1) $\frac{dy}{dt} = kty$ çözümü aşağıdakilerden hangisidir?

a) $y = y_0 e^{kt}$ b) $y = mx + b$ c) $y = a(x-h)^2 + k$

d) $y = A\sin[B(x-C)] + D$

2) $\frac{dy}{dx} = \frac{4x}{y}$ $y(0) = 1$ özel çözümü

a) $y = \sqrt{4x^2 - 4}$ b) $y = 2x^2 + 1$ c) $y = e^{2x^2}$ d) $y = \sqrt{4x^2 + 1}$

3) $\frac{dy}{dx} = e^y(4x^2 - 5x)$ $y = f(x)$ (1,0) noktasından geçen özel çözümü olsun. (1,0) noktasında $f(x)$ in grafiğine teğet ~~olan~~ doğrusunun denklemi $m = \frac{dy}{dx}\Big|_{(1,0)} = e^0(4-5) = -1$ $y - 0 = -1(x-1)$
$y = -x + 1$

$y = x - 1$ $y = 2(x-1)$ $\boxed{y = -(x-1)}$ $y = -2(x-1)$

4) $y'' + y = e^x$ degree ve mert?

1, 2 1, 1 2, 2 2, 1

5) $\frac{dy}{dx} = 2x\sqrt{y}$ $x = 3$ iken $y = 4$ ise y nin çözümü

$2\sqrt{y} = x^2 + c$ $y = (x^2 + 25)^2/4$ $y = x^4/4$ $\boxed{y = \frac{1}{4}(x^2 - 5)^2}$

5Ç: $\frac{2dy}{2\sqrt{y}} = 2x$ $2\sqrt{y} = \frac{2x^2}{2} + c$ $2\sqrt{4} = 9 + c \Rightarrow c = -5$; $2\sqrt{y} = x^2 - 5$

$\sqrt{y} = \frac{x^2 - 5}{2}$

$y = \left(\frac{x^2 - 5}{2}\right)^2$

3Ç: $e^{-y} \cdot dy = (4x^2 - 5x)\,dx$

$-e^{-y} = 4\frac{x^3}{3} - 5\frac{x^2}{2} + c$

$-1 = \frac{4}{3} - \frac{5}{2} + c \Rightarrow -1 + \frac{7}{6} = c$

$c = \frac{1}{6}$ $-e^{-y} = 4\frac{x^3}{3} - \frac{5}{2}x^2 + \frac{1}{6}$

① Ayşe'nin Alfa Bilişim hizmetleri şirketi ile arasındaki ilişkiyi, iş sözleşmesinin unsurları bakımından değerlendiriniz.

7) $\frac{dy}{dt} = -2y$ $t=0$ iken $y=1$ dir. $y = 1/2$ olduğunda t değeri nedir?

Çözüm: $\frac{dy}{y} = -2dt$, $\ln y = -2t + K$, $\ln 1 = -2t + K$, $0 = -2(0) + K$, $K = 0$

$\ln y = -2t$, $y = e^{-2t} \Rightarrow 1/2 = e^{-2t}$, $-2t = \ln 2^{-1}$, $\boxed{t = \frac{+\ln 2}{+2}}$

$-\ln(2)/2$ $\quad -1/4 \quad$ $\ln 2/2$ $\quad \ln 2 \quad$ $\sqrt{2}/2$

8) $\left(\frac{d^2y}{dx^2}\right)^2 + y = 0$ D.D derecesi nedir?

1 $\quad$ 3 $\quad$ 2 $\quad$ 0

9) Aşağıdaki D.D liner veya lin. olmayan, homojen veya homojen olmayan, adi veya kısmi ve onların derece leri şeklinde sınıflandırınız.

a) $\frac{dy}{dt} = ty^2$, 1 M, non lineer, homojen, adi dd.

b) $y'' + y = t - 2$ $\longrightarrow$ 2 M, Lin, hom değil, add

c) $(1-y)\frac{d^2y}{dt^2} - 2\frac{dy}{dt} = \cos 2t$ $\rightarrow$ 2M, non-lin, hom olmayan add

d) $3x'' + 2x' - 4x = 0$ 2 M, linear, hom, ordinary.

10-) $ty' + 2y = \sin t$ $\quad y(\pi/2) = 1$ çözümünü bulunuz. çözümün geçerli olduğu en geniş aralığı ifade ediniz.

