1) $(x\tan(\frac{y}{x}) + y)dx - xdy = 0$ diferansiyel denklemini çözünüz.

$\frac{y}{x} = u \rightarrow y = ux \quad ; \quad y' = u'x + u$

NURAN GÜZEL

$\tan(\frac{y}{x}) + \frac{y}{x} - y' = 0$

$\tan u + u - (u'x + u) = 0$

$x\frac{du}{dx} = \tan u$

$\int \frac{dx}{x} = \int \frac{du}{\tan u} \rightarrow \ln|x| = \ln|\sin u| + c$

$x = c\sin\frac{y}{x}$ //

2) $ye^{2xy}dx + 2bxe^{2xy}dy = -xdx$ diferansiyel denklemi veriliyor. b reel sayısının hangi değeri için bu denklem, tam diferansiyel denklem olur? Bulunan bu b değeri için diferansiyel denklemi çözünüz.

$$\underbrace{(ye^{2xy}+x)}_{M}dx + \underbrace{2bxe^{2xy}}_{N}dy = 0$$

NURAN GÜZEL

$$\frac{\partial M}{\partial y} = e^{2xy} + 2xye^{2xy} \quad ; \quad \frac{\partial N}{\partial x} = 2be^{2xy} + 4bxye^{2xy}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ den } \cancel{e^{2xy}(1+2xy)} = 2b\,\cancel{e^{2xy}(1+2xy)}$$

$$2b = 1 \Rightarrow \boxed{b = \frac{1}{2}} \text{ bulunur.}$$

$f(x,y)=C$ şeklinde çözümü vardır.

$$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = Mdx + Ndy$$

i) $\dfrac{\partial f}{\partial x} = ye^{2xy} + x \rightarrow f(x,y) = \int (ye^{2xy}+x)dx + r(y)$

ii) $\dfrac{\partial f}{\partial y} = xe^{2xy}$

$$f(x,y) = \frac{e^{2xy}}{2} + \frac{x^2}{2} + r(y)$$

$$\frac{\partial f}{\partial y} = xe^{2xy} + r'(y)$$

ii) $\dfrac{\partial f}{\partial y} = xe^{2xy}$

$$xe^{2xy} + r'(y) = xe^{2xy}$$

$$r'(y) = 0 \rightarrow r(y) = C_1$$

Genel çözümü: $f(x,y) = C$

$$\frac{e^{2xy}}{2} + \frac{x^2}{2} + C_1 = C$$

$$e^{2xy} + x^2 = K \qquad (K = 2(C-C_1))$$

4) $2y-2xy'+e^{y'}=0$ diferansiyel denkleminin tüm çözümlerini bulunuz.

$y = xy' - \frac{1}{2}e^{y'}$ clairaut dif. denk

$y'=p$ dersek $y = xp - \frac{1}{2}e^{p}$ olur

$\underset{\downarrow \cancel{p}}{y'} = \cancel{p} + xp' - \frac{1}{2}p'e^{p}$

NURAN GÜZEL

$0 = p'(x - \frac{1}{2}e^{p})$

1°) $p'=0 \rightarrow p=c$ $\left\{\begin{array}{l} y = xp - \frac{1}{2}e^{p} \\ p=c \end{array}\right.$

$\Rightarrow y = xc - \frac{1}{2}e^{c}$ genel çözüm

2°) $x - \frac{1}{2}e^{p} = 0$

$2x = e^{p} \rightarrow p = \ln(2x)$

$\left\{\begin{array}{l} p = \ln(2x) \\ y = xp - \frac{1}{2}e^{p} \end{array}\right. \Rightarrow y = x\ln(2x) - \frac{1}{2}e^{\ln(2x)}$

$y = x\ln(2x) - x$ // Tekil çözüm

3) Bir özel çözümü $y_1(x) = \sin x$ olarak verilen

$2y'\cos x - y^2 = 2\cos^2 x - \sin^2 x$, $y(0) = 2$ başlangıç değer problemini çözünüz.

$y = y_1(x) + \frac{1}{u}$ dönüşümü yapılırsa

$y = \sin x + \frac{1}{u}$; $y' = \cos x - \frac{u'}{u^2}$ dır

Dif. denk. de yazılırsa NURAN GÜZEL

$2\left(\cos x - \frac{u'}{u^2}\right)\cos x - \left(\sin x + \frac{1}{u}\right)^2 = 2\cos^2 x - \sin^2 x$

$\cancel{2\cos^2 x} - \frac{2u'}{u^2}\cos x \cancel{- \sin^2 x} - \frac{2}{u}\sin x - \frac{1}{u^2} = \cancel{2\cos^2 x} \cancel{- \sin^2 x}$

$u' + \tan x\, u = -\frac{1}{2\cos x}$ Lineer dif. denk. $p(x) = \tan x$

$q(x) = -\frac{1}{2\cos x}$

İntegral çarpanı : $\lambda(x) = e^{\int p(x)dx}$

$\lambda(x) = e^{\int \tan x dx} = e^{-\ln \cos x} = \frac{1}{\cos x}$

$u(x) = \frac{1}{\lambda(x)}\left[\int q(x).\lambda(x)\,dx + C\right]$

$u(x) = \cos x\left[\int -\frac{1}{2\cos x}\cdot\frac{1}{\cos x}dx + C\right] = \cos x\left[-\frac{1}{2}\int \sec^2 x dx + C\right]$

$u(x) = \cos x\left[-\frac{1}{2}\tan x + C\right] \rightarrow u(x) = -\frac{1}{2}\sin x + C\cos x$

$y(x) = \sin x + \frac{1}{u} = \sin x + \frac{1}{-\frac{1}{2}\sin x + C\cos x}$ genel çözüm

$y(0) = 2$ den

$2 = \sin 0 + \frac{1}{-\frac{1}{2}\sin 0 + C.\cos 0} \Rightarrow 2 = \frac{1}{C} \Rightarrow C = \frac{1}{2}$ bulunur

O halde $y(x) = \sin x + \frac{2}{\cos x - \sin x}$ dır.

1. $y' = 1 + t^2 - 2ty + y^2$ diferansiyel denkleminin bir özel çözümü $y_1(t) = t$ verildiğine göre genel çözümünü bulunuz.

$$y' = 1 + t^2 - 2ty + y^2 \qquad y_1(t) = t$$

NURAN GÜZEL

$$y = y_1(t) + \frac{1}{u} \Rightarrow y = t + \frac{1}{u}$$

$$y' = 1 - \frac{u'}{u^2}$$ olur. Denklemde yerine yazarsa

$$1 - \frac{u'}{u^2} = 1 + t^2 - 2t\left(t + \frac{1}{u}\right) + \left(t + \frac{1}{u}\right)^2$$

$$= 1 + t^2 - 2t^2 - \frac{2\cancel{t}}{\cancel{u}} + t^2 + \frac{2\cancel{t}}{u} + \frac{1}{u^2}$$

$$\cancel{1} - \frac{u'}{u^2} = \cancel{1} + \frac{1}{u^2} \Rightarrow u' = -1$$

$$u = -t + c$$ bulunur.

$$y = t + \frac{1}{c - t}$$ elde edilir.

