LİNEER DİF. DENKLEMLER

$a_0(x) \cdot y' + a_1(x) y = B(x)$

$a_0(x) \neq 0$

$$y' + \underbrace{\frac{a_1(x)}{a_0(x)}}_{p(x)} \cdot y = \underbrace{\frac{B(x)}{a_0(x)}}_{q(x)} \Rightarrow \boxed{y' + p(x) \cdot y = q(x)}$$

SABİTİN DEĞİŞİMİ YÖNTEMİ

$q(x) = 0$ ise 2. tarafsız dif. denklem denir.

Nasıl çözülür?

$y' + p(x) \cdot y = 0$

$\frac{dy}{dx} = -p(x) y$

$\frac{dy}{y} = -\int p(x) dx$

$\ln y - \ln C = -\int p(x) dx$

$\ln \frac{y}{c} = -\int p(x) dx$

$\frac{y}{c} = e^{-\int p(x) dx}$

$$\Rightarrow \boxed{y = C \cdot e^{-\int p(x) dx}}$$

$y' = C' \cdot e^{-\int p(x) dx} - p(x) \cdot e^{-\int p(x) dx} \cdot C$

Buradan $y' + p(x) y = q(x)$ ifadesine ulaşılır;

$$\underbrace{C' \cdot e^{-\int p(x) dx} - \cancel{p(x) \cdot e^{-\int p(x) dx} \cdot C}}_{y'} + \cancel{p(x)} \underbrace{\cancel{C \cdot e^{-\int p(x) dx}}}_{y} = q(x)$$

$\frac{dc}{dx} \cdot e^{-\int p(x) dx} = q(x)$

$\int dc = \int q(x) \cdot e^{\int p(x) dx} \cdot dx$

$C(x) = \int q(x) \cdot e^{\int p(x) dx} \cdot dx + K$

$y = C \cdot e^{-\int p(x) dx}$ idi. Buna göre;

$$\boxed{y = \left[\int g(x) \cdot e^{\int p(x) dx} \cdot dx + K \right] \cdot e^{-\int p(x) dx}}$$

$\longrightarrow$ Genel Çözüm.

ÖRNEK

$y' = y.\cot x + \sin x$ dif. denkleminin genel çözümünü bulunuz.

$y' - y\cot x = \sin x$

ÇÖZÜM

$y' - y\cot x = 0$

$y' - \cot x . y = \sin x$ *

$\frac{dy}{dx} = y\cot x$

$y' - \cot x . y = 0 \quad (y' + p(x)y = 0)$

$\frac{dy}{y} = \cot x\, dx$

$\frac{dy}{dx} = -p(x)y \Rightarrow \frac{dy}{dx} = \cot x . y$

$\int \frac{dy}{y} = \int \cot x\, dx$

$\int \frac{dy}{y} = \int \cot x\, dx = \text{olur}$

$\ln y - \ln c = \ln \sin x$

$\ln\left(\frac{y}{c}\right) = \ln \sin x$

$\ln y + \ln C = \ln \sin x$

$\frac{y}{c} = \sin x$

$\ln \frac{y}{c} = \ln \sin x$

$y = c.\sin x$ *

$\frac{y}{c} = \sin x \Rightarrow \boxed{y = C.\sin x}$ *

$y' = c'\sin x + c.\cos x$

$\Rightarrow y' = c'.\sin x + \cos x . C$

$c'\sin x + c\cos x - \frac{\cos x}{\sin x} c\sin x -$

* $c'.\sin x + \cancel{\cos x . c} - \frac{\cos x}{\sin x}(c.\sin x) = \sin x$

$c'\sin x = \sin x$

$\frac{dc}{dx} = 1$

$\boxed{c(x) = x + K}$

$y = C.\sin x$ ifadesinden

$\boxed{y = (x+K).\sin x}$ 'e ulaşılır. Genel Çözüm.

ÖRNEK

$x \cdot y' + y = e^{2x} - \sin x$ dif. denkleminin Genel Çözümünü bulunuz.

