2.) Sabitlerin (Parametrelerin) Değişimi Yöntemi (Lagrange Yöntemi)

$$a_0 y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \dots + a_{n-1} y' + a_n y = f(x)$$

$$\underbrace{a_0 r^n + a_1 r^{n-1} + a_2 r^{n-2} + \dots + a_{n-1} r + a_n r = 0}_{n \text{ tane kök}} \qquad y = y(x)$$

$$\boxed{y = c_1 y_1 + c_2 y_2 + \dots + c_n y_n}$$

$c_1 = c_1(x)$
$c_2 = c_2(x)$
$\vdots$
$c_n = c_n(x)$

$$\begin{cases} c_1' y_1 + c_2' y_2 + \dots + c_n' y_n = 0 \\ c_1' y_1' + c_2' y_2' + \dots + c_n' y_n' = 0 \\ c_1' y_1'' + c_2' y_2'' + \dots + c_n' y_n'' = 0 \\ \vdots \\ c_1' y_1^{(n-1)} + c_2' y_2^{(n-1)} + \dots + c_n' y_n^{(n-1)} = \dfrac{f(x)}{a_0} \end{cases}$$

$c_1', c_2', \dots, c_n'$ bilinmeyen fonk.

$i = 1, 2, 3, \dots$

$$c_i' = \underbrace{\frac{dc_i}{dx} = F_i(x)}$$

$$\int dc_i = \int F_i(x)\,dx$$

$$c_i(x) = G_i(x) + K_i$$

int. sabit
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$$y = c_1 y_1 + c_2 y_2 + \dots + c_n y_n$$

$$\boxed{y = y_h + y_p}$$

Örnek: $1.\ y'' + y = \boxed{\operatorname{cosec} x}$ dif. denklemini çözünüz.

$r^2 + 1 = 0 \qquad r_{1,2} = \mp i \qquad y = c_1 \sin x + c_2 \cos x$

$y = c_1 \sin x + c_2 \cos x$

$c_1 = c_1(x) \qquad c_2 = c_2(x)$

$\sin x \,/\ c_1' \sin x + c_2' \cos x = 0 \quad \checkmark \qquad c_1' = ?$

$\cos x \,/\ c_1' \cos x + c_2' (-\sin x) = \dfrac{1}{\sin x} \quad \checkmark \qquad c_2' = ?$

$c_1' \sin^2 x + \cancel{c_2' \sin x \cos x} = 0$

$c_1' \cos^2 x + \cancel{c_2' (-\sin x \cos x)} = \dfrac{\cos x}{\sin x}$

$c_1' (\underbrace{\sin^2 x + \cos^2 x}_{1}) = \dfrac{\cos x}{\sin x} \longrightarrow \boxed{c_1' = \dfrac{\cos x}{\sin x}}$

$c_1' \sin x + c_2' \cos x = 0$

$\dfrac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x} + c_2' \cos x = 0 \Rightarrow c_2' \cancel{\cos x} = -\cancel{\cos x}$

$\boxed{c_2' = -1}$

$c_1' = \boxed{\dfrac{dc_1}{dx}} = \dfrac{\cos x}{\sin x} \Rightarrow \int dc_1 = \int \dfrac{\cos x}{\sin x}\, dx$

$\boxed{c_1(x) = \ln|\sin x| + K_1}$

$c_2' = \dfrac{dc_2}{dx} = -1 \Rightarrow \int dc_2 = \int -1\, dx$

$\boxed{c_2(x) = -x + K_2}$

$y = c_1 \sin x + c_2 \cos x$

$$y = \left[\ln(\sin x) + k_1\right]\sin x + \left[-x + k_2\right]\cos x$$

$$y = \underbrace{k_1 \sin x + k_2 \cos x}_{y_h} + \underbrace{\sin x \ln(\sin x) - x\cos x}_{y_p}$$

Örnek: $y'' + 2y' + y = \boxed{e^{-x}\ln x}$ (Sabitlerin değişimi)

$$r^2 + 2r + 1 = 0$$

$$(r+1)^2 = 0 \Rightarrow r_{1,2} = -1 \quad (2 \text{ katlı})$$

$$y = e^{-x}(c_1 x + c_2) = c_1 x e^{-x} + c_2 e^{-x}$$

$$c_1' \, x \cdot e^{-x} + c_2' e^{-x} = 0$$

$$c_1'(e^{-x} - x e^{-x}) + c_2'(-e^{-x}) = \frac{e^{-x}\ln x}{1}$$

$$c_1' x + c_2' = 0$$

$$c_1'(1-x) - c_2' = \ln x$$

$$c_1' = \ln x \qquad c_2' = -x\ln x$$

$$\frac{dc_1}{dx} = \ln x \qquad \frac{dc_2}{dx} = -x\ln x$$

$$\int dc_1 = \int \ln x \, dx \qquad \int dc_2 = \int -x\ln x \, dx$$

$$c_1(x) = x\ln x - x + k_1 \qquad c_2(x) = -\frac{x^2}{2}\ln x + \frac{x^2}{4} + k_2$$

$$y = c_1 x e^{-x} + c_2 e^{-x} \Rightarrow y = (x\ln x - x + k_1) x e^{-x} + \left(-\frac{x^2}{2}\ln x + \frac{x^2}{4} + k_2\right) e^{-x}$$

$$y = y_h + y_p$$

örnek

$y''' + y' = \sec x$

$y_1 = 1$
$y_2 = \cos x$
$y_3 = \sin x$

$r^3 + r = 0 \rightarrow \quad r(r^2+1) = 0$

$r_1 = 0 \quad r_{2,3} = \mp i$

$$y = C_1 + C_2 \cos x + C_3 \sin x$$

$$\begin{aligned} C_1' \cdot 1 + C_2' \cos x + C_3' \sin x &= 0 \\ C_1' \cdot 0 + C_2'(-\sin x) + C_3' \cos x &= 0 \\ C_1' \cdot 0 + C_2'(-\cos x) + C_3'(-\sin x) &= \frac{1}{\cos x} \end{aligned}$$

$C_1' = \frac{1}{\cos x}$

$\frac{dC_1}{dx} = \frac{1}{\cos x}$

$\int dC_1 = \int \frac{1}{\cos x} dx$

$C_1(x) = \ln(\sec x + \tan x) + K_1$

$C_2' = -1$

$\frac{dC_2}{dx} = -1$

$\int dC_2 = \int - dx$

$C_2(x) = -x + K_2$

$C_3' = \frac{-\sin x}{\cos x}$

$\frac{dC_3}{dx} = \frac{-\sin x}{\cos x}$

$\int dC_3 = \int -\frac{\sin x}{\cos x} dx$

$C_3(x) = \ln(\cos x) + K_3$

$$y = C_1 + C_2 \cos x + C_3 \sin x$$

$$y = \ln(\sec x + \tan x) + K_1 + \cos x(-x + K_2) + \sin x\,(\ln(\cos x) + K_3)$$