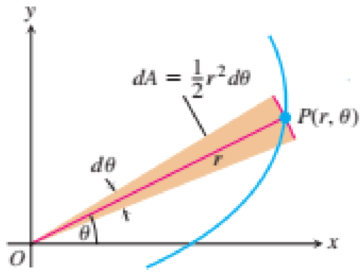
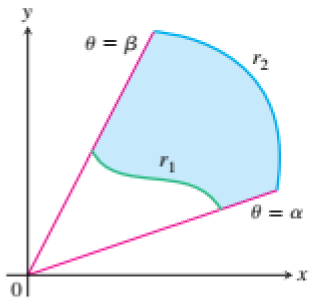


$r = f(\theta);\ \ \alpha \le \theta \le \beta$ için Alan $A = \frac{1}{2}\int\limits_{\theta=\alpha}^{\theta=\beta} \left(f(\theta) \right)^2 d\theta$

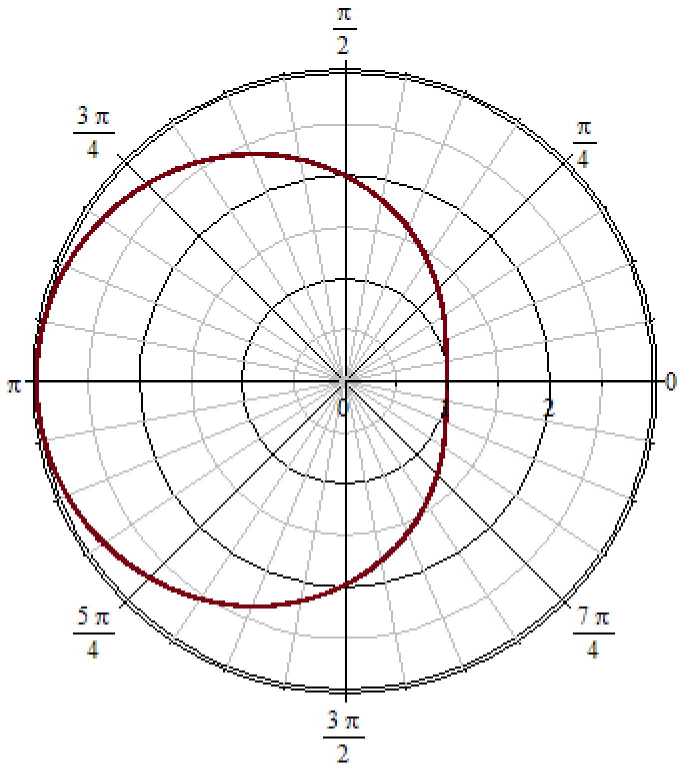


$$A = \int_{\alpha}^{\beta} \frac{1}{2} r_2^2 \, d\theta - \int_{\alpha}^{\beta} \frac{1}{2} r_1^2 \, d\theta = \int_{\alpha}^{\beta} \frac{1}{2} \left(r_2^2 - r_1^2 \right) d\theta$$

Kutupsal eğrinin uzunluğu $r = f(\theta);\ \ \alpha \le \theta \le \beta$ için

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta} \right)^2} \, d\theta .$$

1) $r = 2 - \cos\theta$ kutupsal denklemiyle sınırlanan bölgenin alanını hesaplayınız.



$$A = 2\int_0^{\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{\pi} (2-\cos\theta)^2 \, d\theta = \int_0^{\pi} (4 - 4\cos\theta + \cos^2\theta) \, d\theta = \int_0^{\pi} \left(4 - 4\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \int_0^{\pi} \left(\frac{9}{2} - 4\cos\theta + \frac{\cos 2\theta}{2}\right) d\theta = \left[\frac{9}{2}\theta - 4\sin\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi} = \frac{9}{2}\pi$$