ÇÖZÜM

$* \quad y' + \frac{1}{x} \cdot y = \frac{e^{2x} - \sin x}{x}$

$y' + p(x) y = 0 \Rightarrow p(x) = \frac{1}{x}$

$\frac{dy}{dx} = -\frac{1}{x} y$

$\int \frac{dy}{y} = -\int \frac{1}{x} \cdot dx$

$\ln y - \ln c = -\ln x$

$\boxed{y = \frac{c}{x}}$

$y' = \frac{c' \cdot x - c}{x^2}$

$* \quad \frac{c' \cdot x - c}{x^2} + \frac{1}{x} \cdot \frac{c}{x} = \frac{e^{2x} - \sin x}{x}$

$c' = e^{2x} - \sin x$

$\frac{dc}{dx} = e^{2x} - \sin x$

$\int dc = \int e^{2x} - \sin x \, dx$

$\boxed{C(x) = \frac{e^{2x}}{2} + \cos x + k}$

$y = \frac{c}{x}$ ifadesinden;

$\boxed{y = \frac{1}{x} \cdot \left(\frac{e^{2x}}{2} + \cos x + k \right)}$

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$y' + \frac{y}{x} = \frac{e^{2x} - \sin x}{x}$

$y' + \frac{y}{x} = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

$dy \, x = -y \, dx$

$\frac{dy}{y} = -\int \frac{dx}{x}$

$\ln y - \ln c = \ln x$

$\frac{y}{c} = \frac{1}{x}$

$y \cancel{x} = c$

$y = \frac{c}{x}$

$y' = \frac{c'x - c}{x^2}$

$\frac{c'x - \cancel{c}}{x^2} + \frac{\cancel{c}}{x^2} = \frac{e^{2x} - \sin x}{x}$

$\frac{c' \cancel{x}}{\cancel{x}} = e^{2x} - \sin x$

$c' = e^{2x} - \sin x$

$c = \frac{e^{2x}}{2} + \cos x + k$

İNTEGRASYON ÇARPANI YÖNTEMİ (LINEER DİF. DENKLEM)

$y' + p(x)y = q(x)$

$\frac{dy}{dx} + p(x)y - q(x) = 0$

$\underbrace{[p(x)y - q(x)]}_{M(x,y)} dx + \underbrace{dy}_{N(x,y)=1} = 0$

$$\boxed{\ln \lambda = \int p(x)\,dx}$$

$\frac{\partial M}{\partial y} = p(x) \neq \frac{\partial N}{\partial x} = 0$

$\ln \lambda = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} . dx = \int \frac{p(x) - 0}{1} . dx = \int p(x) dx$

$\Rightarrow \lambda = e^{\int p(x)dx}$

$\Rightarrow e^{\int p(x)dx} . y' + e^{\int p(x)dx} . p(x).y = q(x). e^{\int p(x)dx}$

$\Rightarrow \int \frac{d}{dx}\left(y.e^{\int p(x)dx}\right) = \int q(x).e^{\int p(x)dx} . dx$ (değişkenlerine ayrılabilen dif. denklem)

ÖRNEK $y' = y.\cot g(x) + \sin x$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$* \; y' - \cot x \, y = \sin x \quad (y' + p(x)y = q(x))$

$\ln \lambda = -\int \cot x \, dx$

$\ln \lambda = -\ln(\sin x)$

$\boxed{\lambda = \frac{1}{\sin x}}$

$* \; \frac{1}{\sin x}.y' - \frac{\cos x}{\sin x}.\frac{1}{\sin x}.y = \cancel{\sin x}.\frac{1}{\cancel{\sin x}}$

$\int \frac{d}{dx}\left(\frac{1}{\sin x}.y\right) = \int dx$

$\frac{y}{\sin x} = x + C \Rightarrow \boxed{y = x.\sin x + C.\sin x}$ Genel Çözüm.

ÖRNEK

$x \cdot y' + y = e^{2x} - \sin x$ dif. denkleminin genel çözümünü bulunuz.

