1. MERTEBEDEN DİF. DENKLEMLER

$$F(x, y, y') = 0$$

$$\begin{cases} y' = f(x,y) \\ \dfrac{dy}{dx} = f(x,y) \\ M(x,y)dx + N(x,y)dy = 0 \end{cases}$$

"3 şekilde karşımıza çıkar."

Bunlardan herhangi birinin Genel Çözümü $\Rightarrow G(x, y, \underset{\substack{\downarrow \\ \text{1 tane keyfi parametre}}}{C}) = 0$

DEĞİŞKENLERİNE AYIRILABİLEN DİF. DENKLEMLER

$$\underbrace{M(x,y)}dx + \underbrace{N(x,y)}dy = 0$$

$$f_1(x).g_1(y)\,dx + f_2(x).g_2(y)\,dy = 0$$

$$\frac{f_1(x)}{f_2(x)}dx + \frac{g_1(y)}{g_2(y)}dy = 0$$

$$\boxed{F(x) + G(y) = C}$$

1. mertebeden değişkenlerine ayrılabilen dif. denklemin Genel Çözümü

ÖRNEK

$y' = 2x$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$$\frac{dy}{dx} = 2x$$

$$\int dy = \int 2x\,dx$$

$$\boxed{y = x^2 + C}$$

ÖRNEK

$y \cdot y' + x = 0$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$$y \cdot \frac{dy}{dx} + x = 0$$

$$\int y \cdot dy = -\int x \, dx$$

$$\boxed{\frac{y^2}{2} = -\frac{x^2}{2} + C}$$

$y'y + x = 0$

$y' + \frac{x}{y} = 0$

$\frac{dy}{dx} + \frac{x}{y} = 0$

$dy \cdot y = dx \cdot x$

$\frac{y^2}{2} = -\frac{x^2}{2} + C$

ÖRNEK

$x \cdot y' = 2y$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$y' = \frac{2y}{x}$

$$x \cdot \frac{dy}{dx} = 2y$$

$\frac{dy}{dx} = \frac{2y}{x}$

$$\int \frac{dy}{2y} = \int \frac{dx}{x}$$

$\frac{dy}{2y} = \frac{dx}{x}$

$$\ln y = \ln x^2 + \ln C$$

$$\boxed{y = x^2 \cdot C}$$

$\frac{1}{2}\ln y - \ln x = \ln c$

$\frac{\sqrt{y}}{x} = c$

$y = x^2 c$

ÖRNEK

$y' + y \cdot \cos x = 0$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$$\frac{dy}{dx} + y \cdot \cos x = 0$$

$$\int \frac{dy}{y} = -\int \cos x \, dx$$

$$\ln y = -\sin x + \ln C$$

$$\ln y - \ln C = -\sin x$$

$$\ln \frac{y}{C} = -\sin x$$

$$\frac{y}{C} = e^{-\sin x}$$

$$\Rightarrow \boxed{y = C \cdot e^{-\sin x}}$$

ÖRNEK

$(y+xy)dx+y^2dy=0$ dif. denkleminin $y(1)=1$ koşulu altındaki özel çözümünü bulunuz.

ÇÖZÜM

$y(1+x)dx+y^2dy=0$

$ydx+xydx+y^2dy=0$

$dx+xdx+ydy=0$

$(1+x)dx+ydy=0$

$\int(1+x)dx=-\int ydy$

$x+\frac{x^2}{2}=-\frac{y^2}{2} \Rightarrow \boxed{x+\frac{x^2}{2}+\frac{y^2}{2}=C}$

G. ÇÖZÜM

$y(1)=1$

$\Rightarrow 1+\frac{1^2}{2}+\frac{1^2}{2}=C$

$\Rightarrow C=2$

$\Rightarrow \boxed{x+\frac{x^2}{2}+\frac{y^2}{2}=2}$

Özel Çözüm.