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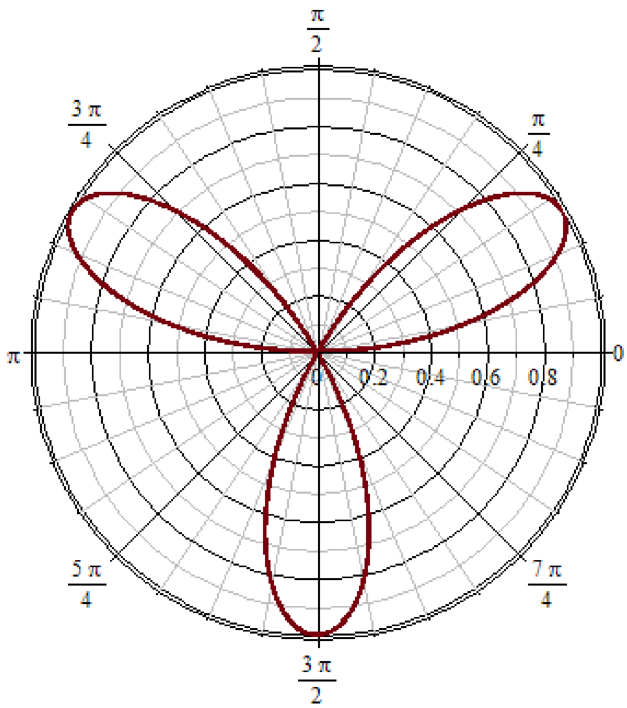
2) $r = \sin 3\theta$ kutupsal denklemiyle verilen gülün bir yaprağının alanını hesaplayınız.



$$A = \int_0^{\pi/3} \frac{1}{2}\left(\sin^2 3\theta\right) d\theta = \int_0^{\pi/3} \left(\frac{1-\cos 6\theta}{2}\right) d\theta = \frac{1}{4}\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\pi/3} = \frac{\pi}{12}$$

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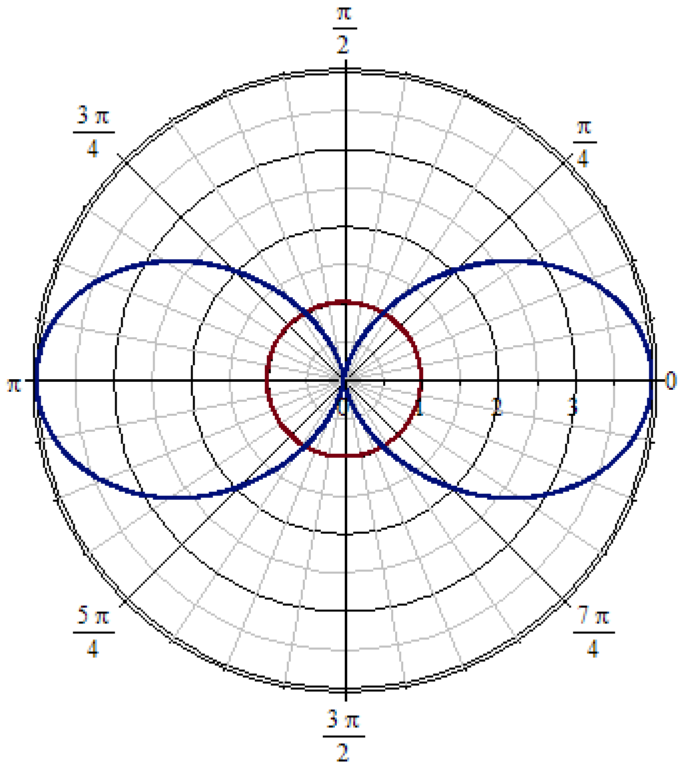
3) $r = 1 + \cos 2\theta$ içinde r=1 dışında kalan kutupsal denklemleriyle verilen alanı hesaplayınız.



