SORU 2.) $y''' + y' = 2\sin x - 2\cos x + 1$ diferansiyel denklemini **Belirsiz Katsayılar Yöntemiyle** çözünüz
(25 Puan)

CEVAP 2) $y''' + y' = 2\sin x - 2\cos x + 1$

$$K(r) = r^3 + r = 0 \Rightarrow r(r^2+1) = 0 \quad r = 0, \quad r = \mp i \quad (2)$$

$$y_h = c_1 + c_2 \cos x + c_3 \sin x \quad (3)$$

$$\boxed{y_1 = x \cdot [A\cos x + B\sin x]} \quad (2)$$

$$y_1' = A\cos x + B\sin x + x \cdot [-A\sin x + B\cos x]$$

$$\boxed{y_1' = (A + Bx)\cos x + (B - Ax)\sin x} \quad (2)$$

$$y_1'' = B\cos x - (A+Bx)\sin x - A\sin x + (B - Ax)\cos x$$

$$= \boxed{(2B - Ax)\cos x + (-2A - Bx)\sin x} \quad (2)$$

$$y_1''' = -A\cos x + (2B - Ax)(-\sin x) - B\sin x + (-2A - Bx)\cos x$$

$$y_1''' = \boxed{(-3A - Bx)\cos x + (-3B + Ax)\sin x} \quad (2)$$

$$(-3A - Bx)\cos x + (-3B + Ax)\sin x + (A + Bx)\cos x + (B - Ax)\sin x = 2\sin x - 2\cos x$$

$$(-2A)\cos x + (-2B)\sin x = 2\sin x - 2\cos x$$

$$-2B = 2 \Rightarrow B = -1$$

$$-2A = -2 \Rightarrow A = 1$$

$$\boxed{y_1 = x(\cos x - \sin x)} \quad (2)$$

$$y_2 = kx, \quad y_2' = k, \quad y_2'' = 0, \quad y_2''' = 0 \quad (3)$$

$$0 + k = 1 \quad \boxed{k = 1} \quad \boxed{y_2 = x} \quad 4$$

$$y = c_1 + c_2\cos x + c_3 \sin x + x(\cos x - \sin x) + x$$

A4 $y'' - 3y' + 2y = x^2 + 2 + e^x - \cos 2x$

$$\boxed{\begin{array}{c} r^2 - 3r + 2 = 0 \\ -2 \\ -1 \\ r_1 = 1,\ r_2 = 2 \\ y_h = c_1 e^x + c_2 e^{2x} \end{array}}$$

a) $y_1 = ax^2 + bx + c$

$y_1' = 2ax + b$

$y_1'' = 2a$

$$\underbrace{2a}_{1}x^2 + \underbrace{(-6a + 2b)}_{0}x + \underbrace{(2a - 3b + 2c)}_{2} =$$

$a = \frac{1}{2}$, $b = \frac{3}{2}$, $c = \frac{9}{4}$

b) $y_2 = Axe^x$

$y_2' = (1+x)Ae^x$

$y_2'' = (2+x)Ae^x$

$\Rightarrow A = -1$

c) $2/\ y_3 = a \sin 2x + b \cos 2x$

$-3/\ y_3' = 2a \cos 2x - 2b \sin 2x$

$1/\ y_3'' = -4a \sin 2x - 4b \cos 2x$

$$\underbrace{(-2a + 6b)}_{0} \sin 2x + \underbrace{(-6a - 2b)}_{-1} \cos 2x = -\cos 2x$$

$a = \frac{3}{20}$ $b = \frac{1}{20}$

$$y = c_1 e^x + c_2 e^{2x} + \left(\frac{1}{2}x^2 + \frac{3}{2}x + \frac{9}{4}\right) - xe^x + \frac{3}{20}\sin 2x + \frac{1}{20}\cos 2x$$

1) $y'' - 3y' + 2y = \dfrac{e^{3x}}{1+e^{x}}$ Par Değ. Yönt.

$r^2 - 3r + 2 = 0 \quad (r-1)(r-2) = 0 \quad r_1 = 1 \quad r_2 = 2$

$y_h = c_1 e^{x} + c_2 e^{2x}$

$y_{öz} = c_1(x).e^{x} + c_2(x).e^{2x}$

$$
\begin{array}{rl}
-1 / & c_1'.e^{x} + c_2'.e^{2x} = 0 \\
+ & c_1' e^{x} + c_2'.2e^{2x} = \dfrac{e^{3x}}{1+e^{x}} \\
\hline
& c_2'.e^{2x} = \dfrac{e^{3x}}{1+e^{x}} \Rightarrow c_2' = \dfrac{e^{x}}{1+e^{x}}
\end{array}
$$

$c_1'.e^{x} + \dfrac{e^{x}}{1+e^{x}}.e^{2x} = 0 \Rightarrow c_1' = -\dfrac{e^{2x}}{1+e^{x}}$

$c_1(x) = -\int \dfrac{e^{2x}}{1+e^{x}}dx = -\int \dfrac{(u-1)}{u}.du = -\int\left(1 - \dfrac{1}{u}\right)du = -u + \ln|u| + k_1$

$1+e^{x} = u \quad e^{x}dx = du$

$c_1(x) = -(1+e^{x}) + \ln(1+e^{x}) + k_1$

$c_2(x) = \int \dfrac{e^{x}}{1+e^{x}}dx = \int \dfrac{du}{u} = \ln|u| + k_2$

$1+e^{x} = u$
$e^{x}dx = du$

$c_2(x) = \ln(1+e^{x}) + k_2$

$y_{öz} = [-(1+e^{x}) + \ln(1+e^{x}) + k_1]e^{x} + [\ln(1+e^{x}) + k_2]e^{2x}$

(A2) $y'' + 6y' + 9y = \dfrac{e^{-3x}}{x^3}$

$r^2 + 6r + 9 = 0 \Rightarrow r_1, r_2 = -3 \Rightarrow y = c_1 e^{-3x} + c_2 x e^{-3x}$

$c_1 = c_1(x), \quad c_2 = c_2(x)$

$y' = \underbrace{c_1' e^{-3x} + c_2' x e^{-3x}}_{0} - 3c_1 e^{-3x} + c_2 e^{-3x} - 3c_2 x e^{-3x}$

$c_1' e^{-3x} + c_2' x e^{-3x} = 0 \Rightarrow \boxed{c_1' + c_2' x = 0}$

$y'' = -3c_1' e^{-3x} + c_2' e^{-3x} - 3c_2' x e^{-3x} + 9c_1 e^{-3x} - 3c_2 e^{-3x} - 3c_2 e^{-3x}$

$-6c_2 e^{-3x}$

0

$+ 9c_2 x e^{-3x} = \dfrac{e^{-3x}}{x^3}$

$(c_2' + 9c_1 - 6c_2 + 9c_2 x)\cancel{e^{-3x}} + 6(-3c_1 + c_2 - 3c_2 x)\cancel{e^{-3x}} +$
$+ 9(c_1 + c_2 x)\cancel{e^{-3x}} = \dfrac{\cancel{e^{-3x}}}{x^3}$