ÖRNEK

$x\cdot \sin y\,dx+(x^2+1)\cos y\,dy=0$ dif. denkleminin özel çözümünü bulunuz.

$\left(y(1)=\frac{\pi}{2}\right)$

ÇÖZÜM

$x\,dx+(x^2+1)\frac{\cos y}{\sin y}dy=0$

$\int\frac{x}{x^2+1}dx=-\int\frac{\cos y}{\sin y}dy$

$\frac{1}{2}\cdot\ln(x^2+1)=-\ln\sin y+\ln C$

$\ln\left[(x^2+1)^{1/2}\cdot\sin y\right]=\ln C$

$\Rightarrow \boxed{C=(x^2+1)^{1/2}\cdot\sin y}$ G. Çözüm

$y(1)=\frac{\pi}{2}$

$C=(1^2+1)^{1/2}\cdot\sin\frac{\pi}{2}$

$C=\sqrt{2}$

$\Rightarrow \boxed{\sin y\cdot(x^2+1)^{1/2}=\sqrt{2}}$ Özel Çözüm.

ÖRNEK

$\sqrt{1-e^{2x}}\cdot y' = (1+y^2)\cdot e^x$ $y(0)=0$ özel çözümünü bulunuz.

ÇÖZÜM

$$\frac{\cancel{\sqrt{1-e^{2x}}}\,dy}{(1+y^2)\cancel{\sqrt{1-e^{2x}}}} = \frac{\cancel{(1+y^2)}\,e^x\,dx}{\cancel{(1+y^2)}\sqrt{1-e^{2x}}}$$

$$\int \frac{dy}{1+y^2} = \int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$$

$e^x = u$

$e^x dx = du$

$\Rightarrow \int \frac{du}{\sqrt{1-u^2}} = \arcsin u$

$\boxed{\arctan y = \arcsin e^x + C}$ G. çözüm

$\arctan 0 = \arcsin e^0 + C$

$0 = \frac{\pi}{2} + C \Rightarrow C = -\frac{\pi}{2}$

$\boxed{\arctan y = \arcsin e^x - \frac{\pi}{2}}$ Özel çözüm.

HOMOJEN DİF. DENKLEMLER

$M(x,y)dx + N(x,y)dy = 0$ dif. denkleminde

$x \to \lambda x \quad y \to \lambda y$

$M(\lambda x, \lambda y)dx + N(\lambda x, \lambda y) = 0$

$M(\lambda x, \lambda y)dx + \lambda^n N(x,y) = 0$

$\lambda^n M(x,y)dx + \lambda^n N(x,y) = 0$ n. dereceden homojen dif.

$f(x,y)$ fonksiyonuna

$\downarrow \quad \downarrow$

$\lambda x \quad \lambda y$ yüklemeleri yapılırsa;

$f(\lambda x, \lambda y) = \lambda^n f(x,y)$ ise $f(x,y)$ n. dereceden homojendir.

ÖRNEK

$f(x,y) = x^2 + xy$

ÇÖZÜM

$f(\lambda x, \lambda y) = \lambda^2 x^2 + \lambda x \lambda y = \lambda^2 (x^2 + xy) = \lambda^2 . f(x,y)$ 2. dereceden homojendir.

$M(x,y)dx + N(x,y)dy = 0$

$y' = f\left(\frac{y}{x}\right)$ buna homojen dif. denklem denir.

$\frac{y}{x} = u$ dersek

$y = u . x$

$y' = u' . x + u$

$u'x + u = f(u)$

$x . \frac{du}{dx} = f(u) - u$

$\int \frac{dx}{x} = \int \frac{du}{f(u) - u}$

$\ln x = F(u) + C$

$\Rightarrow \boxed{\ln x = F\left(\frac{y}{x}\right) + C}$ Genel Çözüm.