Çözüm

$* \quad y' + \frac{1}{x} y = \frac{e^{2x} - \sin x}{x} \qquad (y' + p(x) y = q(x))$

$p(x) = \frac{1}{x}$

$\ln \lambda = \int \frac{1}{x} dx$

$\ln \lambda = \ln x$

$\boxed{\lambda = x}$

$* \quad x \cdot y' + y = e^{2x} - \sin x$

$\int \frac{d}{dx} (xy) = \int e^{2x} - \sin x \, dx$

$xy = \frac{e^{2x}}{2} + \cos x + C.$

$\boxed{y = \frac{1}{x} \cdot \left(\frac{e^{2x}}{2} + \cos x + C \right)}$

ÖRNEK

$y' - \frac{1}{1+x} y = (1+x)^2$ dif. denkleminin genel çözümünü bulunuz.

Çözüm

$p(x) = -\frac{1}{1+x}$

$\ln \lambda = -\int \frac{1}{1+x} dx$

$\ln \lambda = -\ln(1+x)$

$\boxed{\lambda = \frac{1}{1+x}}$

$\left(\frac{1}{1+x}\right) \cdot y' - \left(\frac{1}{1+x}\right) \cdot \left(\frac{1}{1+x}\right) \cdot y = \frac{1}{(1+x)} \cdot \cancel{(1+x)}^2$

$\int \frac{d}{dx} \left(\frac{1}{1+x} \cdot y \right) = \int (1+x) \, dx$

$\boxed{\frac{1}{1+x} \cdot y = x + \frac{x^2}{2} + C}$ Genel Çözüm.

ÖRNEK

$$\frac{dy}{dx} + \frac{2x+1}{x} \cdot y = e^{-2x}$$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$$p(x) = \frac{2x+1}{x}$$

$$\ln \lambda = \int \frac{2x+1}{x} dx$$

$$\int 2 , \frac{1}{x}$$

$$2x + \ln x = \ln \lambda$$

$$\ln e^{2x}$$

$$e^{2x} \cdot x = \lambda$$

$$\ln \lambda = 2x + \ln x$$

$$\ln \frac{\lambda}{x} = 2x$$

$$\frac{\lambda}{x} = e^{2x} \Rightarrow \boxed{\lambda = x \cdot e^{2x}}$$

$$(x \cdot e^{2x}) \cdot y' + (x \cdot e^{2x}) \cdot \frac{2x+1}{x} \cdot y = x \cdot e^{2x} \cdot e^{-2x}$$

$$\int \frac{d}{dx}\left(x \cdot e^{2x} \cdot y\right) = \int x \, dx$$

$$\boxed{x \cdot e^{2x} \cdot y = \frac{x^2}{2} + C.}$$ Genel Çözüm.

$$y' + \frac{2x+1}{x} y = e^{-2x}$$

$$\lambda = e^{\int \frac{2x+1}{x} dx} \quad 2 + 1/x$$

$$\lambda = e^{2x + \ln x} = e^{\ln x}$$

$$\lambda = e^{2x} \cdot x$$

$$y' x e^{2x} + \frac{2x+1}{\cancel{x}} e^{2x} \cdot \cancel{x} y = x$$

$$\frac{d}{dx}(x e^{2x} y) = \int x \, d$$

LİNEER DİF. DENKLEME DÖNÜŞTÜRME

BERNOULLI DİF. DENKLEMİ

$y' + p(x)y = q(x) \cdot y^n \rightarrow$ Bu üslü ifadeden arıyoruz. $\begin{pmatrix} n \neq 0 \\ n \neq 1 \end{pmatrix}$

$$\frac{y'}{y^n} + p(x)\frac{1}{y^{n-1}} = q(x)$$

$\boxed{u = \frac{1}{y^{n-1}}}$ değişken değişimi yapılır. Lineer dif. denklem oluşturulur.