$r = 1 + \cos 2\theta$ and $r = 1 \Rightarrow 1 = 1 + \cos 2\theta \Rightarrow 0 = \cos 2\theta \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$; therefore

$$A = 4\int_0^{\pi/4} \frac{1}{2}\left[(1+\cos 2\theta)^2 - 1^2\right] d\theta = 2\int_0^{\pi/4} (1 + 2\cos 2\theta + \cos^2 2\theta - 1)\, d\theta$$

$$= 2\int_0^{\pi/4} \left(2\cos 2\theta + \frac{1}{2} + \frac{\cos 4\theta}{2}\right) d\theta = 2\left[\sin 2\theta + \frac{1}{2}\theta + \frac{\sin 4\theta}{8}\right]_0^{\pi/4} = 2\left(1 + \frac{\pi}{8} + 0\right) = 2 + \frac{\pi}{4}$$

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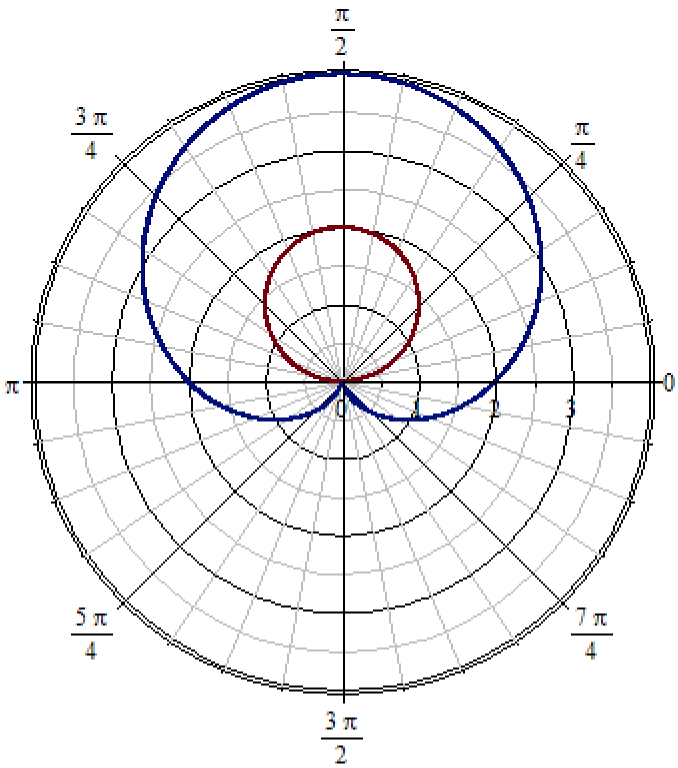
4) $r = 2(1+\sin\theta)$ kardiodinin içinde $r = 2\sin\theta$ çemberinin dışında kalan bölgenin alanını hesaplayınız.



$$A = 2\int_{-\pi/2}^{\pi/2} \frac{1}{2}\,[2(1+\sin\theta)]^2\, d\theta - \pi$$

$$= \int_{-\pi/2}^{\pi/2} 4\,(1 + 2\sin\theta + \sin^2\theta)\, d\theta - \pi = \int_{-\pi/2}^{\pi/2} (6 + 8\sin\theta - 2\cos 2\theta)\, d\theta - \pi = [6\theta - 8\cos\theta - \sin 2\theta]_{-\pi/2}^{\pi/2} - \pi$$