ÖRNEK

GÖZÜM $\underbrace{(y+\sqrt{x^2-y^2})}_{M}.dx - \underbrace{x}_{N}.dy=0$ dif. denkleminin genel çözümünü bulunuz.

$M(x,y)=y+\sqrt{x^2-y^2}$

$M(\lambda x,\lambda y)=\lambda y+\sqrt{\lambda^2x^2-\lambda^2y^2}=\lambda y+\lambda\sqrt{x^2-y^2}$

$=\lambda^{1}.M(x,y)$ (1. dereceden homojen)

$N(x,y)=x$

$N(\lambda x,\lambda y)=\lambda x=\lambda^{1}.N(x,y)$ (1. dereceden homojen)

I. YOL

$u=\dfrac{y}{x}$

$y=u.x$

$* \; y'=u'.x+u$

$\dfrac{dy}{dx}=y'=\dfrac{y+\sqrt{x^2-y^2}}{x}=\dfrac{y}{x}+\sqrt{\dfrac{x^2-y^2}{x^2}}$

$\Rightarrow y'=\dfrac{y}{x}+\sqrt{1-\left(\dfrac{y}{x}\right)^2}$

$u'x+\cancel{u}=\cancel{u}+\sqrt{1-u^2}$

$\dfrac{du}{dx}.x=\sqrt{1-u^2}$

$\Rightarrow \displaystyle\int\frac{du}{\sqrt{1-u^2}}=\int\frac{dx}{x}$

$\arcsin\dfrac{y}{x}=\ln x-\ln C$

$\ln\dfrac{x}{C}=\arcsin u$

$u=\sin\left(\ln\dfrac{x}{C}\right)$

$\boxed{\dfrac{y}{x}=\sin\left(\ln\dfrac{x}{C}\right)}$

II. YOL

$u=\dfrac{y}{x}$

$y=u.x$

$* \; dy=du.x+dx.u$

$(y+\sqrt{x^2-y^2})dx-x.dy=0$

$(u.x+\sqrt{x^2-u^2x^2})dx-x.(du.x+dx.u)=0$

$\cancel{ux\,dx}+x.dx\sqrt{1-u^2}-x^2du-\cancel{x.u.dx}=0$

$\displaystyle\int\frac{1}{x}dx=\int\frac{du}{\sqrt{1-u^2}}$

$\ln x-\ln C=\arcsin u$

$\boxed{\dfrac{y}{x}=\sin\left(\ln\dfrac{x}{C}\right)}$

?

ÖRNEK

$(x^3+y^3)\,dx - xy^2\,dy = 0$ dif. denkleminin genel çözümünü bulunuz.

NOT: 1. mertebeden dif. denklem. Değişkenlere ayrılmıyor. Dönüşüm yapmadığım

3. dereceden homojen dif. denklem.

ÇÖZÜM

$u = \frac{y}{x}$

$y = u.x$

$dy = du.x + dx.u$

$(x^3+u^3x^3)\,dx - x.u^2x^2.(du.x + dx.u) = 0$

$(1+u^3)\,dx - u^2du.x - u^3dx = 0$

$dx + \cancel{u^3dx} - u^2.x.du - \cancel{dx.u^3} = 0$

$\int \frac{dx}{x} = \int u^2\,du$

$$\boxed{\ln x = \frac{1}{3}\cdot\frac{y^3}{x^3} + C}$$

ÖRNEK

$x.\cos\frac{y}{x}.\frac{dy}{dx} = y.\cos\frac{y}{x} + x$ dif denkleminin genel çözümünü bulunuz.

ÇÖZÜM 1. mertebeden homojendir.

