Örnekler

① $y''' + 2(y'')^2 + y' = \cos x$, 3. mertebe, 1. derece

② $\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right)^3 + y = 0$, 2. mertebe, 1. derece

③ $(y'')^{2/3} = 1 + (y')^2$, rasyonel kuvvet tamsayı yapılmalı

$\Rightarrow (y'')^2 = (1 + (y')^2)^3$, 2. mertebe, 2. derece

4. $(1+(y')^2)^{2/3} = y''$

$\Rightarrow (1+(y')^2)^2 = (y'')^3$, 2. mertebe 3. derece

5. $y'' + (y')^2 = \ln y'$, 2. mertebe 1. derece

6. $y'' + (y)^2 = \ln y''$. 2. mertebe, derecesi tanımlı değil

7. $\sin y' = y' + x$, 1. mertebe, derecesi tanımlı değil

8. $e^{y'} + x = 1$

$\Rightarrow e^{y'} = 1 - x$

$\Rightarrow y' = \ln(1-x)$, 1. mertebe, 1. derece

9. $e^{y'} + x = y'$. 1. mertebe, derece tanımlı değil

Her dif denk mertebeye sahiptir fakat dereceye sahip olmayabilir!!

① $y'' + \cos x \, y = \sin x$

2.M, 1.D, değişken katsayılı, lineer, homojen olmayan dif

② $y'' + \cos x \, y y' = \sin x$

2.M, 1.D, lineer olmayan dif denk

③ $y' = 1 + x y^2$

1.M, 1.D, lineer olmayan dif denk

④ $y'' + 5y' + 6y = e^x$

2.M, 1.D, sabit katsayılı, lineer, homojen olmayan

⑤ $(y'')^{1/3} = (1+y')^{5/2} \Rightarrow (y'')^2 = (1+y')^{1}$

2.M, 2.D, lineer olmayan dif denk

(6) $y'' + x \sin y = 0$

2.M, 1.D, lineer olmayan dif denk

(7) $y'' + \sin x \cdot y = 0$

2.M, 1.D, değişken katsayılı, lineer, homojen dif denk

(8) $\sin x \, y''' - \cos x \, y' = 2$

3.M, 1.D, değişken katsayılı, lineer, homojen olmayan dif denk

(9) $\dfrac{d^2x}{dt^2} = -\dfrac{2}{x^2}$

2.M, 1.D, lineer olmayan dif denk.

(10) $\dfrac{d^2x}{dt^2} = -\dfrac{2}{t^2} \qquad \left(\dfrac{d^2x}{dt^2} + \dfrac{2}{t^2} = 0 \right)$

2) $ydx+\left(2xy-e^{-2y}\right)dy=0$ diferansiyel denkleminin, bir $\mu=\mu(y)$ integrasyon çarpanını elde ederek, genel çözümünü bulunuz.

$$y\,dx+(2xy-e^{-2y})dy=0$$ (10)

$$P=y \quad , \quad Q=2xy-e^{-2y}$$

$$\frac{\partial}{\partial y}(\mu(y)P)=\frac{\partial}{\partial x}(\mu(y)Q)$$

$$\frac{\partial}{\partial y}(\mu y)=\frac{\partial}{\partial x}(\mu(2xy-e^{-2y}))$$ (1)

(10)

$$\frac{\partial}{\partial y}(\mu y)=\mu' y+\mu \quad , \quad \frac{\partial}{\partial x}(\mu(2xy-e^{-2y}))=2y\mu$$

(2)

$$\mu' y+\mu=2y\mu \Rightarrow \mu' y=(2y-1)\mu$$

(2)

$$\frac{d\mu}{\mu}=\frac{2y-1}{y}dy \Rightarrow \frac{d\mu}{\mu}=(2-\frac{1}{y})dy \Rightarrow \ln\mu=2y-\ln y$$ (2)

$$\ln(\mu y)=2y \Rightarrow \mu y=e^{2y} \Rightarrow \mu=\frac{1}{y}e^{2y}$$ (3)

Veya

$$\frac{\partial P}{\partial y}=1 \quad \frac{\partial Q}{\partial x}=2y$$ (2)

(3)

$$\ln\lambda=\int\frac{2y-1}{y}dy=\int(2-\frac{1}{y})dy$$

$$\ln\lambda=2y-\ln y$$ (2)

$$\lambda=\frac{e^{2y}}{y}$$ (3)

NURAN GÜZEL

$$\frac{1}{y}\cdot y e^{2y}dx+\frac{1}{y}e^{2y}(2xy-e^{-2y})dy=0$$ (3)

$$e^{2y}dx+(2xe^{2y}-\frac{1}{y})dy=0 \quad , \quad P_y=Q_x \quad \text{Tam dif.}$$

$$U(x,y)=\int e^{2y}dx+h(y) \Rightarrow u=xe^{2y}+h(y)$$ (3)

$$U_y=2xe^{2y}+h'(y) \Rightarrow 2xe^{2y}+h'(y)=2xe^{2y}-\frac{1}{y} \Rightarrow h'=-\frac{1}{y}, h(y)=-\ln y+K$$