ÖRNEK

$y' + y = x \cdot y^3$ dif. denkleminin genel çözümünü bulunuz:

ÇÖZÜM

$$\frac{y'}{y^3} + \frac{1}{y^2} = x$$

$u = y^{-2}$ değişken dönüşümü

$u' = -2 \cdot y^{-3} \cdot y' = -2 \cdot \frac{y'}{y^3}$

$\Rightarrow -\frac{u'}{2} + u = x$

$u' - 2u = -2x \quad (u' + p(x)u = q(x))$

$\ln\lambda = \int -2 \cdot dx$

$\ln\lambda = -2x$

$\boxed{\lambda = e^{-2x}}$

$e^{-2x} \cdot u' - 2e^{-2x} \cdot u = -e^{-2x} \cdot 2x$

$\int \frac{d}{dx}\left(e^{-2x} \cdot u\right) = \int -e^{-2x} \cdot 2x \, dx$

$e^{-2x} \cdot u = x \cdot e^{-2x} + \frac{e^{-2x}}{2} + C$

$\boxed{\frac{1}{y^2} = x + \frac{1}{2} + C_1 e^{2x}}$ Genel Çözüm.

$y'y^{-3} + y^{-2} = x$

$y^{-2} = z$

$-2y^{-3} \cdot y' = z'$

$-\frac{z'}{2} + z = x$

$\frac{z'}{2} - z = -x$

$z' - 2z = -2x$

$\frac{dz}{dx} \quad 2z$

$\ln z \quad \ln z = 2x$

$z/c = e^{2x}$

$z = ce^{2x}$

$V_A = \frac{1}{2}$

ÖRNEK

$x^2 y' = y^2 + 2xy$ dif. denkleminin $y(0)=1$ için özel çözümünü bulunuz.

ÇÖZÜM

$y' + p(x)y = q(x) \cdot y^n \rightarrow$ Bernoulli

$$y' - \frac{2}{x}y = \frac{1}{x^2}y^2$$

$$\frac{y'}{y^2} - \frac{2}{x}\cdot\frac{1}{y} = \frac{1}{x^2}$$

$$u = \frac{1}{y} \Rightarrow u' = -\frac{1}{y^2}\cdot y'$$

$$-u' - \frac{2}{x}u = \frac{1}{x^2} \Longrightarrow u' + \frac{2}{x}u = -\frac{1}{x^2} \quad *$$

$$p(x) = \frac{2}{x}$$

$$\ln\lambda = \int \frac{2}{x}dx$$

$$\ln\lambda = 2\ln x$$

$$\boxed{\lambda = x^2}$$

$$* \; x^2 u' + \frac{2}{\cancel{x}} u \cdot \cancel{x^2} = -\frac{1}{\cancel{x^2}}\cancel{x^2}$$

$$\int \frac{d}{dx}(x^2 u) = \int -1 \, dx$$

$$x^2 u = -x + c.$$

$$\boxed{\frac{1}{y} = -\frac{1}{x} + \frac{c}{x^2}} \quad \text{Genel Çözüm.}$$

Başlangıç Değer Problemi

$x=0$, $y=1$) $c=0$

$$\boxed{x^2 \cdot \frac{1}{y} = -x} \quad \text{Özel Çözüm}$$

$x^2 y' = y^2 + 2xy$

$x^2 y' - y^2 = 2xy$

$y' - \frac{y^2}{x^2} = \frac{2y}{x}$

$\frac{y'}{y} - \frac{y}{x^2} = \frac{2}{x}$

$y'y^{-1} - y \cdot x^{-2} = \frac{2}{x}$

$z = y$

$z' = 1$

$$z' = \frac{c' x^4 - \cancel{2x^3 c}}{2x^{\cancel{8}}}$$

$z =$

$\frac{2\cancel{4}}{\cancel{x^5}}$

$$y' = \frac{y^2}{x^2} + \frac{2y}{x}$$

$$y' - \frac{2y}{x} = \frac{y^2}{x^2}$$

$$y'y^{-2} - 2xy^{-1} = \frac{1}{x^2}$$

$$\frac{c'x^4}{2x^{\cancel{4}}}$$

$$y^{-1} = \frac{c'}{2x^4} = \frac{1}{x^2} \cdot y$$

$$c' = 2x^2$$

$$\frac{1}{y} = z = \frac{2}{3x} + \frac{k}{x^4} \qquad \frac{2\frac{x^3}{3x^4} + k}{x^4}$$

ÖRNEK.

