UYGULAMA

1) $y' = (x+y+1)^2$ dif. denkleminin genel çözümünü bulunuz.

Çözüm

$u^2 = \frac{du}{dx} - 1 \qquad u^2 + 1 = \frac{du}{dx} \qquad u^2 = y'$

$\arctan u = x$

$(x+y+1) = u$

$u' = 1 + y'$

$y' = u' - 1$

$u' - 1 = u^2$

$u' = u^2 + 1$

$\frac{du}{dx} = u^2 + 1$

$\int \frac{du}{u^2+1} = \int dx$

$\arctan u = x + C$

$\boxed{\arctan(x+y+1) = x + C}$ Genel Çözüm.

$x + y + 1 = u$

$\cancel{dx} + dy = du$

$dy + 1 = du$

$\frac{du - 1}{dx} = u^2$

$\frac{dy}{dx} = u^2$

$dx \cdot u^2 \cancel{- du} - 1$

2) $y' = \sin(x-y)$ dif. denkleminin genel çözümünü bulunuz.

Çözüm

$x - y = u$

$u' = 1 - y'$

$y' = 1 - u'$

$1 - u' = \sin(u)$

$1 - \frac{du}{dx} = \sin(u)$

$\int \frac{du}{1 - \sin(u)} = \int dx$

$(1 + \sin(u))$

$\int \frac{(1 + \sin u)}{\cos^2 u} du = x + C$

$\int \frac{1}{\cos^2 u} du + \int \frac{\sin u}{\cos^2 u} du = x + C.$

$\tan u + \frac{1}{\cos u} = x + C$

$\boxed{\tan(x-y) + \frac{1}{\cos(x-y)} = x + C}$

3) $y' = \left(\frac{x-y+3}{x-y+1}\right)^2$ dif. denkleminin genel çözümünü bulunuz.

Çözüm

$(x-y+1) = u$

$u' = 1 - y'$

$y' = 1 - u'$

$1 - u' = \left(\frac{u+2}{u}\right)^2$

$1 - u' = \left(1 + \frac{2}{u}\right)^2$

$\cancel{1} - u' = \cancel{1} + \frac{4}{u} + \frac{4}{u^2}$

$-\frac{du}{dx} = \frac{4.(u+1)}{u^2}$

$\int \frac{u^2}{u+1} du = -4\int dx$

$= -4x + C$

$\int \left(u - 1 + \frac{1}{u+1}\right) du = -4x + C$

$\boxed{\frac{u^2}{2} - u + \ln(u+1) = -4x + C}$ $u = x-y+1$ yerine konulursa.

4) $x^2 y' . \cos y = 2x . \sin y - 1$ dif. denklemini çözünüz.

Çözüm

$\sin y = u$

$u' = y' . \cos y$

$x^2 . u' = 2x . u - 1$

$* \; u' - \frac{2}{x} u = -\frac{1}{x^2}$ Lineer dif. denklem. $(y' + p(x) y = q(x))$

$\ln \lambda = -\int \frac{2}{x} dx$

$\ln \lambda = -2 \ln x$

$\boxed{\lambda = \frac{1}{x^2}}$

$\frac{1}{x^2} . u' - \frac{1}{x^2} . \frac{2}{x} . u = \frac{1}{x^2} . \left(-\frac{1}{x^2}\right)$

$\int \frac{d}{dx}\left(\frac{1}{x^2} . u\right) = \int -\frac{1}{x^4} dx$

$\frac{1}{x^2} . u = -\frac{x^{-3}}{3} + C.$

$u = \frac{1}{3x} + Cx^2$

$\boxed{\sin y = \frac{1}{3x} + Cx^2}$ Genel Çözüm

5) $(1+e^y)\cdot y' + (y+e^y)x = e^{-x^2/2}$. dif. denklemini çözünüz.

<u>Çözüm</u> y'

$y + e^y = u$

$u' = y' + y'\cdot e^y = y'(1+e^y)$

$\Rightarrow u' + u\cdot x = e^{-x^2/2}$ lineer dif. denklem.

$\ln \lambda = \int 1 x\, dx$

$\ln \lambda = \frac{x^2}{2}$

$\boxed{\lambda = e^{x^2/2}}$

$\Rightarrow \underbrace{e^{x^2/2}\cdot u' + e^{x^2/2}\cdot x \cdot u}_{} = 1$

$\int \frac{d}{dx}\left(e^{x^2/2}\cdot u\right) = \int dx$

$u\cdot e^{x^2/2} = x + C$

$\boxed{y + e^y = \frac{x+C}{e^{x^2/2}}}$ Genel çözüm.

6) $4e^{2y}\cdot y'^2 + 2xy' = 1$ dif. denkleminin $u = e^{2y}$ değişken dönüşümünü yaparak;
a) Oluşan dif. denklemin türünü belirtiniz.
b) Bu dif. denk. çözünüz.

Çözüm

$u = e^{2y}$

$u' = 2y'\cdot e^{2y} = 2\cdot u\cdot y'$

$y' = \dfrac{u'}{2u}$

$4u\cdot\left(\dfrac{1}{2u}\cdot u'\right)^2 + \cancel{2}x\cdot\dfrac{u'}{\cancel{2}u} = 1$

$\dfrac{(u')^2}{u} + x\cdot\dfrac{u'}{u} = 1.$

$u = x\cdot u' + (u')^2$ Clairaut Dif. Denklem.

$u' = P$

$u = x\cdot P + P^2$

$\cancel{u'} = \cancel{P} + P'\cdot x + 2P\cdot P'$

$\dfrac{dP}{dx}\cdot(x + 2P) = 0$

1) $\dfrac{dP}{dx} = 0 \Rightarrow P = C.$

$\boxed{e^{2y} = x\cdot C + C^2}$ Genel çözüm.

2) $x + 2P = 0$

$x = -2P$

$u = -2P^2 + P^2$

$u = -P^2$

$P = -\dfrac{x}{2} \Rightarrow P^2 = \dfrac{x^2}{4}$

$u = -\dfrac{x^2}{4} \Rightarrow \boxed{e^{2y} = -\dfrac{x^2}{4}}$ Tekil Çözüm.