(2) (2) (2)

$$U=xe^{2y}-\ln y+K=C_1 \qquad K-c=C$$

$$xe^{2y}-\ln y=C$$ (3)

1) $y=-xy'+(y')^3$ diferansiyel denkleminin genel çözümünü bulunuz. nuran güzel

NURAN GÜZEL

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$\boxed{y'=p}$ 2

$y=-xp+p^3$

$\boxed{y'=-p-xp'+3p^2p'}$ 2

$p=-p-xp'+3p^2p'$

$2p=p'(-x+3p^2)$ 1

$\boxed{2p\cdot\frac{dx}{dp}=3p^2-x}$ 2 $\Rightarrow 2p\frac{dx}{dp}+x=3p^2 \Rightarrow \boxed{\frac{dx}{dp}+\frac{x}{2p}=\frac{3}{2}p}$ (lineer) 3

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$\frac{dx}{dp}+\frac{x}{2p}=0 \Rightarrow \boxed{\frac{dx}{x}=-\frac{dp}{2p}}$ 2 $\Rightarrow \ln x+\frac{1}{2}\ln p=\ln C \Rightarrow \boxed{x=C\ldots}$ 2

$\boxed{c'p^{-1/2}-\cancel{\frac{1}{2}cp^{-3/2}}+\cancel{\frac{cp^{-1/2}}{2p}}=\frac{3}{2}p}$ 2 $\Rightarrow c'p^{-1/2}=\frac{3}{2}p \Rightarrow \boxed{c'=\frac{3}{2}p^{3/2}}$ 2

$c'=\frac{3}{2}p^{3/2} \Rightarrow \boxed{C=\frac{3}{5}p^{5/2}+K}$ 2

$x=\left(\frac{3}{5}p^{5/2}+K\right).p^{-1/2} \Rightarrow \boxed{x=\frac{3}{5}p^2+Kp^{-1/2}}$ 2

$y=-xp+p^3 \Rightarrow y=-\frac{3}{5}p^3-Kp^{1/2}+p^3 \Rightarrow \boxed{y=\frac{2}{5}p^3-Kp^{1/2}}$ 2

Başarılar

$\boxed{x=\frac{3}{5}p^2+Kp^{-1/2} \text{ ve } y=\frac{2}{5}p^3-Kp^{1/2}}$ 1

-) $y'' = (y')^3 + y'$ diferansiyel denkleminin genel çözümünü bulunuz.

$y' = p \quad , \quad y'' = p\dfrac{dp}{dy}$

NURAN GÜZEL

$p\dfrac{dp}{dy} = p^3 + p \Rightarrow$ i) $p = 0$ için $y' = 0 \Rightarrow y = c$

$\dfrac{1}{p^2+1} dp = dy$

$\arctan p = y + c_1$

$p = \tan(y + c_1)$

$y' = \tan(y + c_1)$

$\dfrac{dy}{dx} = \tan(y + c_1)$

$\dfrac{1}{\tan(y+c_1)} dy = dx$

$\dfrac{\cos(y+c_1)}{\sin(y+c_1)} dy = dx \Rightarrow \ln(\sin(y+c_1)) = x + c_2$

T301

4-) $a_0(x)\frac{d^2y}{dx^2}+a_1(x)\frac{dy}{dx}+a_2(x)y=0$ diferansiyel denkleminde a_0, a_1 ve a_2 fonksiyonları $a \le x \le b$ reel aralığında sürekli ve bu aralıktaki her x için $a_0(x) \ne 0$ olsun. f_1 ve f_2 fonksiyonları verilen diferansiyel denklemin lineer bağımsız iki çözümü ve A_1, A_2, B_1, B_2 de $A_1B_2 - A_2B_1 \ne 0$ denklemini sağlayan sabitler olsun. Bu durumda verilen diferansiyel denklemin $A_1f_1 + A_2f_2$ ve $B_1f_1 + B_2f_2$ çözümlerinin de $a \le x \le b$ de lineer bağımsız olduğunu gösteriniz.

NURAN GÜZEL

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$$w(f_1,f_2) = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix} = f_1f_2' - f_1'f_2 \ne 0$$

f_1, f_2 lineer bağımsız olduğundan

$$\begin{vmatrix} A_1f_1+A_2f_2 & B_1f_1+B_2f_2 \\ A_1f_1'+A_2f_2' & B_1f_1'+B_2f_2' \end{vmatrix} = \cancel{A_1B_1f_1f_1'} + A_1B_2f_1f_2' + A_2B_1f_2f_1'$$

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$$+ \cancel{A_2B_2f_2f_2'} - \cancel{A_1B_1f_1'f_1} - A_1B_2f_1'f_2$$

$$- A_2B_1f_2'f_1 - \cancel{A_2B_2f_2'f_2}$$

$$= (A_1B_2 - A_2B_1)f_2'f_1 + (A_2B_1 - A_1B_2)f_1'f_2$$

$$= (A_1B_2 - A_2B_1)(f_2'f_1 - f_1'f_2)$$ 5

0 5

$A_1B_2 - A_2B_1 \ne 0$ ve $f_2'f_1 - f_1'f_2 \ne 0$ olduğundan

$w(A_1f_1+A_2f_2,\ B_1f_1+B_2f_2) \ne 0$ dır dolayısıyla verilen çözüm fonksiyonları lineer bağımsızdırlar.