$y' - \frac{y}{3x} = y^4 . \ln x$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$$\frac{y'}{y^4} - \frac{1}{y^3} . \frac{1}{3x} = \ln x$$

$$u = \frac{1}{y^3} \Rightarrow u' = -3 . y^{-4} . y' = -3 . \frac{y'}{y^4}$$

$$-\frac{1}{3} u' - \frac{1}{3x} . u = \ln x$$

$$* \; u' + \frac{1}{x} u = -3 . \ln x$$

$$p(x) = \frac{1}{x}$$

$$\ln \lambda = \int \frac{1}{x} dx$$

$$\ln \lambda = \ln x$$

$$\boxed{\lambda = x}$$

$$* \; x . u' + u = x . \ln x . (-3)$$

$$\int \frac{d}{dx}(x.u) = \int -3x . \ln x \, dx$$

$$x . u = -3 \left(\ln x . \frac{x^2}{2} - x \right) + C .$$

$$\boxed{\frac{1}{y^3} = -\frac{3}{2} . \ln x . x + 3 + \frac{C}{x}}$$

ÖRNEK

$x^4 \cdot y' + x^3 \cdot y = y^{-3} \cdot \sin x$ dif. denkleminin ; $y(0)=1$ özel çözümünü bulunuz.

Çözüm

$$y' + \frac{1}{x} y = x^{-4} \cdot y^{-3} \cdot \sin x \qquad (y' + p(x) y = q(x) \cdot y^n)$$

$$\frac{y'}{y^{-3}} + \frac{1}{x} \cdot \frac{y}{y^{-3}} = x^{-4} \cdot \sin x$$

$$u = \frac{y}{y^{-3}} = y^4.$$

$$u' = 4 \cdot y^3 \cdot y'$$

$$\frac{u'}{4} + \frac{1}{x} \cdot u = x^{-4} \sin x$$

$$* \; u' + \frac{4}{x} \cdot u = 4 \cdot x^{-4} \cdot \sin x$$

$$p(x) = \frac{4}{x}$$

$$\ln \lambda = \int \frac{4}{x} dx$$

$$\ln \lambda = 4 \cdot \ln x$$

$$\boxed{\lambda = x^4}$$

$$* \; (x^4)\, u' + (x^4) \frac{4}{\cancel{x}} \cdot u = 4 \cdot \cancel{x^{-4}} \cdot \sin x \cdot \cancel{(x^4)}$$

$$\int \frac{d}{dx}\left(x^4 \cdot u\right) = \int 4 \cdot \sin x \, dx$$

$$4 \cdot x^4 = -4 \cdot \cos x + C.$$

$$\boxed{y^4 = \frac{-4 \cdot \cos x + C}{x^4}}$$ Genel çözüm.

$x=0$; $y=1 \Rightarrow C=4$.

$$\boxed{y^4 = \frac{4(1-\cos x)}{x^4}}$$ Özel çözüm.

RICCATI DIF. DENKLEMI

$\boxed{y'+p(x)y+q(x)y^2+r(x)=0}$ ve $\boxed{y_1}$ özel çözümü verilsin.

Bu dif denklem iki yolla çözülebilir;

1) $y=y_1+u \rightarrow$ Bernoulli Dif. Denkleme döner

2) $\boxed{y=y_1+\frac{1}{u}} \rightarrow$ Lineer Dif. Denkleme döner.

ÖRNEK

$(1+x^3)y'-x^2y+y^2+2x=0$ dif. denkleminin bir özel çözümü $y_1=-x^2$ olduğuna göre genel çözümü bulunuz.