$u = \frac{y}{x}$

$y = u.x$

$y' = u'.x + u$

$x.\cos u.(u'x+u) = u.x.\cos u + x$

$\cancel{x.\cos u}.u' + \cancel{u.x\cos u} = \cancel{u.x\cos u} + \cancel{x}$

$\cos u\,du = \frac{1}{x}\,dx$

$$\boxed{\sin\frac{y}{x} = \ln x + C}$$

$u'x + u = \frac{x[u\cos u + 1]}{x\cos u}$

$u'x + \cancel{u} = \cancel{u} + \frac{1}{\cos u}$

$u'x = \frac{1}{\cos u}$

$\cos u\,du = \frac{1}{x}$

$\sin u = \ln x + C$

ÖRNEK

$y^2 + x^2.y' = x.y.y'$ $\quad y(1) = 1$ (özel çözüm).

ÇÖZÜM

$u = \frac{y}{x}$

$y = u.x$

$y' = u'.x + u$

$u^2x^2 + x^2.(u'.x+u) = x.u.x.(u'.x+u)$

$\cancel{u^2x^2} + x^3.u' + x^2u = x^3.u.u' + \cancel{x^2u^2}$

$x.u' + u = x.u.u'$

$u'.(xu - x) = u.$

$u'x(u-1) = u.$

$\int \frac{(u-1)}{u}\,du = \int \frac{dx}{x} \Rightarrow \int\left(1 - \frac{1}{u}\right)du = \ln x$

$u - \ln u = \ln x + C \Rightarrow \boxed{\ln y + C = \frac{y}{x}}$

HOMOJEN HALE GETİRİLEBİLEN DENKLEMLER

$\frac{dy}{dx} = f \cdot \left(\frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2} \right)$ bu tip dif. denklemler, homojen hale getirilebilen dif. denklemlerdir.

LİNEER BAĞIMSIZ İSE ;

$a_1, a_2, b_1, b_2, c_1, c_2 \in \mathbb{R}$

$(a_1x + b_1y + c_1)\,dx + (a_2x + b_2y + c_2)\,dy = 0$

Her soruda ;

$\underbrace{a_1x + b_1y \qquad a_2x + b_2y}_{\text{Lineer Bağımsız}}$ arasında bir bağıntı var mı. İlk analiz budur.

h, k herhangi sabitler olmak üzere;

$\left.\begin{array}{l} x = x_1 + h \\ y = y_1 + k \end{array}\right\}$ değişken değişimi yapılacak. (lineer bağımlı ise bunu yapmaya gerek kalmaz)

$\left.\begin{array}{l} x_1, y_1 \rightarrow \text{yeni değişkenler} \\ dx = dx_1 \\ dy = dy_1 \end{array}\right\}$

$\Rightarrow \left[a_1(x_1+h) + b_1(y_1+k) + c_1\right]dx_1 + \left[a_2(x_1+h) + b_2(y_1+k) + c_2\right]dy_1 = 0$

$\Rightarrow (a_1x_1 + b_1y_1 + \overbrace{a_1h + b_1k + c_1}^{=0})\,dx_1 + (a_2x_1 + b_2y_1 + \overbrace{a_2h + b_2k + c_2}^{=0})\,dy_1 = 0$

$\left.\begin{array}{l} a_1h + b_1k + c_1 = 0 \\ a_2h + b_2k + c_2 = 0 \end{array}\right\} \Rightarrow h, k = ?$

$\boxed{(a_1x_1 + b_1y_1)\,dx_1 + (a_2x_1 + b_2y_1)\,dy_1 = 0}$ Homojen Dif. Denklem.

$\frac{y_1}{x_1} = u \rightarrow y_1 = u \cdot x_1$:

$\left.\begin{array}{l} x_1 = x - h \\ y_1 = y - k \end{array}\right\}$ Bunları yerine yazınız.

ÖRNEK $(x+y-3)\,dx + (-x+y+1)\,dy = 0$ dif. denkleminin genel çözümünü bulunuz?

ÇÖZÜM

$\underbrace{(x+y) \qquad (-x+y)}_{\text{Lineer Bağımsız.}}$ Aralarında bir bağıntı var mı?