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YTÜ

C.1) 2014 YAZ DÖNEMİ DİF. DENKL'İ 2 VİZE SINAV ÇÖZÜM ANAHTARI 2 VİZE

NURAN GÜZEL

25P $y'' - 2y(y')^3 = 2y\cdot y'$ diferansiyel denklemini çözerek y' türevini bulunuz. (y' türevinin tüm çözümlerini bulunuz)

(2) $y' = p \quad y'' = \frac{dp}{dx} = \frac{dp}{dy}\cdot\frac{dy}{dx} = p\cdot\frac{dp}{dy}$ (3)

$p\cdot\frac{dp}{dy} - 2y\cdot p^3 = 2y\cdot p$ (5)

$p\left[\frac{dp}{dy} - 2y\,p^2 - 2y\right] = 0 \qquad p=0 \quad \frac{dy}{dx} = 0$ (2)

$\frac{dp}{dy} - 2y\,p^2 - 2y = 0$ (2) T.14

$\frac{dp}{dy} = 2y(p^2+1)$ (5) $\quad \int\frac{dp}{p^2+1} = \int 2y\,dy$ (2) T.19

$\arctan p = \frac{y^2}{\cancel{2}} + C_2$ (3)

$p = y' = \tan\left(\frac{y^2}{\cancel{2}} + C_2\right)$ (3)

A grubu

(1) $(y+1)y''=(y')^2$ denkleminin genel çözümünü bulunuz.

NURAN GÜZEL

$y'=p \Rightarrow y''=p\cdot\frac{dp}{dy}$ $\phi 3$

$\phi 2$

+2 $(y+1)\cdot p\,\frac{dp}{dy}=p^2 \Rightarrow (y+1)\cdot p\cdot\frac{dp}{dy}-p^2=0$

$p\left((y+1)\frac{dp}{dy}-p\right)=0$

~~1°) $p=0 \Rightarrow y'=0 \Rightarrow \boxed{y=c}$ tekil çözüm.~~ sorulmuyor

2°) $(y+1)\frac{dp}{dy}-p=0 \Rightarrow \frac{dp}{p}-\frac{dy}{y+1}=0$ $\phi 2$

$\phi 5$

$\ln p-\ln(y+1)=\ln c_1$ $\phi 3$

$\Rightarrow \frac{p}{y+1}=c_1 \Rightarrow \boxed{p=(y+1)c_1} \Rightarrow \frac{dy}{dx}=(y+1)c_1$

$\phi 3$

$\Rightarrow \int\frac{dy}{y+1}=\int c_1\,dx \Rightarrow \ln(y+1)=c_1x+c_2$ 25

$\Rightarrow y+1=e^{c_1x+c_2} \Rightarrow y=e^{c_1x+c_2}-1$

b) $\sqrt{1+y'^2}\,y'' = 2(y')^{3/2}\,y$; $y(\arctan\sqrt{8}) = \sqrt{8}$; $y'(\arctan\sqrt{8}) = 9$ başlangıç değer problemini çözünüz (10P)

x 'in bulunmadığı denklem

$y' = p$; $y'' = p\dfrac{dp}{dy}$ 05

Denklemde yerine yazılırsa ; $\sqrt{1+y^2}\; p\dfrac{dp}{dy} = 2p^{3/2}\,y$

$p\left(\sqrt{1+y^2}\,\dfrac{dp}{dy} - 2p^{1/2}y\right) = 0$

$\boxed{\displaystyle\int \frac{dp}{\sqrt{p}} = \int \frac{2y}{\sqrt{1+y^2}}\,dy}$ 10 $\longrightarrow$ $\underline{2\sqrt{p} = 2\sqrt{1+y^2} + c_1}$ 10

Verilen koşulları kullanarak c_1 sabitini bulalım;

$2\sqrt{9} = 2\sqrt{1+(\sqrt{8})^2} + c_1 \longrightarrow \boxed{c_1 = 0}$ 05

$\longrightarrow$ $2\sqrt{p} = 2\sqrt{1+y^2}$ $\quad p = y'$ idi

$\dfrac{dy}{dx} = 1+y^2$

$\displaystyle\int dx = \int \frac{dy}{1+y^2}$ 05

$x = \arctan y + c_2$ koşulları kullanırsak

$\arctan\sqrt{8} = \arctan\sqrt{8} + c_2 \Rightarrow c_2 = 0$ bulunur. 05

$\Rightarrow x = \arctan y \longrightarrow \underline{y = \tan x}$ //

B grubu

4) $(x-1)y'' + y' = (x-1)^2$ G.Ç ?