$\boxed{c_2' = \dfrac{1}{x^3}}$

$\left.\begin{aligned} c_1' + c_2' x &= 0 \\ c_2' &= \dfrac{1}{x^3} \end{aligned}\right\} \Rightarrow \quad c_2 = \int \dfrac{1}{x^3}\,dx = -\dfrac{1}{2x^2} + K_2$

$c_1' + \dfrac{1}{x^2} = 0$

$c_1 = -\int \dfrac{1}{x^2}\,dx = \dfrac{1}{x} + K_1$,

$y = \left(\dfrac{1}{x} + K_1\right) e^{-3x} + \left(-\dfrac{1}{2x^2} + K_2\right) e^{-3x} \cdot x$

$y = K_1 e^{-3x} + K_2 x e^{-3x} + \underbrace{\left(\dfrac{1}{x} - \dfrac{1}{2x}\right)}_{+\frac{1}{2x}} e^{-3x}$

2. $x^2y''+2xy'-1=0,\ x>0$, diferansiyel denkleminin genel çözümünü bulunuz.

CEVAP 2: $x^2y''+2xy'-1=0 \qquad x>0$

$x=e^t$ ∅2 dönüşümü yapılırsa;

$$\cancel{e^{2t}}\,\cancel{e^{-2t}}\,D(D-1)Y+2\cancel{e^{t}}\,\cancel{e^{-t}}\,DY-1=0 \qquad \emptyset 5$$

$$(D^2-D+2D)Y=1$$

$$(D^2+D)Y=1 \quad \text{bulunur.} \qquad \Sigma 7$$

$$K(r)=r^2+r=0 \Rightarrow r(r+1)=0 \qquad r_1=0,\ r_2=-1 \qquad \emptyset 2$$

∅3

$$Y_h=Ae^{0t}+Be^{-t}=A+Be^{-t} \qquad 3$$

$\Sigma 15$

$C=0$ olduğundan;

3 $Y_0=Ct+\cancel{D}$ seçilir.

$$\left.\begin{aligned} Y_0' &= C \\ Y_0'' &= 0 \end{aligned}\right\} \text{Denklemde yazılırsa;} \quad 0+C=1 \qquad C=1 \text{ olur.}$$

∅2

$Y_0=t$

∅2

$Y_c=A+Be^{-t}+t$ elde edilir. Ancak

$x=e^t \Rightarrow t=\ln x$ idi. Buradan

$$Y_G=A+\frac{B}{x}+\ln x \quad \text{elde edilir.} \qquad \emptyset 3$$

Soru 3: $2y''+3y'+y=t^2+3\sin t$ diferansiyel denkleminin genel çözümünü bulunuz.

CEVAP 3

$$2y''+3y'+y=t^2+3\sin t$$

$$K(r)=2r^2+3r+1=0$$

$$r_{1/2}= \begin{cases} -1 \\ -\frac{1}{2} \end{cases}$$

$$y_h=Ae^{-t}+Be^{-\frac{t}{2}}$$

$$\left.\begin{aligned} y_{01}&=At^2+Bt+C \\ y_{01}'&=2At+B \\ y_{01}''&=2A \end{aligned}\right\}$$

$$2(2A)+3(2At+B)+(At^2+Bt+C)=t^2$$

$$4A+6At+3B+At^2+Bt+C=t^2$$

$$\boxed{A=1}$$

$$6A+B=0$$

$$\boxed{B=-6}$$

$$4A+3B+C=0$$

$$4-18+C=0$$

$$\boxed{C=14}$$

$$y_{01}=t^2-6t+14$$

$$y_{02}=A\sin t+B\cos t$$

$$y_{02}'=A\cos t-B\sin t$$

$$y_{02}''=-A\sin t-B\cos t$$

$$2(-A\sin t-B\cos t)+3(A\cos t-B\sin t)+A\sin t+B\cos t=3\sin t$$

$$-2B+3A+B$$

$$(A-B)\sin t+(3A-B)\cos t=3\sin t$$

$$\left.\begin{aligned} -A\pm B&=-3 \\ 3A-B&=0 \end{aligned}\right\}$$

$$2A=-3$$

$$\boxed{A=-\frac{3}{2}}$$

$$B=-\frac{3}{2}-\frac{6}{2}$$

$$\boxed{B=-\frac{9}{2}}$$

$$y_{02}=-\frac{3}{2}\sin t-\frac{9}{2}\cos t$$

Soru 4: $y'' - ty' - y = 0$
diferansiyel denkleminin genel çözümünü seri yöntemi ile bulunuz.

CEVAP 4:

$$y = \sum_{n=0}^{\infty} c_n t^n \Rightarrow y'(t) = \sum_{n=1}^{\infty} n c_n t^{n-1}$$

$$y''(t) = \sum_{n=2}^{\infty} n.(n-1) t^{n-2}$$

değerleri denklemde yerine yazıldığında

$$\sum_{n=2}^{\infty} n.(n-1) c_n t^{n-2} - \sum_{n=1}^{\infty} n.c_n t^n - \sum_{n=0}^{\infty} c_n t^n = 0$$

bulunur. Birinci ve üçüncü serilerin ilk termleri dışarı alınırsa

$$2c_2 + \sum_{n=3}^{\infty} n.(n-1) c_n t^{n-2} - \sum_{n=1}^{\infty} n c_n t^n - c_0 - \sum_{n=1}^{\infty} c_n t^n = 0$$

ur. Birinci seride n yerine $n+2$ yazılırsa

$$2c_2 - c_0 + \sum_{n=1}^{\infty} \{ (n+2)(n+1) c_{n+2} - (n+1) c_n \} t^n = 0 \quad \text{olur.}$$

$$2c_2 - c_0 = 0 \quad \text{ve} \quad (n+2) c_{n+2} - c_n = 0 \quad (n \geq 1)$$

$$\Rightarrow c_2 = 2c_0 \quad \text{ve} \quad c_{n+2} = \frac{1}{n+2} c_n \quad (n \geq 1)$$

$$c_3 = \frac{1}{3} c_1, \quad c_4 = \frac{1}{4} c_2 = \frac{1}{4.2} c_0, \quad c_5 = \frac{1}{5} c_3 = \frac{1}{5.3} c_1$$

$$\vdots$$

$$\ldots \quad c_{2n} = \frac{1}{2.4.6\ldots(2n)} c_0, \quad c_{2n+1} = \frac{1}{3.5.7\ldots(2n+1)} c_1, \quad (n \geq 1)$$

