2. $\int \frac{2}{1-e^{2x}}\,dx = ?$ (25 p)

$$I = \int \frac{2}{1-e^{2x}}\,dx = \int \frac{2}{(1+e^x)(1-e^x)}\,dx$$

5 $\begin{cases} e^x = u \\ e^x\,dx = du \end{cases} \Rightarrow dx = \frac{du}{u}$

$$I = 2\int \frac{du}{u(1+u)(1-u)} \quad 5$$

$A - Au^2 - Bu^2 + Bu + Cu + Cu^2 = 2$

$$\frac{2}{u(1+u)(1-u)} = \frac{A}{u} + \frac{B}{1+u} + \frac{C}{1-u}$$

$1-u^2$ $\quad (1-u)u$ $\quad u(1+u)$

$A = 2,\ B = -1,\ C = 1$

$-A - B + C = 0$
$B + C = 0$
$A = 2$
$-B + C = 2$
$B + C = 0$
$2C = 2$
$C = 1$
$B = -1$

$$I = \int \left(\frac{2}{u} - \frac{1}{1+u} + \frac{1}{1-u}\right) du \quad 5$$

$$= 2\ln|u| - \ln|1+u| - \ln|1-u| + C \quad 5$$

$$= 2x - \ln(1+e^x) - \ln|1-e^x| + C \quad 5$$

---

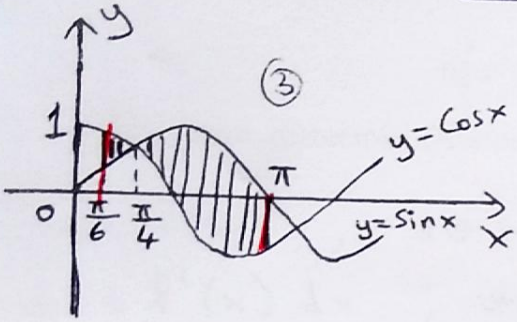
2. yol: $1 - e^{2x} = t$ 5 dönüşümü ile, $\frac{A}{t} + \frac{B}{t-1} = \frac{1}{t(t-1)}$ 5 $At - A + Bt = 1$, $A = -1$, $B = 1$

$$I = \int \frac{dt}{t(t-1)} \quad 5 = \int \left(\frac{1}{t-1} - \frac{1}{t}\right) dt \quad 5 = \ln|t-1| - \ln|t| + C$$

$$= \ln\left[\frac{e^{2x}}{|1-e^{2x}|}\right] + C \quad 5$$ sonuca elde edilir.

Not: Aynı integral $e^{2x} = v$ dönüşümü ile de elde edilir.
Sabiti yazmayandan 2p (iki) kırılacaktır.

3. $\frac{\pi}{6} \le x \le \pi$ olmak üzere, $y = \sin x$ ve $y = \cos x$ eğrileri arasındaki bölgenin alanını hesaplayınız. **(Şekil çiziniz)** *(25 p.)*



3

2

$\sin x = \cos x \Rightarrow x = \frac{\pi}{4} \in \left[\frac{\pi}{6}, \pi\right]$

4 4

$$A = \int_{\pi/6}^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi} (\sin x - \cos x)\,dx$$

4 4

$$= (\sin x + \cos x \Big|_{\pi/6}^{\pi/4} + (-\cos x - \sin x \Big|_{\pi/4}^{\pi}$$

$$= 2\sqrt{2} + \frac{1-\sqrt{3}}{2}.$$

2 2

4. a) f fonksiyonu, $f(x)=\int_0^{x^3} e^{-t^2}\,dt$ integrali ile tanımlanmış olsun.

$$6\int_0^1 x^2 f(x)\,dx - 2\int_0^1 e^{-x^2}\,dx = e^{-1}-1$$

olduğunu gösteriniz. (Kısmi integrasyon tekniği kullanılabilir) *(15 p.)*

$u=f(x)$, $dv=6x^2\,dx$ dersek
$du=f'(x)\,dx$, $v=2x^3$ olur. (3)

Diferansiyel hesabın temel teoreminden,
$f'(x)=3x^2e^{-x^6}$ olur. (3)

$$6\int_0^1 x^2 f(x)\,dx = 2x^3 f(x)\Big|_0^1 - \int_0^1 6x^5 e^{-x^6}\,dx = \quad (3)$$

$$=2\int_0^1 e^{-t^2}\,dt + e^{-x^6}\Big|_0^1 = 2\int_0^1 e^{-t^2}\,dt + e^{-1}-1 \quad (3)$$

Sağ taraftaki integralde t yerine x yazarak sol tarafa geçirirsek,

$$6\int_0^1 x^2 f(x)\,dx - 2\int_0^1 e^{-x^2}\,dx = e^{-1}-1 \quad (3)$$ elde edilir.