$$= [3\pi - (-3\pi)] - \pi = 5\pi$$

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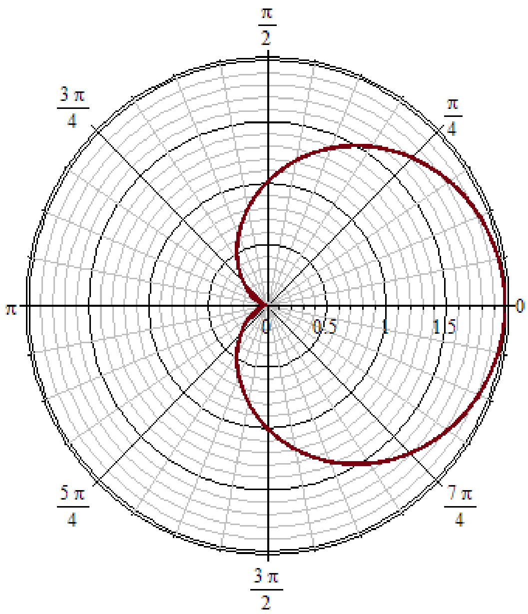
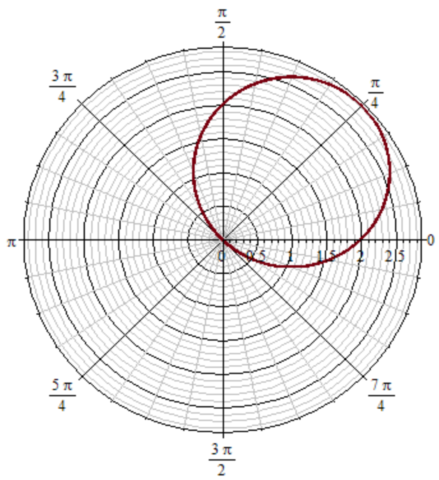
5) $r=-1+\cos\theta$ kutupsal koordinat denklemiyle verilen eğrinin uzunluğunu hesaplayınız.



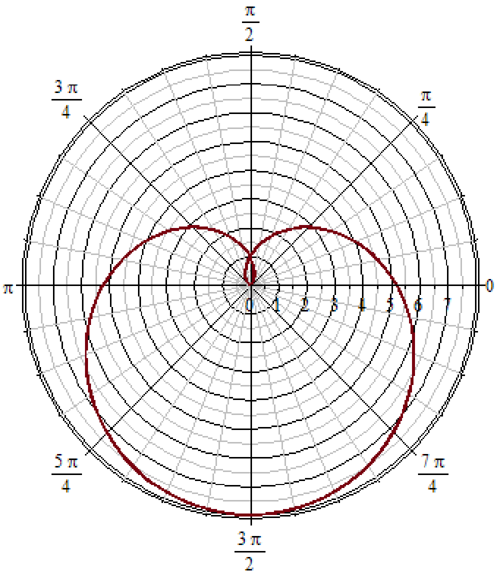
$$r=-1+\cos\theta \Rightarrow \frac{dr}{d\theta}=-\sin\theta; \text{Length}=\int_0^{2\pi}\sqrt{(-1+\cos\theta)^2+(-\sin\theta)^2}\,d\theta=\int_0^{2\pi}\sqrt{2-2\cos\theta}\,d\theta$$
$$=\int_0^{2\pi}\sqrt{\frac{4(1-\cos\theta)}{2}}\,d\theta=\int_0^{2\pi}2\sin\frac{\theta}{2}\,d\theta=\left[-4\cos\frac{\theta}{2}\right]_0^{2\pi}=(-4)(-1)-(-4)(1)=8$$

6) $r=2\cos\theta+2\sin\theta,\ 0\le\theta\le\dfrac{\pi}{2}$ kutupsal koordinat denklemiyle verilen eğrinin uzunluğunu hesaplayınız.



$$r=2\sin\theta+2\cos\theta, 0\le\theta\le\frac{\pi}{2} \Rightarrow \frac{dr}{d\theta}=2\cos\theta-2\sin\theta; r^2+\left(\frac{dr}{d\theta}\right)^2=(2\sin\theta+2\cos\theta)^2+(2\cos\theta-2\sin\theta)^2$$
$$=8\left(\sin^2\theta+\cos^2\theta\right)=8 \Rightarrow L=\int_0^{\pi/2}\sqrt{8}\,d\theta=\left[2\sqrt{2}\,\theta\right]_0^{\pi/2}=2\sqrt{2}\left(\frac{\pi}{2}\right)=\pi\sqrt{2}$$

7) $r = 8\sin^3\dfrac{\theta}{3},\ 0 \le \theta \le \dfrac{\pi}{4}$ kutupsal koordinat denklemiyle verilen eğrinin uzunluğunu hesaplayınız.