$y' + \frac{2}{t}y = \frac{\sin t}{t}$ $\qquad \lambda(t) = e^{\int \frac{2}{t} dt} = e^{2\ln t} = t^2$

$t^2y' + 2ty = t\sin t$

$\frac{d}{dt}(t^2y) = t\sin t$

$t^2y = \int t\sin t\, dt = -t\cos t + \int \cos t\, dt = -t\cos t + \sin t + K$

$t^2y = -t\cos t + \sin t + K$

$t = u$ $\quad \sin t\, dt = dv$

$dt = du$ $\quad -\cos t = v$

$y = -\frac{1}{t}\cos t + \frac{\sin t}{t^2} + \frac{K}{t^2}$

en geniş aralık $(0, +\infty)$

$\dfrac{dy}{dx} = \dfrac{2xy^3}{(3x^2-5)^2}$ $y(0) = \frac{1}{2}$

$3x^2-5 = t$

$6x\,dx = dt$

$3.\underbrace{2x\,dx} = dt$

$2x\,dx = \dfrac{dt}{3}$

$\displaystyle\int \frac{dy}{y^3} = \int \frac{2x\,dx}{(3x^2-5)^2}$

$-\dfrac{1}{2y^2} = \displaystyle\int \frac{dt/3}{t^2}$

$y(0) = 1/2$

$1 = \frac{1}{3}\left(-\frac{1}{5}\right) + K$

$K = 1 + \frac{1}{15} = \frac{16}{15}$

$+\dfrac{1}{2y^2} = \dfrac{1}{3}\left(+\dfrac{1}{3x^2-5}\right) + K$

$\dfrac{1}{2y^2} = \dfrac{1}{3}\,\dfrac{1}{3x^2-5} + \dfrac{16}{15} = \dfrac{15+16(9x^2-15)}{45(3x^2-5)}$

$2y^2 = \dfrac{\cancel{45}^{15}(3x^2-5)}{15+16(\cancel{9}x^2-\cancel{15}_{5})} = \dfrac{15(3x^2-5)}{15+16(x^2-5)}$

Aşağıdaki denklemlerin genel çözümlerini bulunuz

a) $y''-y'-2y=0$ $r^2-r-2=0$ $\;{-2}$ $y = c_1 e^{-2x} + c_2 e^{x}$

b) $y''+4y+4y=0$ $r^2+4r+4=0$ $\;{-2},\,{-2}$ $y = c_1 e^{-2x} + c_2 x e^{-2x}$

c) $y''+4y'+5y=0$ $\Delta = -4 < 0$ $r_{1,2} = -2 \mp i$ $y = e^{-2x}(c_1 \sin x + c_2 \cos x)$

$F(x, y(x), y'(x), \dots y^{(n)}(x)) = 0$ n. mertden d.d. lineer d.d olması için F in genel formu ne olmalıdır

$F = a_0(x) y^{(n)} + a_1(x) y^{(n-1)} + \dots + a_{n-1} y' + a_n y + g(t)$

$a_0(x) \neq 0$

$y''+3y'+2y=0$ $y(0)=0$ $y'(0)=\alpha$ b.d.p çöz.

$r^2+3r+2=0$ $\;{-2},\,{-1}$ $y = c_1 e^{-2x} + c_2 e^{-x}$

$y' = -2c_1 e^{-2x} - c_2 e^{-x}$

$0 = c_1 + \cancel{c_2}$

$+\;\alpha = -2c_1 - \cancel{c_2}$

$\alpha = -c_1$ $c_2 = \alpha$

$y = -\alpha e^{-2x} + \alpha e^{-x}$ //

$e^x - yy' = 0 \quad y(0) = 1$

$$e^x = y\frac{dy}{dx} \qquad \int \quad e^x = \frac{y^2}{2} + C \qquad e^0 = \frac{1}{2} + C \qquad C = 1/2$$

$$e^x\,dx = y\,dy \qquad e^x = \frac{y^2}{2} + \frac{1}{2}$$

$$1 - 2e^x = y^2 \qquad y = \sqrt{2e^x - 1}$$

$(2y - xe^{xy})\,y' = 2 + ye^{xy}$

$$\underbrace{(2y - xe^{xy})}_{N}\,dy - \underbrace{(2 + ye^{xy})}_{M}\,dx = 0$$

$$\frac{\partial M}{\partial y} = -(e^{xy} + xy\,e^{xy}) = -e^{xy}(1 + xy) \qquad \frac{\partial N}{\partial x} = -1e^{xy} - xye^{xy} = -e^{xy}(1 + xy)$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{T.D.D}$$

$$\frac{\partial U}{\partial x} = -2 - ye^{xy} \qquad \frac{\partial U}{\partial y} = 2y - xe^{xy}$$

$$\Downarrow$$

$$U(x,y) = \int(-2 - ye^{xy})\,dx + h(y) = -2x - \cancel{y}\frac{e^{xy}}{\cancel{y}} + h(y)$$

$$U(x,y) = -2x - e^{xy} + h(y) \Rightarrow \frac{\partial U}{\partial y} = -\cancel{xe^{xy}} + h'(y) = 2y - \cancel{xe^{xy}}$$

