$\boxed{1}$ 04/05/2020

OPTIMIZASYON TEKNİKLERİ

$\min f(x) = \frac{2}{3}x_1^3 - 2x_1x_2 - 5x_1 + 2x_2^2 + 4x_2 + 5$

Kritik noktalarını bulunuz. Min, maksimum ve eğer noktaları olarak sınıflandırın.

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 2x_1^2 - 2x_2 - 5 \\ -2x_1 + 4x_2 + 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2x_2 = 2x_1^2 - 5$

$-2x_1 + 4x_2 + 4 = 0$ yerine konursa

$-2x_1 + 4x_1^2 - 10 + 4 = 0$

$4x_1^2 - 2x_1 - 6 = 0$

$2x_1^2 - x_1 - 3 = 0$

$(2x_1 - 3)(x_1 + 1) = 0$

$\boxed{x_1 = 3/2}$ $\boxed{x_1 = -1}$

$2x_2 = 2x_1^2 - 5$ de yerine konursa

$2x_2 = 2\frac{9}{4} - 5 \Rightarrow 2x_2 = \frac{9}{2} - 5 \quad x_2 = \frac{\frac{9}{4} - \frac{5}{2}}{2} = -\frac{1}{4}$

$2x_2 = 2(-1)^2 - 5 \Rightarrow 2x_2 = -3 \quad x_2 = -3/2$

$A(3/2, -1/4) \qquad B = (-1, -3/2)$

$$\nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 4x_1 & -2 \\ -2 & 4 \end{bmatrix} \quad \nabla^2 f(A) = \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}$$

$\Delta_1 = 6$

$\Delta_2 = 20$

$\nabla^2 f(A)$ pozitif definit A noktası Minimum noktasıdır

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$$\nabla^2 f(B) = \begin{bmatrix} 4x_1 & -2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -2 & 4 \end{bmatrix} \quad \Delta_1 = -4 \quad \Delta_2 = -20 \quad \text{İndefinit}$$

$(-1, -3/2)$ Eğer noktasıdır.

<u>Örnek</u> $f(x,y) = 3x^2 - 2xy + 2y^2 - 2x + y$

Başlangıç noktasını $[0,0]^T$ alarak En hızlı düşüş algoritmasıyla 2 aşama giderek f fonksiyonunu Minimize ediniz.

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}$$

$$x_{i+1} = x_i - \alpha g_i$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 6x - 2y - 2 \\ -2x + 4y + 1 \end{bmatrix} \quad \nabla f(x_0) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}^T = g_0$$

$x = 0$
$y = 0$

$$\alpha_0 = \frac{g_0 g_0'}{g_0 Q g_0'} = \frac{[-2, 1]\begin{bmatrix} -2 \\ 1 \end{bmatrix}}{[-2,1]\begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix}} = \frac{5}{[-14, 8]\begin{bmatrix} -2 \\ 1 \end{bmatrix}} = \frac{5}{36}$$

$$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - a_0 g_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \alpha_0 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{5}{36}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} \frac{5}{18} \\ -\frac{5}{36} \end{bmatrix} \qquad g_1 = \nabla f(x_1) = \begin{bmatrix} \frac{30}{18} + \frac{10}{36} - 2 \\ -\frac{10}{18} - \frac{20}{36} + 1 \end{bmatrix} = \begin{bmatrix} -1/18 \\ -1/9 \end{bmatrix}$$

$$\alpha_1 = \frac{[-1/18, -1/9]\begin{bmatrix} -1/18 \\ -1/9 \end{bmatrix}}{[-1/18, -1/9]\begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -1/18 \\ -1/9 \end{bmatrix}} = \frac{\left[\frac{1}{324} + \frac{1}{81}\right]}{\left[\frac{-6}{18} + \frac{2}{9}, \frac{2}{18} - \frac{4}{9}\right]\begin{bmatrix} -1/18 \\ -\frac{1}{9} \end{bmatrix}}$$

$$\alpha_1 = \frac{\left[\frac{5}{324}\right]}{\left[-\frac{1}{9}, -\frac{3}{9}\right]\begin{bmatrix} -1/18 \\ -1/9 \end{bmatrix}} = \frac{\frac{5}{324}}{\left[\frac{1}{162} + \frac{3}{81}\right]} = \frac{5/324}{7/162} = \frac{5}{324} \cdot \frac{162}{7} = 5/14$$

$\boxed{3}$ $x_2 = x_1 - \alpha_1 g_1 = \begin{bmatrix} 5/18 \\ \frac{-5}{36} \end{bmatrix} - \frac{5}{14} \begin{bmatrix} -1/18 \\ -1/9 \end{bmatrix}$