4) $y=(1+y')x+(y')^2$ diferansiyel denkleminin genel çözümünü bulunuz.

10

$\boxed{y'=p}$ 2 $y=(1+p)x+p^2$ $\boxed{y'=p'x+(1+p)+2pp'}$ 2

$\cancel{p}=p'x+1+\cancel{p}+2pp'$ $\boxed{p'(x+2p)+1=0}$ 1

NURAN GÜZEL

$\frac{dp}{dx}(x+2p)=-1$ $\frac{dx}{dp}=-(x+2p)$ 2 3 $\boxed{\frac{dx}{dp}+x=-2p}$ Lineer dif denk.

$\frac{dx}{dp}+x=0 \Rightarrow \boxed{\int\frac{dx}{x}+\int dp=0}$ 2 $\Rightarrow \ln x+p=\ln c$ $\boxed{x=ce^{-p}}$ 2

$x'=c'e^{-p}-ce^{-p}$ denk. yeri yaz. $c'e^{-p}\cancel{-ce^{-p}}+\cancel{ce^{-p}}=-2p$

$\boxed{c'=-2pe^{p}}$ 2 $c=-2\int pe^{p}\,dp$ 1 $p=u$ $dp=du$ $e^{p}dp=dv$ $v=e^{p}$

$c=-2\left[pe^{p}-\int e^{p}dp\right]+k$ 1 $\boxed{c=-2pe^{p}+2e^{p}+k}$ 2

$x=ce^{-p}$

$\boxed{x=-2p+2+ke^{-p}}$ 2

3 $x=-2p+2+ke^{-p}$

$y=(1+p)x+p^2$

3) $\left(y+e^{y-x}\right)dx+\left(1+xe^{y-x}\right)dy=0$ diferansiyel denkleminin genel çözümünü bulunuz.

$$\underbrace{\left(y+e^{y-x}\right)}_{P(x,y)}dx+\underbrace{\left(1+xe^{y-x}\right)}_{Q(x,y)}dy=0$$

$$\frac{\partial P}{\partial y}=1+e^{y-x} \qquad \frac{\partial Q}{\partial x}=e^{y-x}-xe^{y-x} \qquad \frac{\partial P}{\partial y}\neq\frac{\partial Q}{\partial x}$$ (3)

(10)

$$\ln\lambda=\int\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}dx=\int\frac{1+\cancel{e^{y-x}}-\cancel{e^{y-x}}+xe^{y-x}}{1+xe^{y-x}}dx=\int dx=x$$ (3) (2)

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$$\ln\lambda=x \qquad \boxed{\lambda=e^x}$$ (2)

$$\boxed{\left(\underbrace{ye^x+e^y}_{M(x,y)}\right)dx+\left(\underbrace{e^x+xe^y}_{N(x,y)}\right)dy=0}$$ (3) Tam. dif. denk

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=e^x+e^y$$

$$(ye^x+e^y)dx+(e^x+xe^y)dy=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy$$

$$\frac{\partial u}{\partial x}=ye^x+e^y \qquad \frac{\partial u}{\partial y}=e^x+xe^y$$ (2)

(15)

$$u(x,y)=\int(ye^x+e^y)dx+h(y)=\underline{ye^x+xe^y+h(y)}$$ (3)

$$\frac{\partial u}{\partial y}=e^x+xe^y+h'(y)=e^x+xe^y \qquad h'(y)=0 \qquad \boxed{h(y)=k}$$ (2) (1) (1)

$$\boxed{u(x,y)=ye^x+xe^y+k}$$ (2) $u(x,y)=c$

$$ye^x+xe^y+k=c \qquad c-k=C \qquad \boxed{ye^x+xe^y=C}$$ (1)

4) $y=(1+y')x+(y')^2$ diferansiyel denkleminin genel çözümünü bulunuz.

10

$\boxed{y'=p}$ 2 $y=(1+p).x+p^2$ $\boxed{y'=p'x+(1+p)+2pp'}$ 2

$\cancel{p}=p'x+1+\cancel{p}+2pp'$ $\boxed{p'(x+2p)+1=0}$ 1

$\frac{dp}{dx}(x+2p)=-1$ $\frac{dx}{dp}=-(x+2p)$ 2 $\boxed{\frac{dx}{dp}+x=-2p}$ 3 Lineer dif denk

NURAN GÜZEL

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$\frac{dx}{dp}+x=0 \Rightarrow \boxed{\int\frac{dx}{x}+\int dp=0}$ 2 $\Rightarrow \ln x+p=\ln c$ $\boxed{x=ce^{-p}}$ 2

$x'=c'e^{-p}-ce^{-p}$ denk. yer: yaz. $c'e^{-p}\cancel{-ce^{-p}}+\cancel{ce^{-p}}=-2p$

$\boxed{c'=-2pe^{p}}$ 2 $c=-2\int pe^{p}\,dp$ 1 $p=u$ $e^{p}dp=dv$ $dp=du$ $v=e^{p}$

$c=-2\left[pe^{p}-\int e^{p}dp\right]+k$ 1 $\boxed{c=-2pe^{p}+2e^{p}+K}$ 2

$x=ce^{-p}$

$\boxed{x=-2p+2+Ke^{-p}}$ 2

3 $x=-2p+2+Ke^{-p}$

$y=(1+p)x+p^2$

Lecturer		Signature	

YÖK nun 2547 sayılı Kanunun ***Öğrenci Disiplin Yönetmeliğinin*** 9. Maddesi olan ***"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"*** fiili işleyenler **bir veya iki yarıyıl uzaklaştırma** cezası alırlar.

1-) Find the general solution of the differential equation $(1-\sqrt{3})y' + ySecx = y^{\sqrt{3}}Secx$

$y^{-\sqrt{3}}$ / $(1-\sqrt{3})y' + y.Secx = y^{\sqrt{3}}.Secx$ Bernoulli d.d

$(1-\sqrt{3})y^{-\sqrt{3}}.y' + y^{1-\sqrt{3}}.Secx = Secx$

$y^{1-\sqrt{3}} = u$

NURAN GÜZEL

$(1-\sqrt{3})y^{-\sqrt{3}}.y' = u'$

$u' + Secx.u = Secx$ Lineer d.d

$\lambda = e^{\int secx dx} = Secx + tanx$

$(Secx + tanx)u' + (Sec^2x + tanx Secx)u = Sec^2x + Secx tanx$

$\frac{d}{dx}((Secx + tanx).u) = Sec^2x + Secx tanx$

$\int d((Secx + tanx)u) = \int (Sec^2x + Secx.tanx)dx$

$(Secx + tanx).u = tanx + Secx + C$

$u = 1 + \frac{c}{Secx + tanx} = y^{1-\sqrt{3}}$

Good Luck

) Show that the differential equation $y^2dx+(3xy-e^y)dy=0$ is not exact. Solve the given equation by inding an integrating factor.

$$\underbrace{y^2}_{P}dx+\underbrace{(3xy-e^y)}_{Q}dy=0$$

$$\frac{\partial P}{\partial y}=2y \neq \frac{\partial Q}{\partial x}=3y \quad \text{d.d. tam değil}$$

$$\lambda(y)=e^{\int\frac{Q_x-P_y}{P}dy}=e^{\int\frac{3y-2y}{y^2}dy}=e^{\int\frac{dy}{y}}=y$$