$$h'(y) = 2y$$

$$h(y) = 2\frac{y^2}{2} + K$$

$$U(x,y) = -2x - e^{xy} + y^2 + K \Rightarrow \boxed{-2x - e^{xy} + y^2 = C}$$

$ty' = 3y + t \quad y(4) = -1$

$$y' = \frac{3}{t}y + 1 \qquad y' - \frac{3}{t}y = 1 \qquad \lambda(t) = e^{-\int\frac{3}{t}dt} = e^{-3\ln t} = \frac{1}{t^3}$$

$$\frac{1}{t^3}\cdot y = \int\frac{1}{t^3}\,dt + K$$

$$\frac{t^{-3+1}}{-2} \qquad y = t^3\left[-\frac{1}{2}\frac{1}{t^2} + K\right] = -\frac{1}{2}t + Kt^3$$

$$-1 = -\frac{1}{2}4 + K\,64 \qquad 64K = 1 \qquad K = \frac{1}{64} \qquad y = -\frac{1}{2}t + \frac{1}{64}t^3 \;//$$

$y' - \frac{1}{2}y = e^{-t}$ $y(0) = y_0$

$\lambda(t) = e^{-\int \frac{1}{2}dt} = e^{-\frac{1}{2}t}$

$e^{-t/2} \cdot y = \int e^{-t} \cdot e^{-t/2}\, dt + K$

$y = e^{+t/2}\left[-\frac{2}{3}e^{-3/2\, t} + K\right]$

$y = -\frac{2}{3}e^{-t} + K e^{t/2}$

$y_0 = -\frac{2}{3}e^{0} + K e^{0}$ $\left(y_0 + \frac{2}{3}\right) = K$

$y = -\frac{2}{3}e^{-t} + \left(y_0 + \frac{2}{3}\right)e^{t/2}$ //

$\underbrace{\sin t\, y' + \cos t \cdot y} = 3t\sin t$ $y\left(\frac{\pi}{2}\right) = 0$

$\frac{d}{dt}(\sin t \cdot y) = 3t\sin t$

$\sin t \cdot y = 3\int \underbrace{t}_{u} \cdot \underbrace{\sin t\, dt}_{dv} = 3\left[-t\sin t + \sin t\right] + K$

$y = 3(-t+1) + \frac{K}{\sin t} \Rightarrow 0 = 3\left(-\frac{\pi}{2}\cdot 0 + 1\right) + \frac{K}{\underbrace{\sin\frac{\pi}{2}}_{1}}$

$\boxed{K = -3}$

$y = 3(-t+1) - \frac{3}{\sin t}$

\# $\frac{dy}{dx} = \cos^2(2y)\sin^2(x) \qquad y\left(\frac{\pi}{8}\right) = 0$

$$\frac{dy}{dx} = \cos^2(2y)\left(1-\cos(2x)\right)\frac{1}{2}$$

$$\int 2\sec^2(2y)\,dy = \int (1-\cos(2x))\,dx$$

$$(\tan(2y)) = x - \frac{\sin 2x}{2} + K$$

$$y\left(\frac{\pi}{8}\right) = 0 \Rightarrow \underbrace{\tan 0}_{0} = \frac{\pi}{8} - \frac{1}{2}\underbrace{\sin 2\left(\frac{\pi}{8}\right)}_{\frac{1}{\sqrt{2}}} + K$$

$$K = \frac{1}{2\sqrt{2}} - \frac{\pi}{8}$$

$$\tan(2y) = x - \frac{1}{2}\sin 2x + \frac{1}{2\sqrt{2}} - \frac{\pi}{8}$$

$$y = \frac{1}{2}\arctan\left(x - \frac{1}{2}\sin 2x + \frac{1}{2\sqrt{2}} - \frac{\pi}{8}\right)$$

\# $\underbrace{(10x^3y^4 + e^y + 3\sin(4x+3y))}_{N}\,y' + \underbrace{(6x^2y^5 + 4\sin(4x+3y) + 2)}_{M} = 0$

diferansiyel denklemi tam d.d midir?

$$M_y = 30x^2y^4 + 12\cos(4x+3y) = N_x = 30x^2y^4 + 12\cos(4x+3y) \Rightarrow \text{TDD}$$

$$u(x,y) = \int 10x^3y^4 + e^y + 3\sin(4x+3y)\,dy + h(x)$$

$$u(x,y) = \frac{10x^3y^5}{5} + e^y - \cos(4x+3y) + h(x)$$

$$\Downarrow$$

$$\frac{\partial u}{\partial x} = 2\cdot\cancel{3x^2}y^5 + 4\sin(4x+3y) + h'(x) = \cancel{6x^2y^5} + 4\sin\cancel{(4x+3y)} + 2$$

$$h'(x) = 2 \Rightarrow h(x) = 2x + K$$

$$u(x,y) = 2x^3y^5 + e^y - \cos(4x+3y) + 2x + K$$

$$2x^3y^5 + e^y - \cos(4x+3y) + 2x = C$$