$y' = p \qquad y'' = p'$

$(x-1)p' + p = (x-1)^2$

$p' + \frac{1}{x-1}p = (x-1)$ lineer d.d

$\lambda(x) = e^{\int \frac{1}{x-1}dx} = e^{\ln(x-1)} = (x-1)$

$(x-1)p' + p = (x-1)^2$

$\frac{d}{dx}[(x-1)p] = (x-1)^2$

$\int d[(x-1)p] = \int (x-1)^2 dx$

$(x-1)p = \frac{(x-1)^3}{3} + c_1$

$p = y' = \frac{(x-1)^2}{3} + \frac{c_1}{(x-1)}$

$y = \int \left[\frac{(x-1)^2}{3} + \frac{c_1}{(x-1)} \right] dx$

$= \frac{(x-1)^3}{9} + c_1 . \ln|x-1| + c_2$

Soru1 $(3x-2y+2)dy+(2y-3x)dx=0$ denkleminin genel çözümünü bulunuz.

Homojen d.d.

$$3x-2y=u \Rightarrow 3dx-2dy=du$$

$$dy=\frac{3dx-du}{2}$$

NURAN GÜZEL

$$(u+2)dy-udx=0$$

$$(u+2)\left(\frac{3dx-du}{2}\right)-udx=0$$

$$(u+2)(3dx-du)-2u\,dx=0$$

$$(3u-2u+6)dx-(u+2)du=0$$

$$(u+6)dx=(u+2)du$$

$$\int dx=\int\frac{u+2}{u+6}du$$

$$x=\int\left(1-\frac{4}{u+6}\right)du \Rightarrow x=u-4\ln$$

$$x=3x-2y-4\ln|3x-2y+6|+c$$

$$2x-2y-4\ln|3x-2y+6|+c=0$$

$$\boxed{x-y-2\ln|3x-2y+6|+C=0}$$

2

B grubu

Soru 2 $2yy'' = 3yy' + (y')^2$ denkleminin genel çözümünü bulunuz ($y \geq 0$).

$$y' = p \Rightarrow y'' = p\frac{dp}{dy}$$

$$2ypp' = 3yp + p^2 \Rightarrow p(2p'y - 3y - p) = 0$$

NURAN GÜZEL

1) $p = 0 \Rightarrow y' = 0 \Rightarrow y = c$

2) $2p'y - 3y - p = 0 \Rightarrow p' - \frac{3}{2} - \frac{p}{2y} = 0$

$$p' - \frac{p}{2y} = \frac{3}{2} \quad (\text{Lineer d.})$$

$$p' - \frac{p}{2y} = 0 \Rightarrow \frac{dp}{p} - \frac{dy}{2y} = 0$$

$$\ln p - \frac{1}{2}\ln y = \ln c \Rightarrow p = c\sqrt{y}$$

$$p' = c'\sqrt{y} + \frac{c}{2\sqrt{y}}$$

$$c'\sqrt{y} + \frac{c}{2\sqrt{y}} - \frac{c\cancel{\sqrt{y}}}{2\cancel{y}\sqrt{y}} = \frac{3}{2}$$

$$c' = \frac{3}{2}\frac{1}{\sqrt{y}} \Rightarrow \boxed{c = 3\sqrt{y} + k}$$

$$p = 3y + k\sqrt{y}$$

$$\frac{dy}{dx} = 3y + k\sqrt{y} \Rightarrow \int\frac{dy}{3y + k\sqrt{y}} = \int dx \qquad \left(\begin{array}{l} y = t^2 \\ dy = 2t\,dt \end{array}\right.$$

$$x + \ln A = \int\frac{2dt}{3t+k} \Rightarrow x + \ln A = \frac{2}{3}\ln|3t+k|$$

$$x = \ln\frac{|3t+k|^{2/3}}{A}$$

$$(3t+k)^{2/3} = Ae^x \Rightarrow 3t + k = Be^{\frac{3}{2}x} \qquad B = A^{2/3}$$

$$\boxed{3\sqrt{y} = Be^{3/2x} - k}$$

1) $\frac{dy}{dx}+xy=10x^3y^3$ diferansiy... eminin genel çözümünü bulunuz.

$y'+xy=10x^3y^3 \qquad y'+P(x)y=Q(x)y^n$ Bernoulli D.D.

NURAN GÜZEL

$z=y^{-n+1} \Rightarrow z=y^{-2} \Rightarrow z'=\frac{-2y'}{y^3} \rightarrow \frac{y'}{y^3}=-\frac{z'}{2}$

$\frac{y'}{y^3}+\frac{x}{y^2}=10x^3 \Rightarrow -\frac{z'}{2}+xz=10x^3 \Rightarrow \underbrace{z'-2xz=-20x^3}_{\text{Lin. D.D}}$

$z'-2xz=0 \Rightarrow \int\frac{dz}{z}-\int 2x\,dx=\int 0 \Rightarrow \ln z-x^2=\ln c$

$\Rightarrow \frac{z}{c}=e^{x^2} \Rightarrow \boxed{z=ce^{x^2}}$ $\qquad c=c(x)$ olsun.