Böylece

$$y(t) = c_0 + c_1 t + c_2 t^2 + \ldots$$

$$= c_0 \left\{ 1 + \frac{t^2}{2} + \frac{t^4}{4.2} + \ldots \right\} + c_1 \left\{ t + \frac{t^3}{3} + \frac{t^5}{3.5} + \ldots \right\}$$

$$= c_0 \sum_{n=0}^{\infty} \frac{t^{2n}}{2.4.6\ldots(2n)} + c_0 \sum_{n=0}^{\infty} \frac{t^{2n+1}}{3.5.7\ldots(2n+1)}$$

1) $y''+2y'+5y=0, \qquad y(0)=2,\ y'(0)=-1$ başlangıç değer probleminin çözümünü Laplace dönüşümü kullanarak bulunuz.

$$\mathcal{L}\{y''+2y'+5y\}=\mathcal{L}\{0\}$$

$$\boxed{[s^2Y(s)-s\underbrace{y(0)}_{2}-\underbrace{y'(0)}_{-1}]+2[sY(s)-\underbrace{y(0)}_{2}]+5Y(s)=0}$$

3 2 1

$$s^2Y(s)-2s+1+2sY(s)-4+5Y(s)=0$$ 1

$$(s^2+2s+5)Y(s)=2s+3$$

$$\boxed{Y(s)=\frac{2s+3}{s^2+2s+5}}$$ 3

$$\boxed{\frac{2s+3}{(s+1)^2+4}=\frac{2s}{(s+1)^2+4}+\frac{3}{(s+1)^2+4}}$$

1 3 3

$$\mathcal{L}^{-1}\{Y(s)\}=\mathcal{L}^{-1}\left\{\frac{2s+2}{(s+1)^2+4}+\frac{1}{(s+1)^2+4}\right\}$$

$$\boxed{y(t)=2e^{-t}\cos 2t+\frac{1}{2}e^{-t}\sin 2t}$$

4 4

2) $\dfrac{dx}{dt}+\dfrac{dy}{dt}-x-3y=0$

$\dfrac{dx}{dt}+\dfrac{dy}{dt}+x=e^{3t}$ denklem sisteminin genel çözümünü bulunuz.

5 $\left[\begin{array}{l}(D-1)x+(D-3)y=0\\(D+1)x+Dy=e^{3t}\end{array}\right.$ 3 $\quad \Delta=\begin{vmatrix} D-1 & D-3 \\ D+1 & D \end{vmatrix}=D+3$ 2

$x=\dfrac{\begin{vmatrix} 0 & D-3 \\ e^{3t} & D \end{vmatrix}}{\Delta}=\dfrac{3(-e^{3t}+e^{3t})}{\Delta}=0$ 2 $\quad \underline{(D+3)x=0}$ 2 $\quad \underline{x=c_1e^{-3t}}$ 2

$r+3=0$

$r=-3$

$y=\dfrac{\begin{vmatrix} D-1 & 0 \\ D+1 & e^{3t} \end{vmatrix}}{\Delta}=\dfrac{3e^{3t}-e^{3t}}{\Delta}$ 2 $\quad \underline{(D+3)y=2e^{3t}}$ 2 $\quad r=-3$

$\underline{y_h=c_2e^{-3t}}$ 2 $\quad y_ö=Ke^{3t} \quad y_ö'=3Ke^{3t} \quad 3Ke^{3t}+3Ke^{3t}=2e^{3t}$

$K=\dfrac{1}{3} \quad \underline{y_ö=\dfrac{1}{3}e^{3t}}$ 2 $\quad \underline{y=c_2e^{-3t}+\dfrac{1}{3}e^{3t}}$ 1

$(D+1)x+Dy=e^{3t}$ denk. yer. yaz

$-3c_1e^{-3t}+c_1e^{-3t}-3c_2e^{-3t}+\cancel{e^{3t}}=\cancel{e^{3t}}$ 2

$-2c_1e^{-3t}=3c_2e^{-3t}$

$\boxed{c_2=-\dfrac{2}{3}c_1}$ 1

2 $\left\|\begin{array}{l}x=c_1e^{-3t}\\y=-\dfrac{2}{3}c_1e^{-3t}+\dfrac{1}{3}e^{3t}\end{array}\right.$

$$\frac{dx}{dt}+\frac{dy}{dt}-x+3y=6e^{-t}$$

$$\frac{dx}{dt}+\frac{dy}{dt}+2x=0$$

$$D=\frac{d}{dt}$$

$$Dx+Dy-x+3y=6e^{-t} \qquad (D-1)x+(D+3)y=6e^{-t}$$

$$Dx+Dy+2x=0 \qquad (D+2)x+Dy=0$$

$\Rightarrow \begin{vmatrix} D-1 & D+3 \\ D+2 & D \end{vmatrix} = -5D-6 \Rightarrow D$ ye göre 1.dereceden, sadece 1 tane int. sabiti içerecektir.

$$[(D+2)(D+3)-D(D-1)]y=(D+2)6e^{-t}$$

$$(6D+6)y=D(6e^{-t})+12e^{-t}$$

$$6(D+1)y=-6e^{-t}+12e^{-t}$$

$$(D+1)y=e^{-t} \Rightarrow D+1=0 \Rightarrow D=-1 \Rightarrow y_p=Ce^{-t}$$

$$y_0=Ate^{-t} \Rightarrow (D+1)Ate^{-t}=e^{-t}$$

$$A(e^{-t}-te^{-t}+te^{-t})=e^{-t} \Rightarrow A=1$$

$$y_0=te^{-t}$$

$$y=Ce^{-t}+te^{-t}$$

Verilen denklem ler taraf tarafa çıkarılırsa;

$$-x+3y-3x=6e^{-t} \Rightarrow -3x+3y=6e^{-t}$$

$$x=y-2e^{-t}=Ce^{-t}+te^{-t}-2e^{-t}$$

$$x=(C-2)e^{-t}+te^{-t}$$

2) $y''+2y'+y=8e^{-t}$ $y(0)=2,\ y'(0)=-1$ Başlangıç değer problemini Laplace dönüşümü kullanarak çözünüz.

$$\mathcal{L}\{y''+2y'+y\}=\mathcal{L}\{8e^{-t}\}$$

$$s^2Y(s)-s\,y(0)-y'(0)+2(sY(s)-y(0))+Y(s)=\frac{8}{s+1}$$

$$s^2Y(s)+2sY(s)+Y(s)-2s-3=\frac{8}{s+1}$$

$$\underbrace{(s^2+2s+1)}_{(s+1)^2}Y(s)-2s-3=\frac{8}{s+1}$$

$$Y(s)=\frac{8}{(s+1)^3}+\frac{2s+3}{(s+1)^2}$$

$$\frac{2s+3}{(s+1)^2}=\frac{2(s+1)+1}{(s+1)^2}=\frac{2}{s+1}+\frac{1}{(s+1)^2}$$

$$\mathcal{L}^{-1}\{Y(s)\}=\mathcal{L}^{-1}\left\{\frac{8}{(s+1)^3}+\frac{2s+3}{(s+1)^2}\right\}=\mathcal{L}^{-1}\left\{\frac{8}{(s+1)^3}+\frac{2}{s+1}+\frac{1}{(s+1)^2}\right\}$$

$$y(t)=\mathcal{L}^{-1}\{Y(s)\}$$

$$y(t)=8e^{-t}\frac{t^2}{2!}+2e^{-t}\cdot 1+e^{-t}\cdot t$$

$$y(t)=(4t^2+t+2)e^{-t}$$

//

$$\frac{dx}{dt}-\frac{dy}{dt}-y=-e^{t} \quad (1)$$

4) $$\frac{dy}{dt}+x-y=0 \quad (2)$$ denklem sistemini türetme – yok etme yöntemiyle çözünüz.