---

b) $\int_{-1}^{1}\frac{dx}{\sqrt[3]{x}}=?$ *(10 p.)*

$f(x)=\frac{1}{\sqrt[3]{x}}$ (2), $x=0$ da süreksizdir. İntegral improper integral

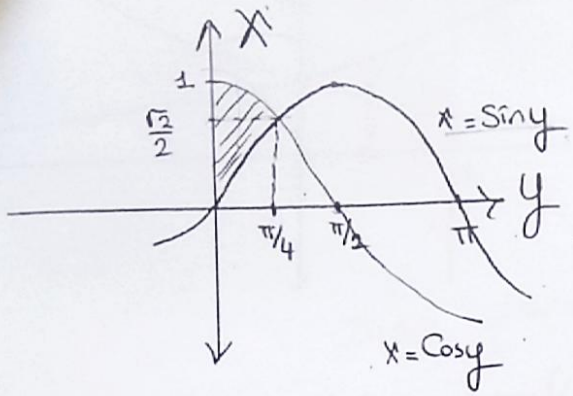
$$\int_{-1}^{1}\frac{dx}{\sqrt[3]{x}} = \int_{-1}^{0} x^{-1/3}\,dx + \int_0^1 x^{-1/3}\,dx = \lim_{b\to 0^-}\int_{-1}^{b} x^{-1/3}\,dx + \lim_{a\to 0^+}\int_a^1 x^{-1/3}\,dx \quad (2)$$

$$=\lim_{b\to 0^-}\frac{3}{2}\sqrt[3]{x^2}\Big|_{-1}^{b} + \lim_{a\to 0^+}\frac{3}{2}\sqrt[3]{x^2}\Big|_a^1 \quad (2)$$

$$=\frac{3}{2}\lim_{b\to 0^-}\left(\sqrt[3]{b^2}-1\right)+\frac{3}{2}\lim_{a\to 0^+}\left(1-\sqrt[3]{a^2}\right)=-\frac{3}{2}+\frac{3}{2}$$

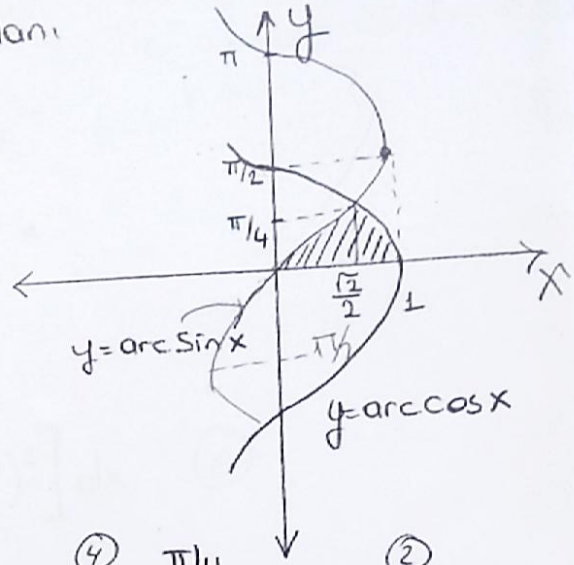
NOT: f, $[-a,a]$ aralığında tanımlanmış, sürekli ve tek fonk. ise
$\int_{-a}^{a} f(x)\,dx=0$ dır. $\Rightarrow \int_{-1}^{1}\frac{dx}{\sqrt[3]{\ }}=0$ ve $\int_{-1}^{1} x^{-1/3}\,dx=\frac{3}{2}x^{2/3}\Big|_{-1}^{1}=0$

2) a) $y = \arccos x$, $y = \arcsin x$ eğrileri ile x-ekse-
-ninin sınırladığı bölgenin alanı



ya da

(2)



