n. mertebeden sabit katsayılı lineer dif. denklemlerde 2. taraflı dif. denklemin çözümü

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = f(x)$$

$f(x) \neq 0 \rightarrow$ 2. taraflı dif dent çözümü $\rightarrow y_ö \ (y_p)$

$f(x) = 0 \rightarrow$ 2. tarafsız dif. dent. çözümü $\rightarrow y_h$

$$\boxed{y = y_h + y_ö} \quad \text{Genel çözüm}$$

$$ay'' + by' + cy = f(x)$$

$$y = y_h + y_ö$$
$$y' = y_h' + y_ö'$$
$$y'' = y_h'' + y_ö''$$

$$a(y_h'' + y_ö'') + b(y_h' + y_ö') + c(y_h + y_ö) = f(x)$$

$$\underbrace{ay_h'' + by_h' + cy_h}_{0} + \boxed{ay_ö'' + by_ö' + cy_ö = f(x)}$$

$y_ö \rightarrow$ Belirsiz katsayılar

1) Belirsiz Katsayılar Yöntemi

$x^n, \ e^{\alpha x}, \ \sin\beta x, \ \cos\beta x \longrightarrow f(x)$

$n \geq 0$ ve $n \in \mathbb{Z}$ $\quad \alpha$ ve $\beta \in \mathbb{R}$

Not: Tablodaki 2. sütunda yer alan 0, α, $\mp i\beta$, $\alpha \mp i\beta$ değerleri karakteristik denklemin p katlı kökü ise 3. sütunda önerilen ifadeler x^p ile çarpılır.

Rezonans

Örnek

$$y''' - y'' - y' + y = \underbrace{5e^x}_{y_{ö_1}} + \underbrace{6x^2e^{-x}}_{y_{ö_2}} - \underbrace{e^x \cos x}_{y_{ö_3}}$$

$r^3 - r^2 - r + 1 = 0$

$r^2(r-1) - (r-1) = 0$

$\underbrace{(r-1)}_{1}\underbrace{(r^2-1)}_{1\,-1} = 0$

$\boxed{\begin{array}{l} r_1 = r_2 = 1 \quad (2 \text{ katlı}) \\ r_3 = -1 \end{array}}$

$y_h = e^x(c_1 x + c_2) + c_3 e^{-x}$

$y_ö = y_{ö_1} + y_{ö_2} + y_{ö_3}$

$f_1(x) = 5e^{1x} \qquad \alpha = 1 = \underbrace{r_1 = r_2}$

$y_{ö_1} = A e^x x^2$ (Rezonans)

$f_2(x) = 6x^2 \cdot e^{-x} \qquad \alpha = -1 = r_3$

$y_{ö_2} = e^{-x}(A_0 x^2 + A_1 x + A_2)\, x$ (Rezonans)

$f_3(x) = -e^x \cdot \cos x$

$y_{ö_3} = e^x(A\cos x + B\sin x)\, X$

Örnek:

$$y''' - 3y'' + 4y' - 12y = 3e^{-2x} - 5\sin 3x + 7x\cos 2x$$

$$r^3 - 3r^2 + 4r - 12 = 0$$

$$r^2(r-3) + 4(r-3) = 0$$

$$(r-3)\underbrace{(r^2+4)}_{\mp 2i} = 0$$

$$3$$

$r_1 = 3$

$r_{2,3} = \mp 2i$ tek kat kök

$$y_h = C_1 e^{3x} + C_2 \sin 2x + C_3 \cos 2x$$

$f_1(x) = 3e^{-2x}$

$y_{ö_1} = Ae^{-2x}$

$f_2(x) = -5\sin 3x$

$y_{ö_2} = A\sin 3x + B\cos 3x$

$y_{ö_3} = 7x\cos 2x$

$$y_{ö_3} = \left[(A_0x + A_1)\cos 2x + (B_0x + B_1)\sin 2x\right] x$$

(rezonans)

Soru:

$$y'' + 4y' + 29y = 4e^{-2x} - 4e^{-2x}\sin 5x + x^2 - 5$$