$$y'' + 4y' + 29y = \underbrace{4e^{-2x}} - \underbrace{4e^{-2x}\sin 5x} + \underbrace{x^2 - 5}$$

$$r^2 + 4r + 29 = 0 \qquad \boxed{r_{1,2} = -2 \mp 5i}$$

$$y_h = e^{-2x}\left(c_1 \cos 5x + c_2 \sin 5x\right)$$

$\underbrace{-2 \mp 5i}$

$f_1(x) = 4e^{-2x}$

$y_{\ddot{o}_1} = A e^{-2x}$

Rezonans yok!

$f_2(x) = -4e^{-2x}\sin 5x$

$y_{\ddot{o}_2} = e^{-2x}\left(A\sin 5x + B\cos 5x\right)x$

rezonans (özel durum)

$f_3(x) = x^2 - 5$

$y_{\ddot{o}_3} = ax^2 + bx + c$ Rezonans yok!

Örnek $y'' + y' - 2y = x^2 - 1$ dif. denk. çözünüz.

$$r^2 + r - 2 = 0 \qquad r_1 = -2,\ r_2 = 1 \qquad y_h = c_1 e^{-2x} + c_2 e^{x}$$

$y_{\ddot{o}} = (ax^2 + bx + c)$

$y_{\ddot{o}}' = 2ax + b$

$y_{\ddot{o}}'' = 2a$

$$2a + 2ax + b - 2ax^2 - 2bx - 2c = x^2 - 1$$

$$\boxed{-2ax^2 + (2a - 2b)x + 2a + b - 2c} \equiv 1 \cdot x^2 - 1$$

$$\begin{cases} -2a = 1 \\ 2a - 2b = 0 \\ 2a + b - 2c = -1 \end{cases}$$

$a = -\frac{1}{2}$

$a = b = -\frac{1}{2}$

$2a + b - 2c = -1$

$c = -\frac{1}{4}$

$$y_{\ddot{o}} = -\frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{4}$$

$y = y_h + y_ö \rightarrow \boxed{y = c_1 e^{-2x} + c_2 e^{x} - \frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{4}}$

Örnek: $y'' - y' = \boxed{2x+1}$

$r^2 - r = 0 \Rightarrow r(r-1) = 0$

$\boxed{r_1 = 0} \quad r_2 = 1$

$y_h = c_1 + c_2 e^{x}$

$y_ö = (ax+b)x = ax^2 + bx$ (Rezonans)

$y_ö' = 2ax + b$

$y_ö'' = 2a$

$2a - 2ax - b = 2x + 1$

$-2a = 2 \rightarrow a = -1$

$2a - b = 1 \rightarrow b = -3$

$y_ö = -x^2 - 3x$

$\boxed{y = y_h + y_ö = c_1 + c_2 e^{x} - x^2 - 3x}$

Örnek: $y'' - 4y' + 3y = 4e^{2x}$

$r^2 - 4r + 3 = 0 \quad r_1 = 1 \quad r_2 = 3 \quad y_h = c_1 e^{x} + c_2 e^{3x}$