1. yol: $A = \int_0^{\pi/4} (\cos y - \sin y)\,dy = (\sin y + \cos y)\Big|_0^{\pi/4} = \sqrt{2} - 1$ (4) (4) (2)

2. yol: $A = \int_0^{\sqrt{2}/2} (\text{Arc}\sin x)\,dx + \int_{\sqrt{2}/2}^{1} (\text{Arc}\cos x)\,dx$ (2) (2)

$\begin{bmatrix} u = \text{Arc}\sin x & dx = dv \\ du = \dfrac{1}{\sqrt{1-x^2}} & x = v \end{bmatrix}$ $\begin{bmatrix} u = \text{Arc}\cos x & dx = dv \\ du = \dfrac{-1}{\sqrt{1-x^2}} & x = v \end{bmatrix}$

$A = x\,\text{Arc}\sin x\Big|_0^{\sqrt{2}/2} - \int_0^{\sqrt{2}/2} \frac{x}{\sqrt{1-x^2}}\,dx + x\,\text{Arc}\cos x\Big|_{\sqrt{2}/2}^{1} + \int_{\sqrt{2}/2}^{1} \frac{x}{\sqrt{1-x^2}}\,dx$ (2) (2)

$1 - x^2 = t^2$

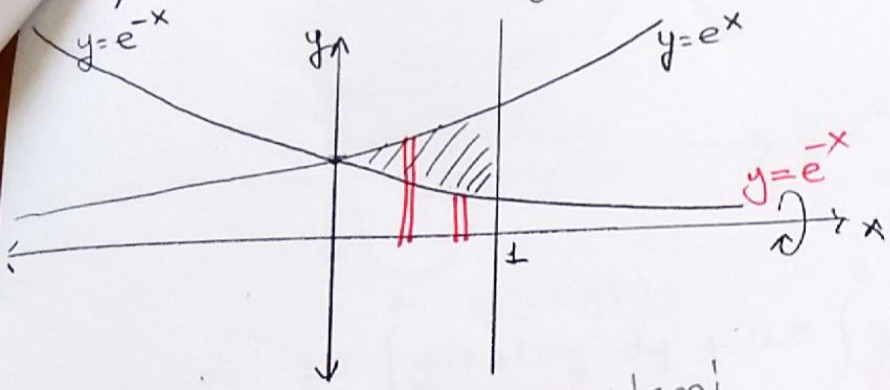
$-2x\,dx = 2t\,dt$

$= \cancel{\frac{\sqrt{2}}{2}\text{Arc}\sin\frac{\sqrt{2}}{2}} + \sqrt{1-x^2}\Big|_0^{\sqrt{2}/2} + \underbrace{\text{Arc}\cos 1}_{0} - \cancel{\frac{\sqrt{2}}{2}\text{Arc}\cos\frac{\sqrt{2}}{2}} - \sqrt{1-x^2}\Big|_{\sqrt{2}/2}^{1}$ (1)

$= \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}} = \sqrt{2} - 1$ (1)

2) b) D: ($y = e^{x}$, $y = e^{-x}$, $x = 1$) alanın Ox ekseni etrafında döndürülmesi ile oluşan cismin hacmi

N. G



Disk yöntemi

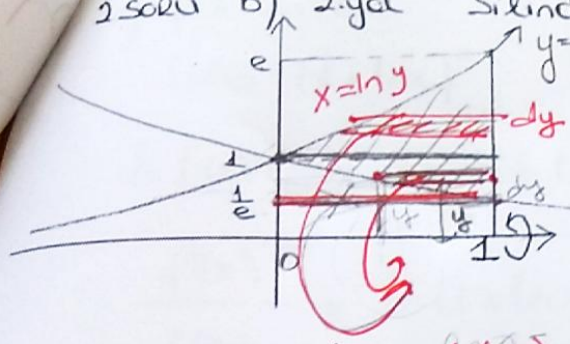
$$V = \int_0^1 \pi\left[(e^{x})^2 - (e^{-x})^2\right] dx \quad (6)$$

$$= \pi \int_0^1 \left(e^{2x} - e^{-2x}\right) dx \quad (3)$$

$$= \pi \left[\frac{e^{2x}}{2} + \frac{e^{-2x}}{2}\right]_0^1 \quad (2)$$

$$= \frac{\pi}{2}\left(e^{2} + e^{-2} - 2\right) \quad (2)$$

2 soru b) 2.yol Silindirik Kabuk Yöntemi $y=e^x$ $(x=\ln y)$ ox eks. etr. dön.