$z'=c'.e^{x^2}+2xce^{x^2}$

$c'e^{x^2}+\cancel{2xce^{x^2}}-\cancel{2xce^{x^2}}=-20x^3 \rightarrow \int c'=\int 20x^3e^{-x^2}$ $\boxed{\begin{matrix}x^2=t\\2xdx=dt\end{matrix}}$

$c=-10\int t.e^{-t}dt+K \rightarrow c=-10\left[t(-e^{-t})+\int e^{-t}dt\right]+K$

$u=t \qquad dv=e^{-t}$

$du=dt \qquad v=-e^{-t}$

$c=10te^{-t}-10e^{-t}+K \Rightarrow c(x)=10x^2e^{-x^2}-10e^{-x^2}+K$

$z=(10x^2e^{-x^2}-10e^{-x^2}+K)e^{x^2}=10x^2-10+Ke^{x^2}$

$\boxed{\frac{1}{y^2}=10x^2-10+Ke^{x^2}} \rightarrow \boxed{y=\frac{1}{\sqrt{10x^2-10+Ke^{x^2}}}}$

Başarılar

$x^2 \frac{dy}{dx} + xy + x^2y^2 = 4$ denkleminin $\frac{k}{x}$ şeklinde bir özel çözüm bularak genel çözümünü bulunuz (Burada k pozitif tamsayıdır)

$y' = -y^2 - \frac{1}{x}y + \frac{4}{x^2}$ Riccati

$y_1 = \frac{k}{x}$, $y_1' = -\frac{k}{x^2}$

NURAN GÜZEL

$-\frac{k}{x^2} = -\frac{k^2}{\cancel{x}^2} - \frac{1}{x}\frac{k}{x} + \frac{4}{x^2}$

$k^2 = 4 \Rightarrow k = \mp 2$, $k=2$ olmalı

$\boxed{y_1 = \frac{2}{x}}$

$y = \frac{2}{x} + \frac{1}{u(x)}$, $y' = -\frac{2}{x^2} - \frac{u'}{u^2}$

$-\frac{2}{x^2} - \frac{u'}{u^2} = -\left(\frac{2}{x} + \frac{1}{u}\right)^2 - \frac{1}{x}\left(\frac{2}{x} + \frac{1}{u}\right) + \frac{4}{x^2}$

$\cancel{-\frac{2}{x^2}} - \frac{u'}{u^2} = \cancel{-\frac{4}{x^2}} - \frac{4}{xu} - \frac{1}{u^2} \cancel{-\frac{2}{x^2}} - \frac{1}{xu} \cancel{+\frac{4}{x^2}}$

$-\frac{u'}{u^2} = -\frac{5}{ux} - \frac{1}{u^2}$, $u' = \frac{5}{x}u + 1$, $u' - \frac{5}{x}u = 1$ lineer)

$= e^{\int \frac{5}{x}dx}\left[\int e^{-\int\frac{5}{x}dx} dx + c\right] = e^{5\ln x}\left[\int e^{-5\ln x} dx + c\right]$

$= x^5\left(\int \frac{dx}{x^5} + c\right) = x^5\left(-\frac{1}{x^4} + c\right) = -x + cx^5$

$= \frac{2}{x} + \frac{1}{-x + cx^5} = \frac{-2x + 2cx^5 + x}{-x^2 + cx^6}$

$\frac{2cx^5 - x}{-x^2 + cx^6} = \frac{2cx^4 - 1}{-x + cx^5}$

DİFERANSİYEL DENKELEMLER I (0252311-0 01.11.2008
YILİÇİ 1.SINAVI (1.seans)

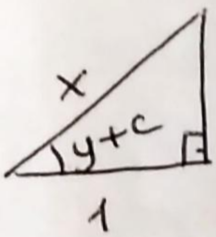
1	2	3	4	Toplam

Ad-Soyad:
Öğrenci no:
Grubu:
İmza:

Soru 1) $x = \sec(y+c)$ eğri ailesinin diferansiyel denklemini kurunuz.

NURAN GÜZEL

$$1 = y' \cdot \sec(y+c)\tan(y+c)$$



$\sqrt{\cancel{1-x^2}}$ x^2-1

$$1 = y' \cdot x \cdot \sqrt{\cancel{1-x^2}} \; x^2-1$$

$$y' = \frac{1}{x\sqrt{\cancel{1-x^2}}} \; x^2-1$$

$$y = \frac{1}{\cos(x+c)}$$

$$y' = \frac{\tan(x+c)}{\cos(x+c)}$$

$$y' = \tan(x-b)$$

$$\cos(x+c) = \frac{1}{y}$$

$$x + c = \arccos(1/y)$$

$$-\frac{k}{x^2} = -\frac{k^2}{k^2} - \frac{1}{x}\frac{k}{x} + \frac{4}{x^2}$$

$k=2$ olmalı

NURAN GÜZEL

Soru 4) $(12xy+6y^2)dx+(3x^2+4xy)dy=0$ diferansiyel denkleminin tam olup olmadığını kontrol ediniz. Tam diferansiyel değil ise tam hale getirip çözünüz.