$$(2)' \Rightarrow \frac{d^2y}{dt^2}+\frac{dx}{dt}-\frac{dy}{dt}=0$$

$$(1) \Rightarrow \frac{dx}{dt}=-e^{t}+\frac{dy}{dt}+y$$ yerine koy

$$\frac{d^2y}{dt^2}-\frac{dy}{dt}-e^{t}+\frac{dy}{dt}+y=0$$

$$\frac{d^2y}{dt^2}+y=e^{t} \qquad r^2+1=0 \qquad r_{1,2}=\mp i$$

$$y=c_1\cos t+c_2\sin t$$

$$y_0=Ae^{t} \qquad y_0'=Ae^{t} \qquad y_0''=Ae^{t}$$

$$Ae^{t}+Ae^{t}=e^{t} \rightarrow 2Ae^{t}=e^{t} \Rightarrow \boxed{A=\frac{1}{2}} \Rightarrow \boxed{y_0=\frac{e^{t}}{2}}$$

$$y=c_1\cos t+c_2\sin t+\frac{e^{t}}{2}$$

$$(2) \Rightarrow x=y-\frac{dy}{dt} \Rightarrow x=c_1\cos t+c_2\sin t+\cancel{\frac{e^{t}}{2}}+c_1\sin t-c_2\cos t-\cancel{\frac{e^{t}}{2}}$$

$$X=(c_1-c_2)\cos t+(c_1-c_2)\sin t$$

3) $y''+y'+xy=0$ denkleminin $x=0$ noktası civarında seri çözümünü elde ediniz.

$$y=\sum_{n=0}^{\infty} c_n x^n \quad , \quad y'=\sum_{n=1}^{\infty} n c_n x^{n-1} \quad y''=\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + \sum_{n=1}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

$$2c_2 + \sum_{n=3}^{\infty} n(n-1) c_n x^{n-2} + c_1 + \sum_{n=2}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

$$(2c_2+c_1) + \sum_{n=3}^{\infty} \left\{ n(n-1)c_n + (n-1)c_{n-1} + c_{n-3} \right\} x^{n-2} = 0$$

$$2c_2+c_1=0 \quad , \quad n(n-1)c_n + (n-1)c_{n-1} + c_{n-3} = 0 \quad (n \geq 3)$$

$$c_2 = -\frac{c_1}{2} \quad , \quad c_n = -\frac{(n-1)c_{n-1} + c_{n-3}}{n(n-1)} \quad (n \geq 3)$$

$$c_2 = -\frac{c_1}{2}$$

$$c_3 = \frac{-2c_2 + c_0}{2\cdot 3} = \frac{c_1 - c_0}{6}$$

$$c_4 = -\frac{3c_3 + c_1}{4\cdot 3} = \frac{c_0 - 3c_1}{24}$$

$$c_5 = -\frac{4c_4 + c_1}{5\cdot 4} = \frac{6c_1 - c_0}{120}$$

$$\vdots$$

$$y = c_0 + c_1 x - \frac{c_1}{2}x^2 + \frac{c_1 - c_0}{6}x^3 + \frac{c_0 - 3c_1}{24}x^4 + \frac{6c_1 - c_0}{120}x^5 + \dots$$

$$y = c_0\left(1 - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120} + \frac{5x^6}{720} - \dots\right) + c_1\left(x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{3x^4}{24} + \frac{6x^5}{120} - \dots\right)$$

gen. çöz.

Başarılar

2) $y'' + 2y' + y = 4e^{-t}$ denkleminin $y(0) = 2$, $y'(0) = -1$ başlangıç değer problemini Laplace dönüşümü kullanarak çözünüz.

$$\mathcal{L}\{y''+2y'+y\} = \mathcal{L}\{4e^{-t}\}$$

$$s^2Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{-1} + 2[sY(s) - \underbrace{y(0)}_{2}] + Y(s) = \frac{4}{s+1}$$

$$s^2Y(s) + 2sY(s) + Y(s) - 2s - 3 = \frac{4}{s+1}$$

$$\underbrace{(s^2+2s+1)}_{(s+1)^2} Y(s) - 2s - 3 = \frac{4}{s+1}$$

$$Y(s) = \frac{4}{(s+1)^3} + \frac{2s+3}{(s+1)^2}$$

$$\frac{2s+3}{(s+1)^2} = \frac{2(s+1)+1}{(s+1)^2} = \frac{2}{s+1} + \frac{1}{(s+1)^2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{4}{(s+1)^3} + \frac{2}{s+1} + \frac{1}{(s+1)^2}\right\}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$y(t) = 4\cdot e^{-t}\cdot\frac{t^2}{2!} + 2e^{-t}\cdot 1 + e^{-t}\cdot t$$

$$y(t) = (2t^2+t+2)e^{-t}$$

Başarılar...

ök

$y'' + 4y = 0 \qquad y(0) = 1 ; \quad y'(0) = -2$

$L\{y(t)\} = Y(s)$

$L\{y'' + 4y\} = L\{y''\} + 4L\{y\} = 0$

$s^2Y(s) - s\underbrace{y(0)}_{1} - \underbrace{y'(0)}_{-2} + 4Y(s) = 0$

$s^2Y - s + 2 + 4Y = 0$

$(s^2+4)Y = s-2 \longrightarrow Y(s) = \frac{s-2}{s^2+4}$

$L^{-1}\{Y(s)\} = y(t)$

$L^{-1}\left\{\frac{s-2}{s^2+4}\right\} = L^{-1}\left\{\frac{s}{s^2+4}\right\} - 2L^{-1}\left\{\frac{1}{s^2+4}\right\} = \cos 2t - \cancel{2}\frac{\sin 2t}{\cancel{2}}$

$y(t) = \cos 2t - \sin 2t$

örnek $y'' - 6y' + 9y = t^2e^{3t}, \quad y(0) = 2, \quad y'(0) = 6$

$L\{y'' - 6y' + 9y\} = L\{t^2e^{3t}\}$

$s^2Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{6} - 6(sY(s) - \underbrace{y(0)}_{2}) + 9Y(s) = \frac{2}{(s-3)^3}$

$s^2Y - 2s - \cancel{6} - 6sY + \cancel{12} + 9Y = \frac{2}{(s-3)^3}$

$s^2 - 6s + 9)Y - 2(s-3) = \frac{2}{(s-3)^3}$

$(s-3)^2Y = 2(s-3) + \frac{2}{(s-3)^3} \rightarrow Y(s) = \frac{2}{(s-3)} + \frac{2}{(s-3)^5}$

$L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{2}{s-3} + \frac{2}{(s-3)^5}\right\} = 2e^{3t} + 2\frac{t^4}{4!}e^{3t}$

$y(t) = 2e^{3t} + \frac{1}{12}t^4e^{3t}$

4) $y''+4y=2+\sin 2x$ diferansiyel denkleminin genel çözümünü bulunuz.