$\left.\begin{array}{l} x = x_1 + h \\ y = y_1 + k \\ dx = dx_1 \\ dy = dy_1 \end{array}\right\}$ $(x_1 + h + y_1 + k - 3)\,dx_1 + (-x_1 - h + y_1 + k + 1)\,dy_1 = 0$

$$\begin{array}{r} h + k - 3 = 0 \\ -h + k + 1 = 0 \\ \hline 2k = 2 \\ k = 1 \\ h = 2 \end{array}$$

$\Rightarrow \boxed{(x_1 + y_1)\,dx_1 + (-x_1 + y_1)\,dy_1 = 0}$ Homojen Dif. Denklem.

$\left.\begin{array}{l} u = \frac{y_1}{x_1} \\ y_1 = u \cdot x_1 \\ dy_1 = du \cdot x_1 + dx_1 \cdot u \end{array}\right\}$ $(x_1 + u \cdot x_1) \cdot dx_1 + (-x_1 + u \cdot x_1) \cdot (du \cdot x_1 + dx_1 \cdot u) = 0$

$x_1(1+u^2)\,dx_1 + x_1^2(u-1)\,du = 0$

$$\int \frac{dx_1}{x_1} = -\int \frac{u-1}{1+u^2}\,du$$

$$\ln x_1 = -\int \left(\frac{u}{1+u^2} - \frac{1}{1+u^2}\right) du$$

$$\ln x_1 = -\frac{1}{2} \cdot \ln(1+u^2) + \arctan u + C$$

$\left.\begin{array}{l} x_1 = x - h = x - 2 \\ y_1 = y - k = y - 1 \end{array}\right\}$ $\boxed{\ln(x-2) + \frac{1}{2} \cdot \ln\left(1 + \left(\frac{y-1}{x-2}\right)^2\right) - \arctan \frac{y-1}{x-2} = C}$

Lineer Bağımlı ise;

$(a_1x+b_1y+c_1)dx+(a_2x+b_2y+c_2)dy=0$

$\underbrace{a_1x+b_1y \qquad a_2x+b_2y}_{\text{Lineer Bağımlı}}$ $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=0$ ise lineer bağımlı

$\lambda(a_1x+b_1y)=a_2x+b_2y$

$(a_1x+b_1y+c_1)dx+(\lambda.(a_1x+b_1y)+c_2)dy=0$

$a_1x+b_1y=t$

$a_1dx+b_1dy=dt$

⋮

ÖRNEK

$(x+5y+1)dx+(2x+10y+7)dy=0$

ÇÖZÜM

$\underbrace{x+5y \qquad 2x+10}_{\text{Lineer Bağımlı}}$

$x+5y=t$

$dx+5dy=dt$

$dy=\frac{1}{5}.(dt-dx)$

$5/(t+1)dx+(2t+7)\frac{1}{5}(dt-dx)=0$

$(5t+5-2t-7)dx+(2t+7)dt=0$

$(3t-2)dx+(2t+7)dt=0$

$\int dx+\int\left(\frac{2t+7}{3t-2}\right)dt=\int 0$

$x+\frac{2t}{3}+\frac{25}{9}.\ln(3t-2)=C$

$\Rightarrow \boxed{x+\frac{2}{3}(x+5y)+\frac{25}{9}\ln(3x+15y-2)=C}$

ÖRNEK

$$(x+2y+1)dx-(2x+4y-3)dy=0$$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$$\underbrace{x+2y \qquad 2x+4y}_{\text{Lineer Bağımlı}}$$