$= \begin{bmatrix} \frac{5}{18} + \frac{5}{18.14} \\ \frac{-5}{36} + \frac{5}{14.9} \end{bmatrix} = \begin{bmatrix} \frac{75}{18.14} \\ \frac{-50}{14.36} \end{bmatrix} = \begin{bmatrix} \frac{25}{84} \\ \frac{-25}{252} \end{bmatrix}$

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<u>Problem</u> $\text{Min} f(x,y) = 3x^2 + \frac{3y^2}{2} - 3x - 3y$

a) Analitik olarak

b) Eşlenik yön algoritmasıyla $x_0 = [0,0]^T$ alarak

hesaplayın.

a) $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 6x-3 \\ \frac{6y}{2}-3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad x = \frac{1}{2}$, $y = 1$

$\nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix}$ $\quad \Delta_1 = 6 > 0$, $\Delta_2 = 18 > 0$

o halde $x = 1/2$ $y = 1$ noktası Min. noktadır.

b) $\alpha_0 = -\frac{g_0 d_0^T}{d_0 Q d_0'} \Rightarrow g_0 = \nabla f(x_0) = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$ $\quad d_0 = -g_0 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$\alpha_0 = -\frac{[-3,-3]\begin{bmatrix} +3 \\ +3 \end{bmatrix}}{[+3,+3]\begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} +3 \\ +3 \end{bmatrix}} = \frac{18}{[18,9]\begin{bmatrix} 3 \\ 3 \end{bmatrix}} = \frac{18}{54+27} = \frac{2}{9}$

$x_1 = x_0 + \alpha_0 d_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \frac{2}{9}\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix}$

$g_1 = \nabla f(x_1) = \begin{bmatrix} 6x-3 \\ 3y-3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\boxed{4}$ $B_1 = \dfrac{g_1 Q d_0}{d_0 Q d_0'} = \dfrac{[1,-1]\begin{bmatrix}6 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}3\\3\end{bmatrix}}{[3,3]\begin{bmatrix}6 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}3\\3\end{bmatrix}}$

$= \dfrac{[6,-3]\begin{bmatrix}3\\3\end{bmatrix}}{[18,9]\begin{bmatrix}3\\3\end{bmatrix}} = \dfrac{18-9}{54+27} = \dfrac{9}{81} = \dfrac{1}{9}$

$d_1 = -g_1 + \beta_0 d_0 = -\begin{bmatrix}1\\-1\end{bmatrix} + \frac{1}{9}\begin{bmatrix}3\\3\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix} + \begin{bmatrix}1/3\\1/3\end{bmatrix} = \begin{bmatrix}-2/3\\4/3\end{bmatrix}$

$\alpha_1 = -\dfrac{g_1 . d_1^T}{d_1 Q d_1^T} = \dfrac{-[1,-1]\begin{bmatrix}-2/3\\4/3\end{bmatrix}}{[-\frac{2}{3}, \frac{4}{3}]\begin{bmatrix}6 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}-2/3\\4/3\end{bmatrix}} = \dfrac{\frac{2}{3}+\frac{4}{3}}{[-4,4]\begin{bmatrix}-2/3\\4/3\end{bmatrix}}$

$= \dfrac{2}{\left[8/3 + \frac{16}{3}\right]} = \dfrac{2}{8} = 1/4$

$x_2 = x_1 + \alpha_1 d_1 = \begin{bmatrix}2/3\\2/3\end{bmatrix} + \frac{1}{4}\begin{bmatrix}-2/3\\4/3\end{bmatrix} = \begin{bmatrix}\frac{2}{3}-\frac{1}{6}\\ \frac{2}{3}+\frac{1}{3}\end{bmatrix}$

$= \begin{bmatrix}1/2\\1\end{bmatrix}$

$\nabla f(x_2) = g_2 = \begin{bmatrix}6x-3\\3y-3\end{bmatrix}_{\substack{x=\frac{1}{2}\\ y=1}} = \begin{bmatrix}0\\0\end{bmatrix}$ elde edilir

o halde $\begin{bmatrix}1/2\\1\end{bmatrix}$ optimumdur.