$r^2+4=0$ $\quad r=\mp 2i$ (2), $\quad y=c_1\cos 2x+c_2\sin 2x$ (3)

(12)

$y_{\ddot{o}_1}=x\,(A\sin 2x+B\cos 2x)$ (4)

$y'_{\ddot{o}_1}=(A\sin 2x+B\cos 2x)+x.(2A\cos 2x-2B\sin 2x)$ (1)

$y''_{\ddot{o}_1}=(2A\cos 2x-2B\sin 2x)+(2A\cos 2x-2B\sin 2x)+x.(-4A\sin 2x-4B\cos 2x)$

$y''_{\ddot{o}_1}=(4A\cos 2x-4B\sin 2x)+x(-4A\sin 2x-4B\cos 2x)$ (2)

$(4A\cos 2x-4B\sin 2x)+x(-4A\sin 2x-4B\cos 2x)+4x(A\sin 2x+B\cos 2x)=\sin 2x$

$4A\cos 2x-4B\sin 2x=\sin 2x$ (1)

$4A=0, \quad 4B=-1$

$A=0$ (2) $\quad B=-\frac{1}{4}$

$y_{\ddot{o}_1}=\frac{-x}{4}\cos 2x$ (2)

(6)

$y_{\ddot{o}_2}=A$ (2), $\quad y'_{\ddot{o}_2}=0, \quad y''_{\ddot{o}_2}=0$

(2)

$0+4A=2 \Rightarrow A=\frac{1}{2}, \quad y_{\ddot{o}_2}=\frac{1}{2}$ (2)

$y=c_1\cos 2x+c_2\sin 2x+\frac{x}{2}-\frac{x}{4}\cos 2x$ (2)

3) $y''-2y'+y=e^x\left(1+\tan^2 x\right)$ diferansiyel denkleminin genel çözümünü sabitin değişimi yöntemini kullanarak bulunuz.

$$y''-2y'+y=e^x(1+\tan^2 x)$$

$$r^2-2r+1=0 \quad r_{1,2}=1 \qquad y=c_1e^x+c_2xe^x \quad (3)$$

(2)

$$(4)\ \begin{cases} c_1'e^x+c_2'xe^x=0 \\ c_1'e^x+c_2'(e^x+xe^x)=e^x(1+\tan^2 x). \end{cases}$$

$$\left.\begin{array}{l} c_2'(1+x)+c_1'=1+\tan^2 x. \\ c_1'+c_2'x=0 \end{array}\right\} \Rightarrow \boxed{c_2'=1+\tan^2 x} \quad (2)$$

$$c_2'=1+\tan^2 x \Rightarrow \int c_2'dx=\int(1+\tan^2 x)dx \Rightarrow \boxed{c_2=\tan x+K_2} \quad (4)$$

$$(2)\ \boxed{c_1'=-c_2'x} \Rightarrow c_1'=\frac{-x}{\cos^2 x} \Rightarrow \boxed{c_1=\int\frac{-x}{\cos^2 x}dx} \quad (5)$$

$x=u$, $dx=du$, $\frac{1}{\cos^2 x}dx=dv$, $v=$

$$c_1=-\left[x\tan x-\int\tan x\,dx\right]$$

$$c_1=-x\tan x+\int\frac{\sin x}{\cos x}dx$$

$$=-x\tan x-\ln|\cos x|+K_1$$

$$y=K_1e^x+K_2xe^x-(x\tan x+\ln|\cos x|)e^x+x\tan x\,e^x \quad (3)$$

4) $y'' + 4y = 2 + \sin 2x$ diferansiyel denkleminin genel çözümünü bulunuz.

$r^2 + 4 = 0$ $\boxed{r = \mp 2i}$ (2), $\boxed{y = c_1\cos 2x + c_2 \sin 2x}$ (3)

$y_{ö_1} = x\,(A\sin 2x + B\cos 2x)$ (4)

$y'_{ö_1} = (A\sin 2x + B\cos 2x) + x\,(2A\cos 2x - 2B\sin 2x)$ (1)

$y''_{ö_1} = (2A\cos 2x - 2B\sin 2x) + (2A\cos 2x - 2B\sin 2x) + x.(-4A\sin 2x - 4B\cos 2x)$

$y''_{ö_1} = (4A\cos 2x - 4B\sin 2x) + x(-4A\sin 2x - 4B\cos 2x)$ (2)

(12) $(4A\cos 2x - 4B\sin 2x) + x(-4A\sin 2x - 4B\cos 2x) + 4x(A\sin 2x + B\cos 2x) = \sin 2x$

$4A\cos 2x - 4B\sin 2x = \sin 2x$ (1)

$4A = 0$, $4B = -1$

$A = 0$ (2) $B = -\frac{1}{4}$

$y_{ö_1} = -\frac{x}{4}\cos 2x$ (2)

(6) $y_{ö_2} = A$ (2), $y'_{ö_2} = 0$, $y''_{ö_2} = 0$

$0 + 4A = 2 \Rightarrow A = \frac{1}{2}$ (2), $y_{ö_2} = \frac{1}{2}$ (2)

$\boxed{y = c_1\cos 2x + c_2\sin 2x + \frac{x}{2} - \frac{x}{4}\cos 2x}$ (2)

1-) $y'' = (y')^3 + y'$ diferansiyel denkleminin genel çözümünü bulunuz.

$y' = p \quad , \quad y'' = p\dfrac{dp}{dy}$

$p\dfrac{dp}{dy} = p^3 + p \quad \Rightarrow \quad$ i) $p = 0$ için $y' = 0 \Rightarrow y = c$

$\dfrac{1}{p^2+1}\,dp = dy$

$\arctan p = y + c_1$

$p = \tan(y + c_1)$

$y' = \tan(y + c_1)$

$\dfrac{dy}{dx} = \tan(y + c_1)$

$\dfrac{1}{\tan(y+c_1)}\,dy = dx$

$\dfrac{\cos(y+c_1)}{\sin(y+c_1)}\,dy = dx \quad \Rightarrow \quad \ln(\sin(y+c_1)) = x + c_2$

3-) $x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+\dfrac{y}{x}=\dfrac{1}{x\sin(\ln x)}$, $x>0$ diferansiyel denkleminin genel çözümünü bulunuz.