NURAN GÜZEL

$$y^3\,dx+(3xy^2-ye^y)dy=0$$

$$\left.\begin{aligned}\frac{\partial u}{\partial x}&=y^3\\ \frac{\partial u}{\partial y}&=3xy^2-y\cdot e^y\end{aligned}\right\} \qquad du(x,y)=0 \qquad u(x,y)=C \quad \text{G.G}$$

$$\frac{\partial u}{\partial x}=y^3 \rightarrow u(x,y)=\int y^3dx+H(y) \Rightarrow u(x,y)=y^3x+H(y)$$

$$\frac{\partial u}{\partial y}=3y^2x+H'(y)=3xy^2-y\cdot e^y \rightarrow H'(y)=-y\cdot e^y$$

$$H(y)=-\int ye^ydy=-\left[y\cdot e^y-e^y\right]+k \qquad H(y)=-ye^y+e^y+k$$

$$u(x,y)=y^3x-ye^y+e^y+k=c$$

$$y^3x-ye^y+e^y=C$$

2) a) Find the differential equation of the family of curves $c_1(x+2)+c_2(x+2)\ln(x+2)=y \quad c_1,c_2 \in R$

$$c_1(x+2)+c_2(x+2)\ln(x+2)=y$$

$$(x+2)\,/ \quad c_1 + c_2 \ln(x+2) + c_2 (x+2)\frac{1}{(x+2)} = y' \quad 4$$

$$(x+2)^2\,/ \quad c_2 \frac{1}{x+2} = y'' \quad 4$$

$$\left.\begin{aligned} c_1(x+2)+c_2(x+2)\ln(x+2) &= y \\ c_1(x+2)+c_2(x+2)\ln(x+2)+c_2(x+2) &= (x+2)y' \\ c_2(x+2) &= (x+2)^2 y'' \end{aligned}\right\} \quad 4$$

NURAN GÜZEL

$$y+(x+2)^2y''=(x+2)y'$$

$$(x+2)^2y''-(x+2)y'+y=0$$

b) Find the general solution of the differential equation $y(x+3)dx+(x+2)(ydx-xdy)=0$.

$$y(x+3)dx+(x+2)y\,dx-x(x+2)dy=0$$

$$y(2x+5)\,dx = x(x+2)dy$$

$$\int\frac{(2x+5)\,dx}{x(x+2)}=\int\frac{dy}{y} \quad 7 \quad -1$$

$$\frac{A}{x}+\frac{B}{x+2}=\frac{2x+5}{x(x+2)}$$

$$A+B=2$$

$$2A=5$$

$$B=2-\frac{5}{2}=\frac{-1}{2}$$

$$\int\left[\frac{5/2}{x}+\frac{-1/2}{x+2}\right]dx=\ln|y|+\ln|c|$$

2 2 1 1

$$\frac{5}{2}\ln|x|-\frac{1}{2}\ln|x+2|=\ln|y|+\ln|c| \quad 6$$

$$\ln\sqrt{\frac{x^5}{x+2}}-\ln|y|=\ln|c|$$

$$\frac{x^5}{y^2\cdot(x+2)}=k$$

$$yx - \frac{y^2}{y'} - x^2 y' + xy = -2$$

4) Consider the differential equation $\left(y - xy'\right)\left(x - \frac{y}{y'}\right) = -2$.

$$2xy - \frac{y^2}{y'} - x^2 y' = -2$$

Find the general solution of this equation and the singular solution of the this equation in cartesian form. (Here $y' > 0$)

$$2xy \cdot y' - y^2 - x^2 y'^2 = -2y'$$

$$(y - xy')(xy' - y) = -2y'$$

$$(y - xy')^2 = 2y'$$

NURAN GÜZEL

$$y - xy' = \pm\sqrt{2}\sqrt{y'} \qquad y = xy' \pm \sqrt{2}\sqrt{y'}$$

Clairaut d.d.

$$y' = p \qquad y = xp \pm \sqrt{2}\sqrt{p}$$

$$p = p + xp' \pm \frac{\sqrt{2}}{2\sqrt{p}} p'$$

$$0 = p'\left(x \pm \frac{\sqrt{2}}{2\sqrt{p}}\right)$$

$$p' = 0 \Rightarrow p = c \Rightarrow y = xc \pm \sqrt{2}\sqrt{c} \qquad \text{G4}$$

$$x \pm \frac{\sqrt{2}}{2\sqrt{p}} = 0 \Rightarrow x = \mp \frac{\sqrt{2}}{2\sqrt{p}}$$

$$y = \mp \frac{\sqrt{2}}{2\sqrt{p}} \cdot p \pm \sqrt{2}\sqrt{p} = \pm \frac{\sqrt{2}}{2}\sqrt{p}$$

$$xy = -\frac{1}{2} \qquad \text{T.4}$$

B-grubu

2) $6(y+1)dx-(x+y+4)dy=$ [...]nsiyel denkleminin genel çözümünü bulunuz.

$\begin{vmatrix} 0 & 6 \\ -1 & -1 \end{vmatrix} = 6 \neq 0$ $\quad x = x_0 + h \quad dx = dx_0$ $\quad y = y_0 + k \quad dy = dy_0$

$6(y_0 + \underbrace{k+1}_{0})dx_0 - (x_0 + y_0 + \underbrace{h+k+4}_{0})dy_0 = 0$

$k+1=0 \Rightarrow k=-1 \qquad h-1+4=0 \quad h=-3$

veya $\quad 6y+6=0 \quad x+y+4=0$

$y=1 \qquad x=-3$

$x = x_0 - 3 \qquad y = y_0 - 1$ dön. uygulanır.

NURAN GÜZEL

$6y_0 dx_0 - (x_0+y_0)dy_0 = 0$ Hom. D. D.

$6\frac{y_0}{x_0}dx_0 - \left(1+\frac{y_0}{x_0}\right)dy_0 = 0 \qquad \frac{y_0}{x_0} = u \qquad y_0 = ux_0$

$dy_0 = u dx_0 + x_0 du$

$6u\, dx_0 - (1+u)(u dx_0 + x_0 du) = 0$

$6u\, dx_0 - u dx_0 - x_0 du - u^2 dx_0 - u x_0 du = 0$

$(6u - u - u^2)dx_0 + (-x_0 - u x_0)du = 0$

$(5u-u^2)dx_0 - x_0(1+u)du = 0$ Değ. Ay. D.D.