$x+2y=t$

$dx+2dy=dt$

$dy=\frac{1}{2}(dt-dx)$

$$2/\;(t+1)dx-(2t-3)\frac{1}{2}\cdot(dt-dx)=0$$

$$(2t+2+2t-3)dx-(2t+3)dt=0$$

$$\boxed{(4t-1)dx-(2t+3)dt=0}$$ → Değişkenlerine ayrılabilen dif. denklem

$$\begin{array}{r|l} 2t+3 & 4t-1 \\ \hline 2t-\frac{1}{2} & \frac{1}{2} \\ \hline \frac{7}{2} & \end{array}$$

$$\int dx-\int\left(\frac{2t+3}{4t-1}\right)dt=\int 0$$

$$\int dx-\int\left(\frac{1}{2}+\frac{\frac{7}{2}}{4t-1}\right)dt=C$$

$$x-\frac{1}{2}t+\frac{7}{8}\cdot\ln(4t-1)=C$$

$$\Rightarrow \boxed{x-\frac{1}{2}(x+2y)+\frac{7}{8}\cdot\ln(4x+8y-1)=C.}$$

TAM DİF. DENKLEMLER

$F(x,y,y')=0$

$y'=f(x,y)$

$\boxed{M(x,y)dx + N(x,y)dy=0}$ → Bu dif. denklem $u=u(x,y)$ fonksiyonunun toplam diferansiyeli ile dif. denkleme TAM DİF. DENK. denir.

$$\boxed{\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = M(x,y)dx + N(x,y)dy}$$ eşit olmaktadır.

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= M(x,y) \\ \frac{\partial u}{\partial y} &= N(x,y) \end{aligned}\right\} \quad \begin{aligned} \frac{\partial^2 u}{\partial y \partial x} &= \frac{\partial M}{\partial y} \\ \frac{\partial^2 u}{\partial x \partial y} &= \frac{\partial N}{\partial x} \end{aligned} \quad \Rightarrow \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \ *$$

Tam dif. ise bu koşul aranır.

$M(x,y)dx + N(x,y)dy=0$	$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
$u(x,y)=C$	
$\frac{\partial u}{\partial x} = M(x,y)$ Bununla başlarsam; $u(x,y) = \int M(x,y)dx + h(y)$	$\frac{\partial u}{\partial y} = N(x,y)$ Bununla başlarsam; $u(x,y) = \int N(x,y)dy + h(x)$

ÖRNEK

$(x^2+\frac{y^2}{x})dx + 2y\ln x\,dy = 0$ dif. denk. genel çözümünü bulunuz.

ÇÖZÜM

$$\frac{\partial M}{\partial y} = \frac{2y}{x} = \frac{\partial N}{\partial x} \checkmark$$

$$u(x,y) = C$$

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}\cdot dy = (x^2+\frac{y^2}{x})dx + 2y\ln x\,dy$$

$$\frac{\partial u}{\partial x} = x^2+\frac{y^2}{x} \rightarrow u(x,y) = \int\left(x^2+\frac{y^2}{x}\right)dx + h(y)$$

$$u(x,y) = \frac{x^3}{3} + y^2\ln x + h(y)$$

$$\frac{\partial u}{\partial y} = 2y\ln x$$

$$\frac{\partial u}{\partial y} = 2y\ln x + \frac{dh}{dy} = 2y\ln x$$

$$\Rightarrow \frac{dh}{dy} = 0$$

$$\Rightarrow \boxed{h(y) = C_1}$$

$$u(x,y) = C$$

$$\frac{x^3}{3} + y^2\ln x + C_1 = C \qquad C - C_1 = K \text{ dersem;}$$

$$\boxed{\frac{x^3}{3} + y^2\ln x = K}$$ Genel Çözüm.

ÖRNEK

$2xy\,dx + (x^2+\cos y)\,dy = 0$ dif. denkleminin genel çözümünü bulunuz.