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\frac{1}{\sin(\ln x)}$$

2 $x=e^t,\ x>0,\ t=\ln x$ $\qquad \dfrac{dy}{dx}=\dfrac{1}{x}\dfrac{dy}{dt}$ veya $\left(\dfrac{dy}{dx}=e^{-t}\dfrac{dy}{dt}\right)$

$$\frac{d^2y}{dx^2}=\frac{1}{x^2}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right) \text{ veya } \left(\frac{d^2y}{dx^2}=e^{-2t}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)\right)$$ 3

2 $x^2\dfrac{1}{x^2}\left(\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}\right)+x\dfrac{1}{x}\dfrac{dy}{dt}+y=\dfrac{1}{\sin t}$ $\quad\left(e^{2t}e^{-2t}\left(\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}\right)+e^{t}e^{-t}\dfrac{dy}{dt}+y=\dfrac{1}{\sin t}\right)$

$$\frac{d^2y}{dt^2}+y=\frac{1}{\sin t}$$ 2 $\quad y(t)=y_h(t)+y_ö(t)$ $\qquad \Sigma\, 9$

$u''+u=0 \Rightarrow m^2+1=0 \Rightarrow m_1=i,\ m_2=-i$

2 $y_h=c_1\cos t+c_2\sin t$ $\qquad y_ö=c_1(x)\cos t+c_2(x)\sin t$ bir özel çözüm olsun

4 $\begin{cases}\cos t\, c_1'+\sin t\, c_2'=0\\ -\sin t\, c_1'+\cos t\, c_2'=\dfrac{1}{\sin t}\end{cases} \Rightarrow W(\cos t,\sin t)=\begin{vmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{vmatrix}=1$

2 $c_1'(t)=\dfrac{\begin{vmatrix}0 & \sin t\\ \frac{1}{\sin t} & \cos t\end{vmatrix}}{1}=-1 \Rightarrow c_1(t)=-t+k_1$ 1

2 $c_2'(t)=\dfrac{\begin{vmatrix}\cos t & 0\\ -\sin t & \frac{1}{\sin t}\end{vmatrix}}{1}=\dfrac{\cos t}{\sin t} \Rightarrow c_2(t)=\ln(\sin t)+k_2$ 1

$y_ö=t\cos t+\ln(\sin t)\sin t$

2 $y(t)=y_h(t)+y_ö(t)=c_1\cos t+c_2\sin t+t\cos t+\ln(\sin t)\sin t$

$y(x)=c_1\cos(\ln x)+c_2\sin(\ln x)+\ln x\cos(\ln x)$

2 $+\ln(\sin(\ln x))\sin(\ln x)$

Başarılar

2-) $y^{(4)}+2y''+y=3+\cos 2t$ diferansiyel denkleminin genel çözümünü bulunuz.

$: y = y_h + y_ö = u + v$

$u^{(4)}+2u''+u=0 \Rightarrow r^4+2r^2+1=0 \quad r^2=m$ denirse

$m^2+2m+1=0$

$(m+1)^2=0 \Rightarrow m_1=m_2=-1$

$r_{1,2}^2=-1 \Rightarrow r_{1,2}=\pm i$

$r_{3,4}^2=-1 \Rightarrow r_{3,4}=\pm i$

$y_h = u = c_1\cos t + c_2\sin t + t(c_3\cos t + c_4\sin t)$ veya

$\left(y_h = (c_1+c_2t)\cos t + (c_3+c_4t)\sin t \right)$

$v^{(4)}+2v''+v=3+\cos 2t$ nin bir özel çözümü

$v = A + B\cos 2t + C\sin 2t$

$v' = -2B\sin 2t + 2C\cos 2t$

$v'' = -4B\cos 2t - 4C\sin 2t$

$v''' = 8B\sin 2t - 8C\cos 2t$

$v^{(4)} = 16B\cos 2t + 16C\sin 2t$

Denklemde yerine yazılırsa

$16B\cos 2t + 16C\sin 2t - 8B\cos 2t - 8C\sin 2t + A + B\cos 2t$
$+ C\sin 2t = 3 + \cos 2t$

$(16B - 8B + B)\cos 2t + (16C - 8C + C)\sin 2t + A = 3 + \cos 2t$

$A = 3, \quad 9B = 1 \Rightarrow B = 1/9, \quad 9C = 0 \Rightarrow C = 0$

$y_ö = v = 3 + \frac{1}{9}\cos 2t$

$y = y_h + y_ö = (c_1+c_2t)\cos t + (c_3+c_4t)\sin t + 3 + \frac{1}{9}\cos 2t$

Başarılar

YTÜ

O252311 DİFERANSİYEL DENKLEMLER

2. VİZE SINAVI

1 Aralık 2007

Adı-Soyadı :

Numara :

Grup :

İmza :

1	2	3	4	5	Toplam

Sınav süresi:75 dk. SADECE 4 SORU CEVAPLANDIRILACAKTIR. ***BAŞARILAR!***

1. $y'' + 2y' + y = 2\cos x$ diferansiyel denkleminin genel çözümünü bulunuz.

$y'' + 2y' + y = 0$

$y = ke^{rx}$, $y' = kre^{rx}$, $y'' = kr^2e^{rx}$

$kr^2e^{rx} + 2kre^{rx} + ke^{rx} = 0 \quad k(r) = r^2 + 2r + 1 = 0$, $r_{1,2} = -1$

$y_h = (C_1 + xC_2)e^{-x}$

$y_{22} = A\cos x + B\sin x$

$y'_{22} = -A\sin x + B\cos x$, $y''_{22} = -A\cos x - B\sin x$

$-A\cos x - B\sin x + 2A\cos x + 2B\sin x + A\cos x + B\sin x = 2\cos x$

$2A = 2$, $A = 1 \quad 2B = 0 \quad B = 0$ $\quad A = 0$

$y_{22} = \cos x \quad y_2 = \sin x$

$B = 1$

$y_0 = y_h + y_{22} = (C_1 + xC_2)e^{-x} + \cos x$

4. $x^2y'' + 3y'x + y = \sin(\ln x)$ diferansiyel denkleminin genel çözümünü bulunuz

$x = e^t$ dönüşümü yapılırsa $t = \ln x$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dx}{dt} = \frac{dy}{dt} \cdot e^{-t}$$

$$\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} e^{-2t} - \frac{dy}{dt} \cdot e^{-2t}$$

bulunur. Denklemde yazarsak

$$e^{2t}\left(\frac{d^2y}{dt^2} e^{-2t}\right) - e^{2t} \frac{dy}{dt} e^{-2t} + 3e^{-t} \frac{dy}{dt} e^{-t} + y = \sin t$$

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = \sin t$$

$r^2 + 2r + 1 = 0 \rightarrow (r+1)^2 = 0 \quad r_{1,2} = -1 \qquad \underline{y_h = (c_1 + c_2 t)e^{-t}}$