$\int \frac{dx_0}{x_0} - \int \frac{1+u}{5u-u^2}du = \int 0$

$\frac{1+u}{u(5-u)} \equiv \frac{A}{u} + \frac{B}{5-u}$

$1+u \equiv 5A - Au + Bu = 5A + (B-A)$

$5A = 1 \rightarrow A = \frac{1}{5} \qquad B - A = 1$

$B = \frac{6}{5}$

$\ln x_0 - \int \left(\frac{1}{5u} + \frac{6}{5(5-u)}\right)du = \ln c$

$\ln x_0 - \frac{1}{5}\ln u + \frac{6}{5}\ln(5-u) = \ln c$

$\frac{x_0 (5-u)^{6/5}}{u^{1/5}} = c \rightarrow x_0 = c \cdot u^{1/5} \cdot (5-u)^{-6/5} \qquad x_0 = x+3,\ y_0 = y+1$

$u = \frac{y_0}{x_0}$

$x = c \cdot \left(\frac{y+1}{x+3}\right)^{1/5} \cdot \left(5 - \frac{y+1}{x+3}\right)^{-6/5} - 3$ //

Başarılar...

⑥ $(y-xy')^2 = 1+y'^2$

Çöz

$y - xy' = \sqrt{1+y'^2}$

$y = xy' + \sqrt{1+y'^2}$ Clairaut. / (Boşverdik)

NURAN GÜZEL

⑦

$y = (2+y')x + y'^2$ $\quad y = x f(y') + g(y')$ Lagrange.

$y' = p$

$y = (2+p)x + p^2$

$y' = \cancel{p} = p'x + (2+\cancel{p}) + 2pp'$

$p'[x+2p] = -2$

$\frac{dp}{dx} = \frac{-2}{x+2p}$

$\frac{dx}{dp} = \frac{x+2p}{-2}$

$\frac{dx}{dp} + \frac{x}{2} = -p$ L.D.D.

$\int \frac{dx}{x} + \int \frac{dp}{2} = \int 0$

$\ln x + \frac{p}{2} = \ln c$

$x = ce^{-p/2}$

$c = c(p)$

$\frac{dx}{dp} = c'e^{-p/2} - \frac{c}{2}e^{-p/2}$

$c'e^{-p/2} \cancel{- \frac{c}{2}e^{-p/2}} + \cancel{\frac{c}{2}e^{-p/2}} = -p$

$c' = -pe^{+p/2}$

$c = -\int pe^{+p/2}\,dp$

$p = u \qquad e^{+p/2}dp = dv$

$dp = du \qquad +2e^{+p/2} = v$

$c = -[+2pe^{+p/2} - 2\int e^{+p/2}dp]$

$c = -2pe^{+p/2} + 4e^{+p/2} + k$

$$\begin{cases} x = -2p + 4 + ke^{-p/2} \\ y = 8 - p^2 + (2+p)ke^{-p/2} \end{cases}$$

c) $y = xy' + \cos y'$ ~~Clairaut~~

$y' = p \Rightarrow y = xp + \cos p$

$y' = \cancel{p} = \cancel{p} + xp' + p'(-\sin p)$

$p'[x - \sin p] = 0$

1°) $p' = 0 \Rightarrow p = c \Rightarrow \boxed{y = xc + \cos c}$ G.Ç.

2°) $x = \sin p \Longrightarrow p = \arcsin x$

NURAN GÜZEL

$y = p\sin p + \cos p$

$y = x\arcsin x + \cos(\arcsin x)$

$y = x\arcsin x + \sqrt{1-x^2}$

) $x^2 - cy^2 = 1$ eğri ailesinin dif. denk kurunuz.

i) $x^2 - cy^2 = 1 \Rightarrow c = \dfrac{x^2-1}{y^2}$

$2x - 2cyy' = 0$

$2x - 2\cdot\dfrac{x^2-1}{y^2}\, \cancel{y}\, y' = 0$

$\boxed{2xy - 2(x^2-1)y' = 0}$

) $(2x-y-7)y' = -(x+2y-1)$ dif. denk. genel çözümünü bulunuz.

Homojen hale getirilebilen dif. denk.

$\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1-4 = -5 \neq 0$

$x = X + h \Rightarrow dx = dX$

$y = Y + k \Rightarrow dy = dY$

Ör. $y = c_1x + c_2(x^2-1) + x^2$ (1) eğri ailesinin dif. d

$y' = c_1 + 2xc_2 + 2x$ (2)

$y'' = 2c_2 + 2$ (3)

$c_2 = \dfrac{y''-2}{2}$

(2) $\rightarrow$ $xy' = c_1x + 2x^2c_2 + 2x^2$

(1) $\quad -y = -c_1x - c_2(x^2-1) - x^2$

$xy' - y = c_2(x^2-1) + x^2$

$xy' - y = (x^2-1)\left[\dfrac{y''-2}{2}\right] + x^2$

$2xy' - 2y = x^2y'' - 2x^2 - y'' + 2 + x^2$

$(x^2-1)y'' - 2xy' + 2y - x^2 + 2 = 0$

4) $\frac{dy}{dx}=\frac{y^2-x^2y-2x}{1-x^3}$ $y_1=-x^2$

$y'=\frac{1}{1-x^3}y^2-\frac{x^2}{1-x^3}y-\frac{2x}{1-x^3}$ Riccati.

$y=-x^2+\frac{1}{u}$, $y'=-2x-\frac{u'}{u^2}$ NURAN GÜZEL

$-2x-\frac{u'}{u^2}=\frac{1}{1-x^3}\left(-x^2+\frac{1}{u}\right)^2-\frac{x^2}{1-x^3}\left(-x^2+\frac{1}{u}\right)-\frac{2x}{1-x^3}$

5

$-2x-\frac{u'}{u^2}=\frac{x^4}{1-x^3}-\frac{2x^2}{(1-x^3)u}+\frac{1}{(1-x^3)u^2}+\frac{x^4}{1-x^3}-\frac{x^2}{(1-x^3)u}-\frac{2x}{1-x^3}$

$-2x(1-x^3)-(1-x^3)\frac{u'}{u^2}=2x^4-3\frac{x^2}{u}+\frac{1}{u^2}-2x$

$\cancel{-2x}+2x^4-(1-x^3)\frac{u'}{u^2}=2x^4-\frac{3x^2}{u}+\frac{1}{u^2}\cancel{-2x}$

$(x^3-1)u'=-3x^2u+1$ 5

$u'+\frac{3x^2}{x^3-1}u=\frac{1}{x^3-1}$ linee

$u=e^{-\int\frac{3x^2}{x^3-1}dx}\left[\int\frac{1}{x^3-1}e^{\int\frac{3x^2}{x^3-1}dx}dx+c\right]$

$u=e^{-\ln(x^3-1)}\left[\int\frac{1}{x^3-1}e^{\ln(x^3-1)}dx+c\right]$ 5

$u=\frac{1}{x^3-1}\left[\int\frac{x^3-1}{x^3-1}dx+c\right]$ 5

$u=\frac{1}{x^3-1}\left[x+c\right]=\frac{x}{x^3-1}+\frac{c}{x^3-1}$

$y=-x^2+\left(\frac{x+c}{x^3-1}\right)^{-1}=-x^2+\frac{x^3-1}{x+c}$ 5

10 Tekil Çözüm

3) $\frac{dy}{dx} = -y^2 + 2x^2y + 2x - x^4$ denkleminin bir özel çözümü $y_1 = x^2$ olduğuna göre genel çözümünü bulunuz.