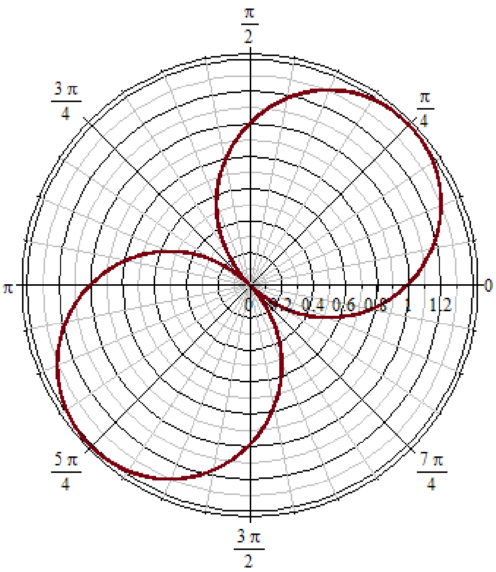


$$r = 8\sin^3\left(\tfrac{\theta}{3}\right), 0 \le \theta \le \tfrac{\pi}{4} \Rightarrow \tfrac{dr}{d\theta} = 8\sin^2\left(\tfrac{\theta}{3}\right)\cos\left(\tfrac{\theta}{3}\right); r^2 + \left(\tfrac{dr}{d\theta}\right)^2 = \left[8\sin^3\left(\tfrac{\theta}{3}\right)\right]^2 + \left[8\sin^2\left(\tfrac{\theta}{3}\right)\cos\left(\tfrac{\theta}{3}\right)\right]^2$$

$$= 64\sin^4\left(\tfrac{\theta}{3}\right) \Rightarrow L = \int_0^{\pi/4}\sqrt{64\sin^4\left(\tfrac{\theta}{3}\right)}\,d\theta = \int_0^{\pi/4} 8\sin^2\left(\tfrac{\theta}{3}\right)d\theta = \int_0^{\pi/4} 8\left[\tfrac{1-\cos\left(\frac{2\theta}{3}\right)}{2}\right]d\theta$$

$$= \int_0^{\pi/4}\left[4 - 4\cos\left(\tfrac{2\theta}{3}\right)\right]d\theta = \left[4\theta - 6\sin\left(\tfrac{2\theta}{3}\right)\right]_0^{\pi/4} = 4\left(\tfrac{\pi}{4}\right) - 6\sin\left(\tfrac{\pi}{6}\right) - 0 = \pi - 3$$

8) $r = \sqrt{1+\cos 2\theta},\ -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$ kutupsal koordinat denklemiyle verilen eğrinin uzunluğunu hesaplayınız.



$$r = \sqrt{1 + \cos 2\theta} \Rightarrow \frac{dr}{d\theta} = \frac{1}{2}(1 + \cos 2\theta)^{-1/2}(-2 \sin 2\theta) = \frac{-\sin 2\theta}{\sqrt{1 + \cos 2\theta}} \Rightarrow \left(\frac{dr}{d\theta}\right)^2 = \frac{\sin^2 2\theta}{1 + \cos 2\theta}$$

$$\Rightarrow r^2 + \left(\frac{dr}{d\theta}\right)^2 = 1 + \cos 2\theta + \frac{\sin^2 2\theta}{1 + \cos 2\theta} = \frac{(1 + \cos 2\theta)^2 + \sin^2 2\theta}{1 + \cos 2\theta} = \frac{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}{1 + \cos 2\theta}$$

$$= \frac{2 + 2\cos 2\theta}{1 + \cos 2\theta} = 2 \Rightarrow L = \int_{-\pi/2}^{\pi/2} \sqrt{2}\, d\theta = \sqrt{2}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \sqrt{2}\,\pi$$