$y_0 = A\cos t + B\sin t$

$y_0' = -A\sin t + B\cos t$

$y_0'' = -A\cos t - B\sin t$

$-A\cancel{\cos t} - B\cancel{\sin t} + 2(-A\sin t + B\cos t) + A\cancel{\cos t} + B\cancel{\sin t} = \sin t$

$-2A\sin t + 2B\cos t = \sin t$

$-2A = 1 \rightarrow A = -\frac{1}{2} \qquad B = 0$

$y_0 = -\frac{1}{2}\cos t$

$y = (c_1 + c_2 t)e^{-t} - \frac{1}{2}\cos t$

$y = (c_1 + c_2 \ln x)\frac{1}{x} - \frac{1}{2}\cos(\ln x)$

2. $y''' + y'' = 3e^{-x}$ diferansiyel denkleminin genel çözümünü bulunuz

$\underline{y_h}$

$r^3 + r^2 = 0 \qquad r^2(r+1) = 0 \quad \rightarrow r_{1,2} = 0 \quad r_3 = -1 \qquad \boxed{y_h = c_1 + c_2 x + c_3 e^{-x}}$

$\underline{y_{0i}}$

$y_{0i} = Axe^{-x}$ $(r_3 = -1$ 'e α olduğundan$)$

$y_{0i}' = Ae^{-x} - Axe^{-x}$

$y_{0i}'' = -Ae^{-x} + Axe^{-x} - Ae^{-x} = -2Ae^{-x} + Axe^{-x}$

$y_{0i}''' = 2Ae^{-x} + Ae^{-x} - Axe^{-x} = 3Ae^{-x} - Axe^{-x}$

denklemde yazarsak

$3Ae^{-x} - A\cancel{xe^{-x}} - 2Ae^{-x} + A\cancel{xe^{-x}} = 3\cancel{e^{-x}} \rightarrow A = 3 \rightarrow y_{0i} = 3xe^{-x}$

$\boxed{\text{Genel Çözüm: } y = y_h + y_{0i} = c_1 + c_2 x + c_3 e^{-x} + 3xe^{-x}}$

3. $y'' + 4y = \sec 2x$ diferansiyel denkleminin genel çözümünü bulunuz.

$y'' + 4y = \sec 2x = \dfrac{1}{\cos 2x}$

$y'' + 4y = 0 \qquad r^2 + 4 = 0 \quad \Rightarrow \quad r_{1,2} = \mp 2i$

$y = c_1 \cos 2x + c_2 \sin 2x$

$\sin 2x / \; c_1' \cos 2x + c_2' \sin 2x = 0$

$\cos 2x / \; -2c_1' \sin 2x + 2c_2' \cos 2x = \dfrac{1}{\cos 2x}$

$2c_1' \sin 2x \cancel{\cos 2x} + 2c_2' \sin^2 2x = 0$

$-2c_1' \sin 2x \cancel{\cos 2x} + 2c_2' \cos^2 2x = 1$

$2c_2' = 1 \quad \Rightarrow \quad c_2' = \dfrac{1}{2} \qquad \int dc_2 = \dfrac{1}{2}\int dx \rightarrow \boxed{c_2 = \dfrac{x}{2} + k_2}$

$c_1' \cos 2x + \dfrac{1}{2}\sin 2x = 0$

$c_1' = -\dfrac{1}{2}\dfrac{\sin 2x}{\cos 2x} \qquad \int dc_1 = -\dfrac{1}{2}\int \dfrac{\sin 2x}{\cos 2x}\,dx$

$\boxed{c_1 = \dfrac{1}{4}\ln \cos 2x + k_1}$

$y = \left(\dfrac{1}{4}\ln \cos 2x + k_1\right)\cos 2x + \left(\dfrac{x}{2} + k_2\right)\sin 2x$

5. $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+4y=x+x^2\ln x$ diferansiyel denkleminin genel çözümünü bulunuz.

Euler Df Denk.

(1) $x=e^t$; $\dfrac{dx}{dt}=e^t$, $\dfrac{dt}{dx}=e^{-t}$

$$\frac{dy}{dx}=\frac{dy}{dt}\,\frac{dt}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\frac{dy}{dt}}{e^t}=e^{-t}\frac{dy}{dt}$$ (1)

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(e^{-t}\frac{dy}{dt}\right)=\frac{d}{dt}\left(e^{-t}\frac{dy}{dt}\right)\frac{dt}{dx}=\frac{-e^{-t}\frac{dy}{dt}+e^{-t}\frac{d^2y}{dt^2}}{e^t}$$

$$\frac{d^2y}{dx^2}=-e^{-2t}\frac{dy}{dt}+e^{-2t}\frac{d^2y}{dt^2}$$ (3)

$$e^{2t}\left[-e^{-2t}\frac{dy}{dt}+e^{-2t}\frac{d^2y}{dt^2}\right]-3e^t\left(e^{-t}\frac{dy}{dt}\right)+4y=e^t+e^{2t}.t$$

$$-\frac{dy}{dt}+\frac{d^2y}{dt^2}-3\frac{dy}{dt}+4y=e^t+te^{2t}$$ (3)

$$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+4y=e^t+te^{2t}$$

$k(r)=r^2-4r+4=0$, $r_{1,2}=2$

$y_h=(c_1+tc_2)e^{2t}$ (2) , $y_{21}=ke^t$ (2), $y_2'=ke^t$, $y_2''=ke^t$

$ke^t-4ke^t+4ke^t=e^t$ $k=1$, $y_{21}=e^t$ (3)

$y_{22}=$ (2) ze^{2t}, $y_{22}'=z'e^{2t}+2ze^{2t}$, $y_2''=z''e^{2t}+2z'e^{2t}+4ze^{2t}$?

$(z''+2z'+4z)e^{2t}+(-4z'-8z)e^{2t}+4ze^{2t}=te^{2t}$

$\cancel{z''-2z'=t}$ (2) $\quad z=t(At+B)=At^2+Bt \quad 2A-4At-2B=t$

$\cancel{r^2-2r=0}$ $\quad z'=2At+B \quad -4A=1 \quad A=-1/4$

$r_1=0, r_2=2 \quad z''=2A \quad 2A-2B=0 \quad B=-1/4$

$z''=t$

$z=\cancel{-\frac{1}{4}t^2-\frac{1}{4}t}+\frac{1}{6}t^3$ $\quad y_{22}=\left(\cancel{-\frac{1}{4}t^2-\frac{1}{4}t}\ \frac{1}{6}t^3\right)e^{2t}$ (3)

$y_G=y_h+y_{21}+y_{22}$, $y_{GG}=(c_1+tc_2)e^{2t}+e^t+\left(-\frac{1}{4}t^2-\frac{1}{4}t\right)e^{2t}$ (2)

x'li yaz. (3)

SORU 2.) $y''' + y' = 2\sin x - 2\cos x + 1$ diferansiyel denklemini **Belirsiz Katsayılar Yöntemiyle** çözünüz (25 Puan)