$y = x^2 + \frac{1}{u}$ dönüşümü uygulanırsa, $y' = 2x - \frac{u'}{u^2}$ elde edili

$\Rightarrow 2x - \frac{u'}{u^2} = -\left(x^2 + \frac{1}{u}\right)^2 + 2x^2\left(x^2 + \frac{1}{u}\right) + 2x - x^4$

$\Rightarrow \cancel{2x} - \frac{u'}{u^2} = -\cancel{x^4} - \frac{1}{u^2} - \cancel{\frac{2x^2}{u}} + \cancel{2x^4} + \cancel{\frac{2x^2}{u}} + \cancel{2x} - \cancel{x^4}$

$\Rightarrow \boxed{u' = 1} \Rightarrow du = dx \Rightarrow \boxed{u = x + c}$

$$\boxed{\Rightarrow y = x^2 + \frac{1}{x+c}}$$

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4) $(x+1)y' - y = e^x (x+1)^2$ dif. denk. nin genel çözümünü bulunu

(lin. dif. denk.)

$y = u.v \Rightarrow y' = u'v + uv' \Rightarrow (x+1)(u'v + uv') - uv = e^x (x+1)^2$

$\Rightarrow xu'v + xuv' + u'v + uv' - uv = e^x (x+1)^2 \Rightarrow \boxed{v(xu' + u' - u) + v'(xu + u) = e^x (x+1}$

$\Rightarrow u'(x+1) - u = 0 \Rightarrow \frac{du}{u} - \frac{dx}{x+1} = 0 \Rightarrow \ln u = \ln(x+1) \Rightarrow \boxed{u = x+1}$

$\Rightarrow v'(x(x+1) + x + 1) = e^x (x+1)^2 \Rightarrow v'\cancel{(x+1)^2} = e^x \cancel{(x+1)^2} \Rightarrow \boxed{v = e^x + c}$

$\Rightarrow \boxed{y = (x+1)(e^x + c) = xe^x + e^x + cx + c}$

5) $y = xy' + \cos y'$ dif. denk.'in çözümlerini bulunuz.

(genel ve tekil)

(clairaut dif. denk.)

$y' = p \Rightarrow y = xp + \cos p$

$\Rightarrow y' = \cancel{p} + p'x - \sin p \cdot p' = \cancel{p}$

$\Rightarrow p'(x - \sin p) = 0$

a) $p' = 0 \Rightarrow p = c$ $\boxed{y = xc + \cos c}$ Genel Çözüm

b) $x - \sin p = 0 \Rightarrow x = \sin p \Rightarrow p =$ ~~$y = p\sin p + \cos p$~~ $\Rightarrow p = \arcsin x$

$y = xp + \cos p$

$y = x \arcsin x + \cos(\arcsin x)$

$\Rightarrow y = (\arcsin x)(\sin(\arcsin x)) + \cos(\arcsin x) \Rightarrow \boxed{y = x\arcsin x + \sqrt{1-x^2}}$

tekil çözüm

Tekil Çözüm

$2xy^2y' = x^2 + y^3$ dif. denk.nin genel çözümünü bulunuz.

$\Rightarrow y' - \frac{1}{2x} y = \frac{1}{2} x y^{-2}$ Bernoulli dif. denk.

$n = -2$

$\boxed{z = y^3} \Rightarrow z' = 3y^2y' \Rightarrow y^2y' = \frac{z'}{3}$

$\frac{z'}{3} - \frac{z}{2x} = \frac{1}{2}x \Rightarrow \boxed{z' - \frac{3z}{2x} = \frac{3x}{2}}$ lin.dif.den

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$P(x) = -\frac{3}{2x}$ ve $Q(x) = \frac{3}{2}x$

$z = e^{-\int P(x)dx} \cdot \left[\int Q(x) . e^{\int P(x)dx} dx + k \right]$

$\Rightarrow z = e^{\frac{3}{2}\int \frac{dx}{x}} \cdot \left[\frac{3}{2} \int x . e^{-\frac{3}{2}\int \frac{dx}{x}} dx + k \right]$

$= x^{3/2} \cdot \left[\frac{3}{2} \int x^{1-3/2} dx + k \right]$

$= x^{3/2} \cdot \left[\frac{3}{\cancel{2}} \cdot \cancel{2}\sqrt{x} + k \right] = 3x^2 + kx^{3/2}$

$z = y^3 \Rightarrow \boxed{y^3 = 3x^2 + kx^{3/2}}$ Genel çözüm

4) 1) $y^3y' + \frac{1}{x}y^4 = \frac{\sin x}{x^4} \Rightarrow y' + \frac{1}{x}y = \frac{\sin x}{x^4}y^{-3}$ Bernoulli

$$\left.\begin{aligned} y^4 = u \rightarrow 4y^3y' = u' \\ y^3y' = \frac{u'}{4} \end{aligned}\right\} \Rightarrow \frac{u'}{4} + \frac{u}{x} = \frac{\sin x}{x^4}$$

$$u' + 4\frac{u}{x} = 4\frac{\sin x}{x^4} \quad \text{LDD}$$

$$\left.\begin{aligned} P(x) = \frac{4}{x} \\ q(x) = \frac{4\sin x}{x^4} \end{aligned}\right\} \Rightarrow \lambda = e^{\int \frac{4}{x}dx} \Rightarrow \lambda = x^4$$

$$u = \frac{1}{x}\left[\int q(x)\,\lambda\,dx + c\right]$$

$$u = \frac{1}{x^4}\left[\int \frac{4\sin x}{x^4}\,x^4 dx + c\right]$$

$$u = \frac{1}{x^4}\left(-4\cos x + c\right)$$

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$(2\sqrt{xy} - y)\,dx + x\,dy = 0$ Homojen dif. denk.

$\frac{y}{x} = u \Rightarrow y = ux \Rightarrow dy = u\,dx + x\,du$

$(2\sqrt{x^2u} - ux)\,dx + x(u\,dx + x\,du) = 0$

$(2\sqrt{x^2u} \cancel{-ux} \cancel{+ux})\,dx + x^2du = 0$

$$\frac{2x\sqrt{u}\,dx}{x^2\sqrt{u}} + \frac{x^2du}{x^2\sqrt{u}} = \frac{0}{x^2\sqrt{u}} \quad \begin{pmatrix} x \neq 0 \\ u \neq 0 \end{pmatrix}$$

$$\int \frac{2dx}{x} + \int \frac{du}{\sqrt{u}} = \int 0$$

$$2\ln x + 2\sqrt{u} = c$$

$$2\ln x + 2\sqrt{\frac{y}{x}} = c$$

4) $(x - y\ln y + y\ln x)dx + x(\ln y - \ln x)dy = 0$

2. yol: $\underbrace{(x - y\ln y + y\ln x)}_{M(x,y)}dx + \underbrace{x(\ln y - \ln x)}_{N(x,y)}dy = 0$

$\frac{\partial M}{\partial y} = -\ln y - 1 + \ln x$ ② $\neq$ ① $\frac{\partial N}{\partial x} = \ln y - \ln x - 1$ ②

$\ln\lambda = \int \frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{N}dx$ ① $= \int \frac{(-\ln y - \cancel{1} + \ln x - \ln y + \ln x + \cancel{1})dx}{x(\ln y - \ln x)}$ ②