$$P = 12xy + 6y^2 \qquad Q = 3x^2 + 4xy$$

$$P_y = 12x + 12y \qquad Q_x = 6x + 4y$$

$P_y \neq Q_x$ tam dif değil

$$\ln\lambda = \int \frac{P_y - Q_x}{Q}\,dx = \int \frac{12x+12y-6x-4y}{3x^2+4xy}\,dy$$

$$= \int \frac{2(3x+4y)}{x(3x+4y)}\,dx = \int \frac{2}{x}\,dx$$

$$\lambda = x^2$$

$$(12x^3y + 6x^2y^2)dx + (3x^4 + 4x^3y)dy = 0$$

$$\frac{\partial u}{\partial y} = 3x^4 + 4x^3y$$

$$u(x,y) = \int (3x^4 + 4x^3y)\,dy$$

$$u(x,y) = 3x^4y + \frac{4x^3}{2}y^2 + R(x)$$

$$12x^3y + 6x^2y^2 = 12x^3y + 6x^2y^2 + R'(x)$$

$$R'(x) = 0 \qquad R(x) = c$$

$$u(x,y) = 3x^4y + 2x^3y^2 = C$$

Soru 2) $xy' = y - x\cos^2\left(\frac{y}{x}\right)$ diferansiyel denkleminin genel çözümünü bulunuz.

NURAN GÜZEL

$$y = ux \qquad y' = u'x + u$$

$$u'x + u = \frac{ux}{x} - \frac{x}{x}\cos^2 u$$

$$u'x = -\cos^2 u$$

$$\frac{du}{\cos^2 u} + \frac{dx}{x} = 0$$

$$\operatorname{tg} u + \ln x = \ln c$$

$$x = c e^{-\operatorname{tg} u}$$

$$x = c e^{-\operatorname{tg} y/x}$$

x

∓ 2 , $k=2$

nuz.

Soru 3) $y' - y = \cos x . e^{-x} . y^2$ diferansiyel denkleminin genel çözümünü bulunuz.

$u = \frac{1}{y}$ $\quad u' = \frac{-y'}{y^2}$ Bernoulli

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$-u' - u = \cos x . e^{-x}$

$u' + u = -\cos x . e^{-x}$ Lineer dif

$u' + u = 0$

$\frac{du}{dx} = -u \quad \frac{du}{u} = -dx$

$\ln u = -x + \ln c$

$u = ce^{-x}$

$c'e^{-x} - e^{-x} . c + ce^{-x} = -\cos x . e^{-x}$

$c' . e^{-x} = -\cos x . e^{-x}$

$c' = -\cos x \quad \frac{dc}{dx} = -\cos x$

$c = -\sin x + k_1$

$u = (-\sin x + k_1)e^{-x}$

$\frac{1}{y} = (-\sin x + k_1)e^{-x}$

$y = \frac{e^x}{-\sin x + k_1}$

1) $(y - y^2 lnx)dx = xlnxdy$ diferansiyel denkleminin genel çözümünü bulunuz.

2 $x\ln x . y' - y = -y^2 \ln x$ Bernouilli: D.D

4 $\left.\begin{array}{l} y^{-1} = z \\ -y^{-2} y' = z' \end{array}\right\}$ $x \ln x y' . y^{-2} - y^{-1} = -\ln x$

$-x\ln x . z' - z = -\ln x$

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$\boxed{z' + \frac{z}{x\ln x} = \frac{1}{x}}$ Lineer D.D 5

$\lambda(x) = e^{\int \frac{dx}{x\ln x}} = e^{\ln(\ln x)} = \ln x$

$\underbrace{z' \ln x + \frac{z}{x}} = \frac{\ln x}{x}$

$\frac{d}{dx}\left(z . \ln x\right) = \frac{\ln x}{x} \rightarrow z . \ln x = \int \frac{\ln x}{x} dx$

$= \frac{\ln^2 x}{2} + c$

12

$z = \frac{\ln x}{2} + \frac{c}{\ln x}$

$y = \frac{1}{z} = \frac{1}{\frac{\ln x}{2} + \frac{c}{\ln x}}$ 2

Başar

2) $y = xy' + \sqrt{(y')^2 + 1}$ diferansiyel denklemini çözünüz.

2

$y' = p \rightarrow y = xp + \sqrt{p^2+1}$ Clairaut D.D

5

$$\underbrace{y'}_{p} = p + xp' + \frac{2pp'}{2\sqrt{p^2+1}}$$

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$$xp' + \frac{pp'}{\sqrt{p^2+1}} = 0 \rightarrow p'\left[x + \frac{p}{\sqrt{p^2+1}}\right] = 0$$ 3

$y = xc_1 + \sqrt{c_1^2+1}$

1) $p' = 0 \rightarrow p = c_1 \rightarrow \boxed{y = c_1 x + c_2}$ Genel çözüm 5

2) $x = \dfrac{-p}{\sqrt{p^2+1}}$

$y = \dfrac{-p^2}{\sqrt{p^2+1}} + \sqrt{p^2+1} = \dfrac{1}{\sqrt{p^2+1}}$

$\boxed{x^2 + y^2 = 1}$ Tekil çözüm 10

Cevap Anahtarı

Soru 1 $x^2y' + x^2y^2 - 3xy + 3 = 0$ denkleminin bir özel çözümü $y_1 = \frac{1}{x}$ olduğuna göre genel çözümünü bulunuz.