CEVAP 2) $y''' + y' = 2\sin x - 2\cos x + 1$

$K(r) = r^3 + r = 0 \Rightarrow r(r^2+1) = 0$; $r = 0$, $r = \mp i$ ②

$y_h = c_1 + c_2 \cos x + c_3 \sin x$ ③

$\boxed{y_1 = x \cdot [A\cos x + B\sin x]}$ ②

$y_1' = A\cos x + B\sin x + x \cdot [-A\sin x + B\cos x]$

$\boxed{y_1' = (A + Bx)\cos x + (B - Ax)\sin x}$ ②

$y_1'' = B\cos x - (A+Bx)\sin x - A\sin x + (B - Ax)\cos x$

$= \boxed{(2B - Ax)\cos x + (-2A - Bx)\sin x}$ ②

$y_1''' = -A\cos x + (2B - Ax)(-\sin x) - B\sin x$

$+ (-2A - Bx)\cos x$

$y_1''' = \boxed{(-3A - Bx)\cos x + (-3B + Ax)\sin x}$ ②

$(-3A - \cancel{Bx})\cos x + (-3B + \cancel{Ax})\sin x + (A + \cancel{Bx})\cos x + (B - \cancel{Ax})\sin x$

$\equiv 2\sin x - 2\cos x$

$(-2A)\cos x + (-2B)\sin x \equiv 2\sin x - 2\cos x$ ③

$-2B = 2 \Rightarrow B = -1$

$-2A = -2 \Rightarrow A = 1$

$\boxed{y_1 = x(\cos x - \sin x)}$ ②

$y_2 = kx$; $y_2' = k$; $y_2'' = 0$; $y_2''' = 0$; $0 + k = 1$; $\boxed{k = 1}$; $\boxed{y_2 = x}$ 4

$y = c_1 + c_2\cos x + c_3 \sin x + x(\cos x - \sin x) + x$ [illegible]

C.1) 2014 YAZ DÖNEMİ DİF. DENKL. 2. SINAV
ÇÖZÜM ANAHTARI 2 VİZE

25P $y'' - 2y(y')^3 = 2y \cdot y'$ diferansiyel denklemini çözerek y' türevini bulunuz. (y' türevinin tüm çözümlerini bulunuz)

2 $y' = p \quad y'' = \frac{dp}{dx} = \frac{dp}{dy} \cdot \frac{dy}{dx} = p \cdot \frac{dp}{dy}$ 3

$p \cdot \frac{dp}{dy} - 2y \cdot p^3 = 2y \cdot p$ 5

$p\left[\frac{dp}{dy} - 2y p^2 - 2y\right] = 0 \quad p = 0 \quad \frac{dy}{dx} = 0$ 2

$\frac{dp}{dy} - 2y \cdot p^2 - 2y = 0$ 2 T.14

$\frac{dp}{dy} = 2y(p^2+1)$ 5 $\quad \int \frac{dp}{p^2+1} = \int 2y\,dy$ 2 T.19

$\arctan p = y^2 + C_2$ 3

$p = y' = \tan(y^2 + C_2)$ 3

S.2) $y''' + y' = -2\sin x + 2\cos x + 1$.
diferansiyel denklemini belirsiz katsayılar yöntemiyle çözünüz.

C.2)

$K(r) = r^3 + r = 0 \quad r_1 = 0 \quad r_{2,3} = \mp i$

$y_h = C_1 + C_2\cos x + C_3\sin x$ (5) //

$y_{2ö_1} = x(A\cos x + B\sin x)$ (2) //

$y'_{2ö_1} = A\cos x + B\sin x - Ax\sin x + Bx\cos x$ (2) //

$y''_{2ö_1} = -A\sin x + B\cos x - A\sin x - Ax\cos x + B\cos x - Bx\sin x$ (2) //

$-A\sin x + B\cos x - A\sin x - \cancel{Ax\cos x} + B\cos x - \cancel{Bx\sin x}$
$+ \cancel{Ax\cos x} + \cancel{xB\sin x} = -2\sin x + 2\cos x$

$-2A\sin x + 2B\cos x = -2\sin x + 2\cos x$ (2) //

$-2A = -2 \quad A = 1$ // $\quad 2B = 2 \quad \boxed{B = 1}$

$\boxed{y_{2ö_1} = x(\cos x + \sin x)}$ (3) //

$y_{2ö_2} = kx \quad y'_{2ö_2} = k, \quad y''_{2ö_2} = 0, \quad y'''_{2ö_2} = 0$

$0 + k = 1 \quad \boxed{k = 1}$

$y_{2ö_2} = x$ (5) //

$y_G = y_h + y_{2ö_1} + y_{2ö_2} = C_1 + C_2\cos x + C_3\sin x +$
$+ x(\cos x + \sin x) + x$

(3) //

S.3) $y'' + y = \sec x + \operatorname{cosec} x$ diferansiyel denklemini sabitin değişimi (Parametrelerin değişimi) (Lagrange) yöntemiyle çözünüz.

C.3) $K(r) = r^2 + 1 = 0 \quad r_{1,2} = \mp i$ (2) //

$y_G = C_1 \cos x + C_2 \sin x$ (3) //

$y_G' = C_1' \cos x + C_1(-\sin x) + C_2' \sin x + C_2 \cos x$

$\boxed{C_1' \cos x + C_2' \sin x = 0}$ 1. Denklem (3) //

$y_G' = -C_1 \sin x + C_2 \cos x$

$y_G'' = -C_1' \sin x - C_1 \cos x + C_2' \cos x - C_2 \sin x$

$-C_1' \sin x - \cancel{C_1 \cos x} + C_2' \cos x - \cancel{C_2 \sin x} + \cancel{C_1 \cos x} + \cancel{C_2 \sin x} = \sec x + \operatorname{cosec} x$

$\boxed{-C_1' \sin x + C_2' \cos x = \sec x + \operatorname{cosec} x}$ 2. denklem (4) //

$\sin x \,/\, C_1' \cos x + C_2' \sin x = 0 \,/\, \sin x$

$\cos x \,/\, -C_1' \sin x + C_2' \cos x = \sec x + \operatorname{cosec} x \,/\, \cos x$

$(\underbrace{\sin^2 x + \cos^2 x}_{1}) C_2' = 1 + \cot x$

$\int dC_2 = \int (1 + \cot x) dx$

$C_2 = x + \ln|\sin x| + K_2$ (5) //

$C_1' \cos x + \sin x (1 + \cot x) = 0$

$C_1' \cos x = -\sin x - \cos x$

$C_1' = -\tan x - 1$ (5) //

$\int dC_1 = \int (-\tan x - 1) dx \Rightarrow C_1 = \ln|\cos x| - x + K_1$

$y_G = [(\ln|\cos x| - x + K_1)] \cos x + [x + \ln|\sin x| + K_2] \sin x$ (3) //