$\ln\lambda = \int -\frac{2}{x}dx \Rightarrow \boxed{\lambda = \frac{1}{x^2}}$ ②

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$\underbrace{\left(\frac{1}{x} - \frac{y}{x^2}\ln y + \frac{y}{x^2}\ln x\right)}_{M_1(x,y)}dx + \underbrace{\frac{1}{x}(\ln y - \ln x)}_{M_2(x,y)}dy = 0$

$\left(\frac{\partial M_1}{\partial y} = -\frac{1}{x^2}\ln y - \frac{1}{x^2} + \frac{1}{x^2}\ln x\right)$ ② $= \left(\frac{\partial M_2}{\partial x} = -\frac{1}{x^2}\ln y + \frac{1}{x^2}\ln x - \frac{1}{x^2}\right)$ ② Tam dif $\Rightarrow$

$u = u(x,y)$ fonksiyonu vardır ki;

$\frac{\partial u}{\partial x} = \frac{1}{x} - \frac{y}{x^2}\ln y + \frac{y}{x^2}\ln x$ ① ve $\frac{\partial u}{\partial y} = \frac{1}{x}(\ln y - \ln x)$ ①

$u(x,y) = \frac{1}{x}(y\ln y - y - y\ln x) + h(x)$ ②

$\frac{\partial u}{\partial x} = -\frac{1}{x^2}(y\ln y - y - y\ln x) + \frac{1}{x}\left(-\frac{y}{x}\right) + \frac{dh}{dx} = -\frac{y}{x^2}\ln y + \frac{y}{x^2}\ln x + \frac{dh}{dx}$ ④

$\cancel{-\frac{y}{x^2}\ln y} + \cancel{\frac{y}{x^2}\ln x} + \frac{dh}{dx} = \frac{1}{x} - \cancel{\frac{y}{x^2}\ln y} + \cancel{\frac{y}{x^2}\ln x}$ ①

$\frac{dh}{dx} = \frac{1}{x} \Rightarrow h(x) = \ln x + c_1$ ①

$u(x,y) = \frac{y}{x}(\ln y - 1 - \ln x) + \ln x + c_1 = k$

① $\frac{y}{x}(\ln y - 1 - \ln x) + \ln x = \underbrace{k - c_1}_{c}$

A GRUBU

Soru 1: C_1, C_2, C_3 birer parametre olmak üzere $y=: C_1e^{-x}+ C_2Sinx+ C_3Cosx$ fonksiyonu ile verilen eğri ailesinin diferansiyel denklemini bulunuz.

$$y = C_1 e^{-x} + C_2 \sin x + C_3 \cos x$$
$$y' = -C_1 e^{-x} + C_2 \cos x - C_3 \sin x$$
$$y'' = C_1 e^{-x} - C_2 \sin x - C_3 \cos x$$
$$y''' = -C_1 e^{-x} - C_2 \cos x + C_3 \sin x$$

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$$y''' + y'' + y' + y = 0$$

$$\left[y.(1+e^{x/y})dx + (y-x).e^{x/y}.dy = 0 \right]$$

$$1 + 1e^{x/y} - \frac{x}{y^2} \cdot e^{x/y} \cdot y$$

$$\frac{\partial M}{\partial y} = e^{x/y} - \frac{x}{y} e^{x/y} \qquad \frac{\partial N}{\partial x} = -1e^{x/y} - \frac{x}{y^2} e^{\frac{x}{y}} (y-x) + e^{x/y} + \frac{x}{y} e^{x/y} - \frac{x^2}{y^2} e^{x/y}$$

$$dy = \sqrt{2x+y+1}\, dx \qquad 2x+y+1=u \qquad 2 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{dy}{dx} = \sqrt{2x+y+1}$$

$$\frac{du}{dx} - 2 = \sqrt{u}$$

$$\frac{du}{dx} = \sqrt{u} + 2 \qquad u = t^2$$

$$\int \frac{du}{\sqrt{u}+2} = \int dx$$

$$\int \frac{2t\,dt}{t+2} = x + C$$

$$2\int \frac{t+2-2}{t+2}\,dt = x + C$$

$$2\int \left[1 - \frac{2}{t+2}\right] = x + c$$

$$2\left(t - 2\ln(t+2)\right) = x + C$$

$$2.\sqrt{u} - 4\ln(\sqrt{u}+2) = x + C$$

$$2\sqrt{2x+y+1} - 4 \ldots$$

$$(y + ye^{x/y})dx = (x-y)e^{x/y}dy$$

$$\frac{dx}{dy} = \frac{(x-y)e^{x/y}}{y(1+e^{x/y})}$$

$$\frac{dx}{dy} = \frac{(\frac{x}{y}-1)e^{x/y}}{1+e^{x/y}}$$

$$\frac{x}{y} = u \qquad x = y.u \qquad \frac{dx}{dy} = 1.u + yu'$$

$$u + yu' = \frac{(u-1)e^u}{1+e^u} - u$$

$$yu' = \frac{ue^u - e^u - u - ue^u}{1+e^u}$$

$$-\frac{1+e^u}{u+e^u} = \frac{dy}{y}$$

$$-\ln(u+e^u) = \ln y + \ln c$$

$$-\ln\left(\frac{x}{y} + e^{x/y}\right) = y.c$$

Soru 2: $\frac{dy}{dx}=\frac{3x^2+2xy-y^2}{3y^2+2xy-x^2}$ diferansiyel denkleminin genel çözümünü bulunuz

$(3x^2+2xy-y^2)dx-(3y^2+2xy-x^2)dy=0$

Homojen d.d. dir.

$y=ux$ değişken dönüşümü uygulayalım.

$dy=xdu+udx$

$(3x^2+2ux^2-u^2x^2)dx-(3u^2x^2+2ux^2-x^2)(xdu+udx)=0$

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$(3+2u-u^2)dx-x(3u^2+2u-1)du-(3u^3+2u^2-u)dx=0$

$(-3u^3-3u^2+3u+3)dx-x(3u^2+2u-1)du=0$

$$\int\frac{dx}{x}+\int\frac{-3u^2-2u+1}{-3u^3-3u^2+3u+3}du=\int 0$$

$t=-3u^3-3u^2+3u+3$

$\frac{dt}{3}=(-3u^2-2u+1)du$

$$\ln|x|+\int\frac{dt/3}{t}=C$$

$$\ln|x|+\frac{1}{3}\ln|t|=C$$

$$\ln|x|+\frac{1}{3}\ln\left|-3\frac{y^3}{x^3}-3\frac{y^2}{x^2}+3\frac{y}{x}+3\right|=C$$

Soru 3: $x^2y + (y - x^3)y' = 0$ diferansiyel denklemini $\lambda = \lambda(y)$ şeklinde y'ye bağlı integrasyon çarpanı bularak çözünüz.