ÇÖZÜM:

$$\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x} \quad ✓$$

$$u(x,y) = C$$

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = 2xy\,dx + (x^2+\cos y)\,dy$$

$$\frac{\partial u}{\partial x} = 2xy \qquad u(x,y) = \int (x^2+\cos y)\,dy + h(x)$$

$$\frac{\partial u}{\partial y} = x^2+\cos y \qquad u(x,y) = x^2y + \sin y + h(x)$$

$$\frac{\partial u}{\partial x} = \cancel{2xy} + \frac{dh}{dx} = \cancel{2xy}$$

$$\frac{dh}{dx} = 0 \Rightarrow \boxed{h(x) = C_1}$$

$$u(x,y) = C$$

$$x^2y + \sin y + C_1 = C \qquad (C - C_1 = K)$$

$$\Rightarrow \boxed{x^2y + \sin y = K}$$

ÖRNEK $(2y-x)y' + 2x - y = 0 \qquad y(-2) = 1$ özel çözümünü bulunuz.

ÇÖZÜM $(2x-y)\,dx + (2y-x)\,dy = 0$

$$\frac{\partial M}{\partial y} = -1 = \frac{\partial N}{\partial x} \quad ✓$$

$$u(x,y) = C$$

$$\frac{\partial u}{\partial x} = 2x-y \qquad u(x,y) = \int (2x-y)\,dx + h(y)$$

$$u(x,y) = x^2 - yx + h(y)$$

$$\frac{\partial u}{\partial y} = 2y-x \qquad \frac{\partial u}{\partial y} = \cancel{-x} + \frac{dh}{dy} = 2y \cancel{-x} \Rightarrow \boxed{h(y) = y^2 + C_1}$$

$$u(x,y) = C$$

$$x^2 - yx + y^2 + C_1 = C \qquad (C - C_1) = K$$

$\boxed{x^2 - yx + y^2 = K}$ Genel Çözüm.

$y(-2) = 1$ için $K = 7$

$\boxed{x^2 - yx + y^2 = 7 \text{ özel çözüm.}}$

ÖRNEK

$$(2xy+e^x)\,dx+(-1+x^2)\,dy=0$$ G. çözümünü bulunuz?

ÇÖZÜM

$$\frac{\partial M}{\partial y}=2x=\frac{\partial N}{\partial x}$$ ✓

$u(x,y)=C$

$$\frac{\partial u}{\partial x}=2xy+e^x \rightarrow u(x,y)=\int(2xy+e^x)\,dx+h(y)$$

$$\frac{\partial u}{\partial y}=x^2-1 \qquad u(x,y)=x^2y+e^x+h(y)$$

$$\frac{\partial u}{\partial y}=\cancel{x^2}+\frac{dh}{dy}=\cancel{x^2}-1 \Rightarrow \boxed{h(y)=-y+C_1}$$

$u(x,y)=C$

$x^2y+e^x-y+C_1=C \qquad (C-C_1=K)$

$\boxed{x^2y+e^x-y=K}$ Genel Çözüm

TAM DİF. HALİNE DÖNÜŞTÜRME : İNTEGRASYON ÇARPANI

$M(x,y)dx + N(x,y)dy = 0$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ tam dif. olmadığını gördük.

$$\underbrace{\lambda(x,y) \cdot M(x,y)\,dx}_{M_1(x,y)} + \underbrace{\lambda(x,y) \cdot N(x,y)\,dy}_{N_1(x,y)} = 0$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\lambda(x,y)$ integrasyon çarpanı

$$\Rightarrow \frac{\partial \lambda}{\partial y} \cdot M(x,y) + \frac{\partial M}{\partial y} \cdot \lambda(x,y) = \frac{\partial \lambda}{\partial x} N(x,y) + \frac{\partial N}{\partial x} \cdot \lambda(x,y)$$

$$\Rightarrow \lambda \left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right] = \frac{\partial \lambda}{\partial x} \cdot N - \frac{\partial \lambda}{\partial y} M$$

1) $\lambda = \lambda(x)$

$$\frac{d\lambda}{dx} N = \lambda \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\int \frac{d\lambda}{\lambda} = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\, dx$$

$$\boxed{\ln \lambda = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\, dx}$$

X'e göre integrasyon çarpanı

2)
$\lambda = \lambda(y)$

$$-\frac{\partial \lambda}{\partial y} \cdot M = \lambda \cdot \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\boxed{\ln \lambda = \int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} \, dy}$$

y'e göre integrasyon çarpanı

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ÖRNEK

$(xy+1)y\,dx + (2y-x)dy = 0$ G.Ç= ?

