frac{1}{2}\ln(1-x^2)$$

$$1-x^2 = u$$

$$-2x\,dx = du$$

$$x\,dx = -\frac{1}{2}du$$

$$\ln a + \ln b = \ln(a\cdot b)$$

SORU

$$\underbrace{x\sin y\,dx + (x^2+1)\cos y\,dy = 0}_{\text{Genel çözüm } \checkmark \quad \text{Özel çözüm } \times}\ ; \quad \underbrace{y(1)=\frac{\pi}{2}}_{x=1 \quad y=\pi/2} \Rightarrow y = ?$$

Genel çözüm ✓

Özel çözüm ✗

$x=1 \quad y=\pi/2$

$c=?$ ✓

? $\boxed{(x^2+1)\sin^2 y = 2}$ Cevap