$$\underbrace{x^2y}_{M(x,y)}\,dx + \underbrace{(y - x^3)}_{N(x,y)}\,dy = 0 \qquad \frac{\partial M}{\partial y} = x^2 \qquad \frac{\partial N}{\partial x} = -3x^2$$

$$\ln\lambda = \int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\,dy = \int \frac{-3x^2 - x^2}{x^2 y}\,dy$$

$$\ln\lambda = \int -\frac{4}{y}\,dy = -4\ln y$$

$$\Rightarrow \lambda = \frac{1}{y^4}$$

$$\frac{1}{y^4}x^2y\,dx + \frac{1}{y^4}(y - x^3)\,dy = 0$$

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$$\underbrace{\frac{x^2}{y^3}}_{P}\,dx + \underbrace{\left(\frac{1}{y^3} - \frac{x^3}{y^4}\right)}_{Q}\,dy = 0$$

$$\left.\begin{aligned} \frac{\partial P}{\partial y} &= -\frac{3x^2}{y^4} \\ \frac{\partial Q}{\partial x} &= \frac{-3x^2}{y^4} \end{aligned}\right\} \text{Tam dif.}$$

$$du = \frac{\partial u}{\partial x}\,dx + \frac{\partial u}{\partial y}\,dy = P\,dx + Q\,dy = 0 \qquad \begin{aligned} du &= 0 \\ u &= K_1 \end{aligned}$$

$$\frac{\partial u}{\partial x} = \frac{x^2}{y^3} \qquad \int \partial u = \int \frac{x^2}{y^3}\,dx \qquad u = \frac{x^3}{3y^3} + R(y)$$

$$\frac{\partial u}{\partial y} = -\frac{x^3}{y^4} + R'(y)$$

$$\cancel{-\frac{x^3}{y^4}} + R'(y) = \frac{1}{y^3} - \cancel{\frac{x^3}{y^4}}$$

$$R'(y) = \frac{1}{y^3} \Rightarrow R(y) = -\frac{1}{2y^2} + K_2$$

$$u = \frac{x^3}{3y^3} - \frac{1}{2y^2} + K_2 = K_1 \Rightarrow \frac{x^3}{3y^3} - \frac{1}{2y^2} = C$$

1-) a) c_1, c_2 keyfi sabitler ve $x > 0$ olmak üzere, $y = c_1 x^{-1/2} + c_2 x^{-1}$ eğri ailesinin ait diferansiyel denklemi bulunuz. (13P)

$$y = c_1 x^{-1/2} + c_2 x^{-1}$$

$$y' = -\frac{1}{2} c_1 x^{-3/2} - c_2 x^{-2} \quad 4$$

$$y'' = \frac{3}{4} c_1 x^{-5/2} + 2c_2 x^{-3} \quad 4$$

$$5x \cdot y' = -\frac{5}{2} c_1 x^{-1/2} - 5c_2 x^{-1} \quad 2$$

$$2x^2 \cdot y'' = \frac{3}{2} c_1 x^{-1/2} + 4c_2 x^{-1} \quad 2$$

$+$

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$$5xy' + 2x^2 y'' = -c_1 x^{-1/2} - c_2 x^{-1} = -y \quad 1$$

$$5xy' + 2x^2 y'' + y = 0$$

b) Yüksek merteбeden sabit katsayılı bir lineer homojen diferansiyel denklemin karakteristik denkleminin kökleri $r_1 = i$, $r_2 = -i$, $r_3 = 2$, $r_4 = 2$, $r_5 = 3$ olsun. Buna göre bu diferansiyel denklemi ve $y(x)$ genel çözümünü yazınız. (12P)

2 $(r^2+1)(r-2)^2(r-3) = 0$

3 $r^5 - 7r^4 + 17r^3 - 19r^2 + 16r - 12 = 0$ karakteristik denklem

2 $y^{(5)} - 7y^{(4)} + 17y''' - 19y'' + 16y' - 12y = 0$ dif. denklem.

5 $y(x) = c_1 \cos x + c_2 \sin x + c_3 e^{2x} + c_4 x e^{2x} + c_5 e^{3x}$ genel çözüm

Başarılar dilerim.

Başarılar...

3) $xy' = 3xy^{4/3} - 6y$ diferansiyel denkleminin genel çözümünü bulunuz. (25P)

(2) $y' + \frac{6}{x} y = 3y^{4/3}$ Bernoulli D.D.

$n = \frac{4}{3}$, $z = y^{1-n} = y^{-1/3}$ (2) $\Rightarrow z' = -\frac{1}{3} \cdot y' \cdot y^{-4/3}$ (2)

$y' \cdot y^{-4/3} + \frac{6}{x} \cdot y^{-1/3} = 3$ (2)

$-3z' + \frac{6}{x} \cdot z = 3$ (2)

$z' - \frac{2}{x} z = -1$ (3) Lineer D.D.

1. yol:

$\lambda(x) = e^{-\int \frac{2}{x} dx}$ (2) $= e^{-2\ln x} = \frac{1}{x^2}$ (2)

$z = \frac{1}{\lambda} \left[\int \lambda . Q(x) dx + c \right]$

$z = x^2 \left[\int \frac{1}{x^2} \cdot (-1) dx + c \right]$ (3)

$z = x^2 \left(\frac{1}{x} + c \right) = x + cx^2$ (3)

2. yol:

$\lambda(x) = e^{-\int \frac{2}{x} dx}$ (2) $= e^{-2\ln x} = \frac{1}{x^2}$ (2)

$\frac{1}{x^2} z' - \frac{2}{x^3} z = -\frac{1}{x^2}$

$\frac{d}{dx} \left[z \cdot \frac{1}{x^2} \right] = -\frac{1}{x^2}$ (3)

$\frac{z}{x^2} = -\int \frac{1}{x^2} dx + c$

$z = x^2 \left(\frac{1}{x} + c \right) = x + cx^2$ (3)

$z = x + cx^2$, $z = y^{-1/3}$

$y^{-1/3} = x + cx^2 \Rightarrow y = \frac{1}{(x + cx^2)^3}$ (2)

2) $(y\tan y-2y)dx+(xy\sec^2 y+1)dy=0,\ y(0)=1$ şeklinde verilen başlangıç değer problemini çözünüz. (25P)

$$\underbrace{(y\tan y-2y)}_{M(x,y)}dx+\underbrace{(xy\sec^2 y+1)}_{N(x,y)}dy=0$$

$$\frac{\partial M}{\partial y}=\tan y+y\sec^2 y-2 \quad ② \quad \neq ① \quad \frac{\partial N}{\partial x}=y\sec^2 y \quad ②$$

$\lambda=\lambda(y)$; $\quad \ln\lambda=\int\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}dy=\int\left[\frac{y\sec^2 y-\tan y-y\sec^2 y+2}{y\tan y-2y}\right]dy$ ① ②

$$\ln\lambda=-\int\frac{1}{y}dy=-\ln y$$

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$\boxed{\lambda=1/y}$ ② $\underbrace{(\tan y-2)}_{M_1(x,y)}dx+\underbrace{(x\sec^2 y+\frac{1}{y})}_{N_1(x,y)}dy=0$

$\left(\frac{\partial M_1}{\partial y}=\sec^2 y\right)$ ② $=\left(\frac{\partial N_1}{\partial x}=\sec^2 y\right)$ ② Tam D.D $\Rightarrow u(x,y)$ fonk. vardır ki:

$\frac{\partial u}{\partial x}=\tan y-2$ ① ve $\frac{\partial u}{\partial y}=x\sec^2 y+\frac{1}{y}$ ①

$u(x,y)=x\tan y-2x+h(y)$ ②

$\frac{\partial u}{\partial y}=x\sec^2 y+\frac{dh}{dy}=x\sec^2 y+\frac{1}{y}$ ②

$\frac{dh}{dy}=\frac{1}{y}\Rightarrow h(y)=\ln y+c_1$ ②

$u(x,y)=x\tan y-2x+\ln y+c_1=k$ ①

2. yol:

$u(x,y)=x.\tan y+\ln y+\alpha(x)$ ②

$\frac{\partial u}{\partial x}=\tan y+\frac{d\alpha}{dx}=\tan y-2$ ②

$\frac{d\alpha}{dx}=-2\Rightarrow\alpha(x)=-2x+c_1$ ②

$u(x,y)=x\tan y+\ln y-2x+c_1=k$ ①

$$x\tan y-2x+\ln y=\underbrace{k-c_1}_{c}$$

$x\tan y-2x+\ln y=c\quad,\ y(0)=1$

$0-2.0+\ln 1=c\Rightarrow c=0$ ①

$x\tan y-2x+\ln y=0$ ①

Başarılar dilerim.

4) $(x - y\ln y + y\ln x)dx + x(\ln y - \ln x)dy = 0$ diferansiyel denkleminin genel çözümünü bulunuz. (25P)

$$(x - y(\ln y - \ln x))dx + x(\ln y - \ln x)dy = 0$$

$$\left(x - y.\ln\left(\frac{y}{x}\right)\right)dx + x.\ln\left(\frac{y}{x}\right)dy = 0 \quad (2)$$

$$\left(1 - \frac{y}{x}.\ln\left(\frac{y}{x}\right)\right)dx + \ln\left(\frac{y}{x}\right)dy = 0 \quad (2)$$ Homojen D.D.

(2) $y = ux \quad , \quad dy = udx + xdu \quad (2)$

$$(1 - u.\ln u)dx + \ln u(udx + xdu) = 0 \quad (2)$$

$$(1 - \cancel{u\ln u} + \cancel{u\ln u})dx + x.\ln u du = 0$$ NURAN GÜZEL

$$dx + x\ln u du = 0 \quad (4)$$

$$\int \frac{dx}{x} + \int \ln u du = \int 0 \quad (2)$$

$$\overset{(2)}{\ln x} + u(\overset{(4)}{\ln u} - 1) = C^{(1)}$$

$$\ln x + \frac{y}{x}\left(\ln\left(\frac{y}{x}\right) - 1\right) = C \quad (2)$$

ii.) Cayley-Hamilton Teoremini kullanarak A^5 değerini hesaplayınız.

3) $(3xy+y^2+1)dx + (x^2+xy)dy=0$ diferansiyel denklemini çözünüz.

Çözüm: $m(x,y) = 3xy+y^2+1 \Rightarrow \frac{\partial M}{\partial y} = 3x+2y$ > ≠

$N(x,y) = x^2+xy \Rightarrow \frac{\partial N}{\partial x} = 2x+y$

$$\mu = \mu(x) = e^{\int \frac{3x+2y-2x-y}{x^2+xy}dx} = e^{\int \cancel{\frac{x+y}{x(x+y)}}dx} = e^{\int \frac{1}{x}dx} = x$$

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$$x\left[(3xy+y^2+1)dx + (x^2+xy)dy\right] = 0 \Rightarrow$$

$$(3x^2y+xy^2+x)dx + (x^3+x^2y)dy = 0$$ şeklindedir.

$$\left[\begin{array}{l} \frac{\partial m}{\partial y} = 3x^2+2xy \\ \frac{\partial N}{\partial x} = 3x^2+2xy \end{array}\right] =$$

Genel çözüm $F(x,y)=c$

$$\left.\begin{array}{l} \frac{\partial F}{\partial x} = 3x^2y+xy^2+x \\ \frac{\partial F}{\partial y} = x^3+x^2y \end{array}\right\} \Rightarrow$$

$$F(x,y) = \int (3x^2y+xy^2+x)dx$$

$$f(x,y) = x^3y + \frac{x^2y^2}{2} + \frac{x^2}{2} + k(y)$$

$$\frac{\partial f}{\partial y} = x^3 + x^2y + k'(y)$$

$$k'(y)=0 \Rightarrow k(y)=c_1$$

$$x^3y + \frac{x^2y^2}{2} + \frac{x^2}{2} + c_1 = c \Rightarrow x^3y + \frac{x^2y^2}{2} + \frac{x^2}{2} = c$$

$$\Rightarrow \boxed{x^3y + \frac{x^2}{2}(y^2+1) = c}$$

3) $(2x-y-7)y' = -(x+2y-1)$ diferansiyel denkleminin genel çözümünü bulunuz.

$(x+2y-1)dx + (2x-y-7)dy = 0$ Homojen hale getirilebilen dif denklem

$x+2y-1=0 \quad 5x=15 \quad y=-1$

$2x-y-7=0 \quad x=3$

$x = x_1+3 \rightarrow dx = dx_1$

$y = y_1-1 \rightarrow dy = dy_1$

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$(x_1+3+2y_1-2-1)dx_1 + (2x_1+6-y_1+1-7)dy_1 = 0$

$(x_1+2y_1)dx_1 + (2x_1-y_1)dy_1 = 0$ Homojen dif denk

$\left(1+2\frac{y_1}{x_1}\right) + \left(2-\frac{y_1}{x_1}\right)\frac{dy_1}{dx_1} = 0 \qquad \frac{y_1}{x_1} = u \rightarrow y_1 = u x_1$

$y_1' = u'x_1 + u$

$(1+2u) + (2-u)(u'x_1+u) = 0$

$1+2u+2u'x_1+2u-u'u x_1-u^2 = 0$

$u'(2x_1-ux_1) - u^2+4u+1 = 0$

$\frac{du}{dx_1}x_1(2-u) = u^2-4u-1$

$\frac{(2-u)du}{u^2-4u-1} = \frac{dx_1}{x_1}$

$\ln x_1 = \int \frac{2-u}{u^2-4u-1}du = -\frac{1}{2}\int\frac{2u-4}{u^2-4u-1}du + \ln c_1$

$\ln x_1 = -\frac{1}{2}\ln(u^2-4u-1) + \frac{1}{2}\ln c_1$

$2\ln x_1 + \ln(u^2-4u-1) = \ln c_1$

$x_1^2(u^2-4u-1) = c_1$

$(x-3)^2\left(\left(\frac{y+1}{x-3}\right)^2 - 4\frac{y+1}{x-3} - 1\right) = c_1$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ tam dif

$2 = 2$

$f(x,y) = \int(x+2y-1)dx$

$f(x,y) = \frac{x^2}{2} + 2xy - x + g(y)$

$\frac{\partial f}{\partial y} = (2x-y-7)$

$2x + g'(y) = 2x-y-7$

$g'(y) = -y-7$

$g(y) = -\frac{y^2}{2} - 7y$

$\frac{x^2}{2} + 2xy - x - \frac{y^2}{2} - 7y$