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Çözüm

(4) $y = \frac{1}{x} + \frac{1}{u}$

$y' = -\frac{1}{x^2} - \frac{u'}{u^2}$

$$x^2\left(-\frac{1}{x^2} - \frac{u'}{u^2}\right) + x^2\left(\frac{1}{x^2} + \frac{2}{ux} + \frac{1}{u^2}\right) - 3x\left(\frac{1}{x} + \frac{1}{u}\right) + 3$$

(4)

$$-1 - \frac{u'x^2}{u^2} + 1 + \frac{2x}{u} + \frac{x^2}{u^2} - 3 - \frac{3x}{u} + 3 = 0$$

(5) $\boxed{u' + \frac{u}{x} = 1}$ Lineer dif. denk.

$$\lambda(x) = e^{\int \frac{dx}{x}} = e^{\ln x} = x \quad (3)$$

$$u'x + u = x \Rightarrow \frac{d}{dx}(u.x) = x$$

$$ux = \frac{d}{dx}(x) = \frac{x^2}{2} + c$$

(3)

$$\boxed{u = \frac{x}{2} + \frac{c}{x}} \quad (4)$$

$$\boxed{y = \frac{1}{x} + \frac{2x}{x^2 + 2c}} \quad (2)$$

3) $2yy'(1+x)+2x-x^2+y^2=0$ d.d. çözünüz

Cevap: $2y(1+x)$ ile bölelim

$$y'+\frac{y^2}{2y(1+x)}+\frac{2x-x^2}{2y(1+x)}=0 \Rightarrow \underbrace{y'+\frac{1}{2(1+x)}y=\frac{x^2-2x}{2(1+x)}y^{-1}}_{\text{Bernoulli d.d.}}$$

$z=y^2$ dön. yapalım

$z'=2yy'$

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$$2y/\;y'+\frac{1}{2(1+x)}y=\frac{x^2-2x}{2(1+x)}y^{-1} \Rightarrow 2yy'+\frac{1}{1+x}\cdot y^2=\frac{x^2-2x}{1+x}$$

$$\Rightarrow z'+\frac{1}{1+x}\cdot z=\frac{x(x-2)}{1+x}\Big\}\text{ l.d.d.} \Rightarrow \mu(x)=e^{\int\frac{1}{1+x}dx}$$

$$\Rightarrow \boxed{\mu(x)=1+x} \Rightarrow (1+x).z'+z=x^2-2x$$

$$\Rightarrow \left[(1+x).z\right]'=x^2-2x \Rightarrow (1+x).z=\int(x^2-2x)dx$$

$$\Rightarrow (1+x).z=\frac{x^3}{3}-x^2+\frac{c}{3} \Rightarrow z=\frac{x^3-3x^2+c}{3.(1+x)}$$

$$\Rightarrow \boxed{y^2=\frac{x^3-3x^2+c}{3(1+x)}} \text{ genel çözüm}$$

Dif. Denk. II. Yıliçi Sınavı
23 Kasım 2013

1) $\dfrac{dy}{dx} = \dfrac{(x+y)^2}{2x^2}$ d.d. çözünüz

Ç: $\dfrac{y}{x} = u \Rightarrow y = ux \Rightarrow y' = u'.x + u$

$$u'.x + u = \frac{(x+ux)^2}{2x^2} \Rightarrow u'x + u = \frac{\cancel{x^2}(1+u)^2}{2\cancel{x^2}}$$

$$\Rightarrow u'x + u = \frac{(1+u)^2}{2} \Rightarrow \frac{du}{dx}.x = \frac{(1+u)^2}{2} - u$$

$$\Rightarrow \frac{du}{dx}.x = \frac{1+u^2}{2} \Rightarrow \frac{dx}{x} = \frac{2}{1+u^2}\,du$$

$$\Rightarrow \ln|x| = 2.\,tg^{-1}u + \ln c \Rightarrow \boxed{\ln\left|\frac{x}{c}\right| = 2\,tg^{-1}\frac{y}{x}}$$ genel çözüm

$$\frac{dy}{dx} = \sin^2(3x-3y+1) \quad \text{d.d. çözünüz}$$

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: $3x-3y+1=u \Rightarrow \frac{du}{dx} = 3 - 3\cdot\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 1 + \frac{1}{3}\cdot\frac{du}{dx}$

Dif. Denk. de yerine yazalım

$$1 + \frac{1}{3}\cdot\frac{du}{dx} = \sin^2 u \Rightarrow \frac{1}{3}\cdot\frac{du}{dx} = \sin^2 u - 1$$

$$\Rightarrow \frac{du}{1-\sin^2 u} = -3dx \Rightarrow \frac{du}{\cos^2 u} = -3dx \Rightarrow \int \sec^2 u\, du = -3\int dx$$

$$\tan u = -3x + c \Rightarrow \left\{ \boxed{\tan(3x-3y+1) = -3x+c} \right\} \text{g.ç.}$$

7)

4) $y = x.\left[\left(\frac{dy}{dx}\right)^2 + 2\left(\frac{dy}{dx}\right)\right] - \left[\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} - 1\right]$ diferansiyel denkleminin çözümünü bulunuz.