$\boxed{5}$ $\quad Min f(x) = x_1^2 + x_2^2$

$$x_1^2 + 2x_2^2 = 1 \Rightarrow g(x) = x_1^2 + 2x_2^2 - 1$$

$$L = f(x) + \lambda g(x)$$

$$L = x_1^2 + x_2^2 + \lambda(x_1^2 + 2x_2^2 - 1)$$

(1) $\dfrac{\partial L}{\partial x_1} = 2x_1 + 2\lambda x_1 = 0$ $\qquad$ (1) $2x_1(1+\lambda) = 0$

a) $x_1 = 0 \lor \lambda = -1$ (b)

(2) $\dfrac{\partial L}{\partial x_2} = 2x_2 + 4\lambda x_2 = 0$

(3) $x_1^2 + 2x_2^2 - 1 = 0$

$x_1 = 0 \Rightarrow$ (2) $2x_2^2 - 1 = 0$ $\quad x_2 = \frac{1}{\sqrt{2}}$ $\quad x_2 = -\frac{1}{\sqrt{2}}$

(2)den $2x_2(1+2\lambda) = 0$ $\quad x_2 = \frac{1}{\sqrt{2}}$ konsada $x_2 = -\frac{1}{\sqrt{2}}$ konsada

$\lambda = -1/2$ elde edilir.

ohulde $A\left(x_1 = 0 \quad x_2 = \frac{1}{\sqrt{2}} \quad \lambda = -1/2\right)$

$B\left(x_1 = 0 \quad x_2 = -\frac{1}{\sqrt{2}} \quad \lambda = -1/2\right)$

$\lambda = -1$ ise (1) kısıtı sağlanır. (2) den

$2x_2 - 4x_2 = 0$ $\quad -2x_2 = 0$ $\quad x_2 = 0$ olur. (3)de

yerine konursa $\quad x_1^2 + 2 \cdot 0^2 - 1 = 0$ $\quad x_1 = 1$ $\quad x_1 = -1$

elde edilir.

$\boxed{6}$ $C(x_1=1,\ x_2=0,\ \lambda=-1)$ $D(x_1=-1\ x_2=0\ \lambda=-1)$

$$\nabla^2 L = \begin{bmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 2+2\lambda & 0 \\ 0 & 2+4\lambda \end{bmatrix}$$

$$\Delta^2 L(A) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \Delta^2(B) \qquad \Delta_1 = 1 > 0 \quad \Delta_2 = 0$$

pozitif semidefinit A ve B noktaları minimum noktalardır.

$$[h_1\ h_2]\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = [h_1\ 0]\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = h_1^2 > 0$$

$$\Delta^2 L(B) = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} = \Delta^2(D)$$

$$[h_1\ h_2]\begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = [0,\ -2h_2]\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = -2h_2^2 < 0$$

negatif definit C, ve D noktaları maksimum noktalardır.

$\boxed{7}$ <u>problem</u>

$$\max f = 5-(x_1-2)^2-2(x_2-1)^2$$

$$x_1+4x_2-3=0$$

$L = f + \lambda(x_1+4x_2-3)$ Zira Maxf olduğundan Minimuma çevirmiştik

$$L = -f + \lambda(x_1+4x_2-3)$$

$$L = (x_1-2)^2+2(x_2-1)^2-5+\lambda(x_1+4x_2-3)$$

$$\frac{\partial L}{\partial x_1} = 2(x_1-2)+\lambda = 0 \quad (1)$$

$$\frac{\partial L}{\partial x_2} = 4(x_2-1)+4\lambda = 0 \quad (2)$$

$$x_1+4x_2-3=0 \quad (3)$$

(1) ve (2)den λ ları yok edelim.

$2x_1-4+\lambda=0$ (−4) ile çarp.

$4x_2-4+4\lambda=0$

$-8x_1+16-\cancel{4\lambda}=0$

$4x_2-4+\cancel{4\lambda}=0$

$-8x_1+4x_2+12=0$

$-2x_1+x_2+3=0$ elde edilir.

2 ile çarp. $x_1+4x_2-3=0$ (3) denk.

$-2x_1+x_2+3=0$

$2x_1+8x_2-6=0$

$9x_2-3=0$

$\boxed{x_2=1/3}$

$\boxed{8}$ $-2x_1 + x_2 + 3 = 0$ $x_2 = 1/3$ konursa

$\cancel{-2/3 + \lambda x_2 + 3 = 0}$ $-2x_1 + \frac{1}{3} + \frac{9}{3} = 0$

$\cancel{x_2 = 3 + \frac{2}{3} = 7/3}$ $-2x_1 = -10/3$

$\boxed{x_1 = +5/3}$

(1) denkleminde $x_1 = 5/3$ konursa
$2(5/3 - 2) + \lambda = 0$ $\lambda = 2/3$ elde edilir.
$A\left(x_1 = 5/3,\ x_2 = 1/3\ \lambda = 2/3\right)$

$$\nabla^2 L = \begin{bmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

$\Delta_1 = 2$ $\Delta_2 = 8$ pozitif definit $A(-f)$in minimum olup ancak A f'in maksimumu olur.