Çözüm

$$y=-x^2+\frac{1}{u}$$

$$y'=-2x-\frac{u'}{u^2}$$

$$(1+x^3).\left(-2x-\frac{u'}{u^2}\right)-x^2\left(-x^2+\frac{1}{u}\right)+\left(-x^2+\frac{1}{u}\right)^2+2x=0$$

$$\left(\cancel{-2x}-\frac{u'}{u^2}\cancel{-2x^4}-x^3.\frac{u'}{u^2}\cancel{+x^4}-\frac{x^2}{u}\cancel{+x^4}-2\frac{x^2}{u}+\frac{1}{u^2}\cancel{+2x}\right)=0$$

$$-\frac{u'}{u^2}\left(1+x^3\right)-\frac{3x^2}{u}+\frac{1}{u^2}=0$$

$$u'(1+x^3)-3x^2u-1=0$$

$$* \quad u'+\frac{3x^2}{1+x^3}u=\frac{1}{1+x^3} \qquad \left(y'+p(x)y=q(x)\right)$$

$$\ln\lambda=\int\frac{3x^2}{1+x^3}dx$$

$$\ln\lambda=\ln(1+x^3)$$

$$\boxed{\lambda=1+x^3}$$

$$(1+x^3).u'+(1+x^3).\frac{3x^2}{1+x^3}u=1.$$

$$\int\frac{d}{dx}\left((1+x^3).u\right)=\int dx$$

$$(1+x^3).u=x+c$$

$$u=\frac{x+c}{1+x^3}\Rightarrow\boxed{y=-x^2+\frac{1+x^3}{x+c}} \text{ Genel Çözüm.}$$

ÖRNEK

$y' - \frac{y}{x} + 9x^2 = y^2$ dif. denkleminin bir özel çözümü $y_1 = 3x$ olduğuna göre G.çözümü bulunuz.

Çözüm

$y = 3x + \frac{1}{u}$

$y' = 3 - \frac{u'}{u^2}$ dönüşümü

$$\left(3 - \frac{u'}{u^2}\right) - \frac{1}{x}\cdot\left(3x + \frac{1}{u}\right) - \left(3x + \frac{1}{u}\right)^2 = -9x^2$$

$$\cancel{3} - \frac{u'}{u^2} - \cancel{3} - \frac{1}{x\cdot u} - \cancel{9x^2} - \frac{6x}{u} - \frac{1}{u^2} = -\cancel{9x^2}$$

$* \; u' + \frac{6x^2+1}{x} u = -1 \quad (y' + p(x)y = q(x))$ (Lineer dif. denklem..

$p(x) = \frac{6x^2+1}{x}$

$\ln\lambda = \int \frac{6x^2+1}{x} dx$

$\ln\lambda = 3x^2 + \ln x$

$\frac{\lambda}{x} = e^{3x^2} \Rightarrow \boxed{\lambda = x\cdot e^{3x^2}}$

$* \; (x\cdot e^{3x^2})\cdot u' + (e^{3x^2}\cdot \cancel{x})\cdot\left(\frac{6x^2+1}{\cancel{x}}\right)\cdot u = -1\cdot(x\cdot e^{3x^2})$

$\int \frac{d}{dx}\left(x\cdot e^{3x^2}\cdot u\right) = -\int (x\cdot e^{3x^2})\, dx$

$x\cdot e^{3x^2}\cdot u =$

$\frac{du}{dx} = -\frac{6x^2+1}{x} u$

$\frac{du}{u} = -\left(\frac{6x^2+1}{x}\right) dx$

$\ln u - \ln c = -\left(3x^2 + \ln x\right)$

$\frac{u}{c} = -3x^2 - \ln x$

$u = (-3x^2 - \ln x)c$

$\left(-6x - \frac{1}{x}\right) c$

$u'x + u = -x - 6ux^2$

$u' + \frac{u}{x} = -1 - 6ux$

$u' + u\left(\frac{1}{x} + 6x\right) = -1$