$y = x(p^2+2p) - (p^2+2p-1)$ 4

$y' = p^2+2p + x(2p+2)\frac{dp}{dx} - (2p+2)\frac{dp}{dx}$ 4

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$p-2p-p^2 = \frac{dp}{dx}\left[x(2p+2) - (2p+2)\right]$

$-p(p+1)\frac{dx}{dp} = 2(p+1)(x-1)$ 6

$\frac{dx}{dp} = \frac{2(p+1)}{-p(p+1)}(x-1)$

$\frac{dx}{dp} + \frac{2}{p}x = \frac{2}{p}$ lineer

$\lambda(p) = e^{\int \frac{2}{p}dp} = e^{2\ln p} = p^2$

$x = \frac{1}{p^2}\left[\int \frac{2}{p}p^2\,dp + c\right]$

$x = \frac{1}{p^2}\left[p^2 + c\right]$

$x = 1 + \frac{c}{p^2}$ 7

$\begin{cases} y = x(p^2+2p) - (p^2+2p-1) \\ x = 1 + \frac{c}{p^2} \end{cases}$

$y = \left(1+\frac{c}{p^2}\right)(p^2+2p) - (p^2+2p-1)$

$y = p^2+2p + c + \frac{2c}{p} - p^2 - 2p + 1$

$y = c + 1 + \frac{2c}{p}$

Genel çözümün parametrik denklemleri

$\begin{cases} x = 1 + \frac{c}{p^2} \\ y = c + 1 + \frac{2c}{p} \end{cases}$ 4

3) $x \sin y \, dx + (x^2+1)\cos y \, dy = 0$, $(1, \frac{\pi}{2})$ b.d.prob.

2   2, ..., 10   5

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$$\frac{x}{x^2+1}dx + \frac{\cos y}{\sin y}dy = 0 \quad \cancel{5}$$

$$\frac{1}{2}\ln(x^2+1) + \ln|\sin y| = \ln c \quad \cancel{10} \; \cancel{(5+0)}$$

$$\ln\sqrt{x^2+1} + \ln|\sin y| = \ln c$$

$$\ln\left(\sqrt{x^2+1}\,.\,|\sin y|\right) = \ln c$$

$$\sqrt{x^2+1}\,.\,|\sin y| = c \quad \cancel{5}$$

$$y(1) = \frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = 1$$

$$\Rightarrow \sqrt{1+1}\,.\,1 = c \Rightarrow c = \sqrt{2} \quad \cancel{5}$$

Aranan çözüm

$$\boxed{(x^2+1)\sin^2 y = 2}$$

$y' - 3y = -2x e^{3x}$

$\ln \lambda = \int 3\,dx \quad \ln \lambda = -3x + C$

$\lambda = e^{-3x+C}$

$e^{-3x+c} y' - 3e^{-3x+c} y = -2x e^{c}$

$e^{-3x+c} y = \int -2x e^{c}\,dx$

$e^{-3x+c} y = -x^2 e^{c}$

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$e^{-3x} y = -x^2 + K$

$-e^{-3x} y - x^2 = K$

$-e^{-3x} y = K + x^2$

$y = \dfrac{x^2 + K}{-e^{-3x}}$

$y = \dfrac{-x^2 - K}{e^{-3x}}$

$z'' = 0$

3) $y'' + y' + xy = 0$ denkleminin $x = 0$ noktası civarında seri çözümünü elde ediniz.

A $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = -2e^{3x} - 6xe^{3x}$

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$r^2 - 3r = 0 \quad r(r-3) = 0 \quad r = 0 \; r = 3 \quad \boxed{y = c_1 + c_2 e^{3x}}$

$\boxed{y_{ö_1} = kxe^{3x}} \quad y'_{ö_1} = ke^{3x} + 3kxe^{3x} \quad y''_{ö_1} = 3ke^{3x} + 3ke^{3x} + 9kxe^{3x}$

$\cancel{3ke^{3x}} + 3k\cancel{e^{3x}} + 3ke^{3x} + \cancel{9kxe^{3x}} - \cancel{3kxe^{3x}} = -2e^{3x} \quad 3k = -2 \quad k = -\frac{2}{3} \quad y_{ö_1} = -\frac{2}{3}$

$\boxed{y_{ö_2} = z(x)e^{3x}} \quad y'_{ö_2} = z'e^{3x} + 3ze^{3x} \quad y''_{ö_2} = z''e^{3x} + 6z'e^{3x} + 9ze^{3x}$

$z'' + 6z' + \cancel{9z} - 3z' - \cancel{9z} = -6x \quad z'' + 3z' = -6x \quad r^2 + 3r = 0 \quad r(r+3) = 0 \quad r =$

$z_ö = (ax+b)x = ax^2 + bx \quad z' = 2ax + b \quad z'' = 2a$

$2a + 6ax + 3b = -6x \quad a = -1 \quad 2a + 3b = 0 \quad -2 + 3b = 0 \quad 3b = 2 \quad b = \frac{2}{3} \quad z_ö = -x$

$y_{ö_2} = \left(-x^2 + \frac{2}{3}x\right)e^{3x}$

$y = c_1 + c_2 e^{3x} - \cancel{\frac{2}{3}xe^{3x}} + \left(-x^2 + \cancel{\frac{2}{3}x}\right)e^{3x}$

$y = c_1 + c_2 e^{3x} - x^2 e^{3x}$