ÇÖZÜM

$\frac{\partial M}{\partial y} = 2xy+1 \neq \frac{\partial N}{\partial x} = -1$ tam dif. değil

$$\ln\lambda = \int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\cdot dy = \int \frac{-2\cancel{(1+xy)}}{y\cdot\cancel{(1+xy)}}\cdot dy =$$

$$\ln\lambda = \ln y^{-2}$$

$\boxed{\lambda = \frac{1}{y^2}}$ integrasyon çarpanı

$$\underbrace{\frac{1}{y^2}\cdot(xy^2+y)\,dx}_{} + \underbrace{\frac{1}{y^2}\cdot(2y-x)\,dy}_{} = 0$$

$$\frac{\partial M_1}{\partial y} = -\frac{1}{y^2} = \frac{\partial N_1}{\partial x}$$

$u(x,y) = C$

$\frac{\partial u}{\partial x} = x + \frac{1}{y}$

$\frac{\partial u}{\partial y} = \frac{2}{y} - \frac{x}{y^2}$

$u(x,y) = \int (x+\frac{1}{y})dx + h(y)$

$u(x,y) = \frac{x^2}{2} + \frac{1}{y}x + h(y)$

$\frac{\partial u}{\partial y} = -\cancel{\frac{x}{y^2}} + \frac{dh}{dy} = \frac{2}{y} - \cancel{\frac{x}{y^2}}$

$\boxed{h(y) = 2.\ln y + C_1}$

$u(x,y) = C$

$\boxed{\frac{x^2}{2} + \frac{1}{y}x + 2.\ln y = K}$ $(C - C_1 = K)$

Genel Çözüm

ÖRNEK

$2x.\sin y\,dx + \cos y\,dy = 0$ dif denkleminin genel çözümünü bulunuz.

ÇÖZÜM

$\frac{\partial M}{\partial y} = 2x.\cos y \neq \frac{\partial N}{\partial x} = 0$ tam dif. değildir.

$$\ln\lambda = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\,dx = \int \frac{2x.\cos y - 0}{\cos y}.dx = \int 2x\,dx$$

$\ln\lambda = x^2$

$\boxed{\lambda = e^{x^2}}$ (integrasyon çarpanı)

2 $\frac{2x\cos y}{(0)}$ 0 $\int 2x$ x^2

$\underbrace{e^{x^2}.2x.\sin y}\,dx + \underbrace{e^{x^2}\cos y}\,dy = 0$

$\frac{\partial M_1}{\partial y} = \cos y.e^{x^2}.2x = \frac{\partial N_1}{\partial x}$

$\Rightarrow u(x,y) = C$

$\frac{\partial u}{\partial x} = e^{x^2}.2x.\sin y$

$\frac{\partial u}{\partial y} = e^{x^2}.\cos y$

$u(x,y) = \int (e^{x^2}.\cos y)\,dy + h(x)$

$u(x,y) = \sin y.e^{x^2} + h(x)$

$\frac{\partial u}{\partial y} = \cos y.e^{x^2} + \frac{dh}{dx} = e^{x^2}.\cos y$

$\frac{2y}{2y-x}$

$\frac{dh}{dx} = 0 \Rightarrow \boxed{h(x) = C_1}$

$u(x,y) = C$

$\sin y.e^{x^2} + C_1 = C$ $\quad (C - C_1 = K)$

$\Rightarrow \boxed{e^{x^2}.\sin y = K}$

$y. -2y^2 \ln(2y-x) - x \ln(2y-x)$