* $y_ö = Ae^{2x}$
* $y_ö' = 2Ae^{2x}$
* $y_ö'' = 4Ae^{2x}$

$4Ae^{2x} - 8Ae^{2x} + 3Ae^{2x} = 4e^{2x}$

$\cancel{e^{2x}}(4A - 8A + 3A) = 4\cancel{e^{2x}}$

$-A = 4$

$A = -4$

$y_ö = -4e^{2x}$

$\boxed{y = y_h + y_ö = c_1 e^{x} + c_2 e^{3x} - 4e^{2x}}$

Örnek

$$y'' - 5y' + 6y = 5e^{2x} \quad \text{tek kat}$$

$$r^2 - 5r + 6 = 0 \qquad r_1 = 2, \quad r_2 = 3$$

$$y_h = C_1 e^{3x} + C_2 e^{2x}$$

$$y_ö = Axe^{2x} \quad (\text{rezonans})$$

$$y_ö' = \left(Ae^{2x} + 2Axe^{2x}\right)$$

$$y_ö'' = 4Ae^{2x} + 4Axe^{2x}$$

$$4Ae^{2x} + 4Axe^{2x} - 5Ae^{2x} - 10Axe^{2x} + 6Axe^{2x} = 5e^{2x}$$

$$4A + 4Ax - 5A - 10Ax + 6Ax = 5$$

$$-A = 5 \Rightarrow A = -5$$

$$y = y_h + y_ö$$

$$\boxed{y = C_1 e^{2x} + C_2 e^{3x} - 5xe^{2x}}$$

Soru

$$y'' - 4y' + 4y = 8e^{2x}$$

Örnek

$$y'' + 5y' + 6y = 2\cos x$$

$$r^2 + 5r + 6 = 0 \qquad r_1 = -2, \quad r_2 = -3$$

$$(r + 3)(r + 2)$$

$$y_h = C_1 e^{-2x} + C_2 e^{-3x}$$

$$y_{\ddot{o}} = A\cos x + B\sin x$$
$$y_{\ddot{o}}' = -A\sin x + B\cos x$$
$$y_{\ddot{o}}'' = -A\cos x - B\sin x$$
$$-A\cos x - B\sin x - 5A\sin x + 5B\cos x + 6A\cos x + 6B\sin x = 2\cos x$$

$$(5A+5B)\cos x + (-5A+5B)\sin x = 2\cos x$$

$$\left.\begin{aligned} 5A + 5B &= 2 \\ -5A + 5B &= 0 \end{aligned}\right\} \quad 10B = 2 \rightarrow B = \frac{1}{5}$$
$$A = \frac{1}{5}$$

$$y_{\ddot{o}} = \frac{1}{5}\cos x + \frac{1}{5}\sin x$$
$$y = y_h + y_{\ddot{o}} = c_1 e^{-2x} + c_2 e^{-3x} + \frac{1}{5}(\cos x + \sin x)$$

Örnek

$$y'' + 4y = \sin 2x$$
$$r^2 + 4 = 0 \Rightarrow r^2 = -4 \rightarrow \underbrace{r_{1,2} = \mp 2i}_{\text{tek kat kök}}$$
$$y_h = c_1\cos 2x + c_2 \sin 2x$$
$$y_{\ddot{o}} = (A\sin 2x + B\cos 2x)x \quad (\text{rezonans})$$
$$A(x\sin 2x) + B(x\cdot\cos 2x)$$

$$y_{\ddot{o}}' = A\sin 2x + 2A(x\cdot\cos 2x) + B\cos 2x - 2B(x\cdot\sin 2x)$$
$$y_{\ddot{o}}'' = 4A\cos 2x - 4Ax\sin 2x - 4B\sin 2x - 4Bx\cos 2x$$

$$4A\cos 2x - \cancel{4Ax\sin 2x} - 4B\sin 2x - \cancel{4Bx\cos 2x}$$
$$+ \cancel{4Ax\sin 2x} + \cancel{4Bx\cos 2x} = \sin 2x$$

$4A = 0 \rightarrow A = 0$

$-4B = 1 \rightarrow B = -\frac{1}{4}$

$y_ö = \frac{-x}{4}\cos 2x$

$y = y_h + y_ö$

$$\boxed{y = C_1\cos 2x + C_2\sin 2x - \frac{x}{4}\cos 2x}$$

Örnek

$y'' + y = \boxed{\sin^2 x}$ $\quad \sin^2 x = \frac{1-\cos 2x}{2}$

$r^2 + 1 = 0 \Rightarrow r_{1,2} = \mp i \quad y_h = C_1\sin x + C_2\cos x$

$$* \quad y'' + y = \underbrace{\frac{1}{2}}_{y_{ö_1}} - \frac{1}{2}\underbrace{\cos 2x}_{y_{ö_2}}$$

$y_{ö_1} = A$

$y_{ö_1}' = 0$

$y_{ö_1}'' = 0$

$0 + A = \frac{1}{2}$

$A = \frac{1}{2}$ $\quad y_{ö_1} = \frac{1}{2}$

$y_{ö_2} = A\sin 2x + B\cos 2x$

$y_{ö_2}' = 2A\cos 2x - 2B\sin 2x$

$y_{ö_2}'' = -4A\sin 2x - 4B\cos 2x$

$-4A\sin 2x - 4B\cos 2x + A\sin 2x + B\cos 2x = -\frac{1}{2}\cos 2x$

$-3A\sin 2x - 3B\cos 2x = -\frac{1}{2}\cos 2x$

$-3A = 0 \rightarrow A = 0$ $\quad y_{ö_2} = \frac{1}{6}\cos 2x$

$-3B = -\frac{1}{2} \rightarrow B = \frac{1}{6}$

$$y_{\breve{o}} = y_{\breve{o}_1} + y_{\breve{o}_2} = \frac{1}{2} + \frac{1}{6}\cos 2x$$

$$\boxed{y = y_h + y_{\breve{o}} = C_1 \sin x + C_2 \cos x + \frac{1}{2} + \frac{1}{6}\cos 2x}$$