$y=e^{-x}$ $(x=-\ln y)$

$\ln y=-x$ $e^{-x}=e^{x}$ $1=e^{2x} \Rightarrow x=0$

$$V=2\pi\underbrace{\int_{1/e}^{1} \underbrace{y}_{K.Y.}\underbrace{(1+\ln y)}_{K.Ylk.}dy}_{\text{alttaki Kabuk}}+2\pi\underbrace{\int_{1}^{e} \underbrace{y}_{K.Yar.}\underbrace{(1-\ln y)}_{K.Ylk.}dy}_{\text{üstteki Kabuk}}$$

$1-(-\ln y)$ ⑥

$$=2\pi\left[\int_{1/e}^{e} y\,dy+\int_{1/e}^{1} y\ln y\,dy-\int_{1}^{e} y\ln y\,dy\right]$$

$\ln y=u$ $y\,dy=dv$
$\frac{1}{y}dy=du$ $\frac{y^2}{2}=v$

③ $$=2\pi\left[\left(\frac{y^2}{2}\right)_{1/e}^{e}+\left(\frac{1}{2}y^2\ln y-\frac{1}{4}y^2\right)_{1/e}^{1}-\left(\frac{1}{2}y^2\ln y-\frac{1}{4}y^2\right)_{1}^{e}\right]$$

② $$=2\pi\left[\frac{1}{2}\left(e^2-\frac{1}{e^2}\right)+\left(-\frac{1}{4}+\frac{3}{4e^2}\right)-\left(\frac{1}{4}e^2+\frac{1}{4}\right)\right]$$

$$=2\pi\left[\frac{1}{4}e^2+\frac{1}{4e^2}-\frac{1}{2}\right]$$

② $$=\frac{\pi}{2}\left(e^2+e^{-2}-2\right)$$

b) $g(x) = \int_{x}^{f(x)} \cos^{12} t \, dt$ $\qquad g'(1) = ?$

3

$g'(x) = f'(x) \cdot \cos^{12} f(x) - 1 \cos^{12} x$ (5)

$g'(1) = f'(1) \cdot \cos^{12} f(1) - \cos^{12} 1$ (3)

$= 1 \cdot \left[\ln(1 + \ln 1) + 1 \right] \cdot \cos^{12} \left[(1 + \ln 1)^1 \right] - \cos^{12} 1$ (3)

$= \cos^{12} 1 - \cos^{12} 1 = 0$ (2)

$\ldots (e^h - 1) \quad e^h - 1 \,]$

a) N.G $I = \int \frac{e^{3x}}{e^{2x}+e^x-2}\,dx = \int \frac{t^2}{t^2+t-2}\,dt$ ①

$\left(e^x = t,\ e^x dx = dt\right)$ ① $\left(\frac{t^2}{t^2+t-2} = 1 - \frac{t-2}{t^2+t-2}\right)$ ③

$I = \int \left(1 - \frac{t-2}{(t-1)(t+2)}\right)dt$ ② $= \int \left(1 - \frac{1/3}{t-1} + \frac{4/3}{t+2}\right)dt$

$\left(\frac{A}{t-1} + \frac{B}{t+2} = \frac{t-2}{(t-1)(t+2)}\right)$ $At + 2A + Bt - B \equiv t - 2$

$(t+2)$ $(t-1)$

$t \to A + B = 1$

sbt $\to 2A - B = -2$ $\Big\}$ $A = -\frac{1}{3}$ ② , $B = \frac{4}{3}$ ②

$I = t - \frac{1}{3}\ln|t-1| + \frac{4}{3}\ln|t+2| + c$ ③

$= e^x - \frac{1}{3}\ln|e^x - 1| + \frac{4}{3}\ln|e^x + 2| + c$ ②

N.G b) $\int_0^\infty e^{-\sqrt{x}}\,dx = I$ $t = u$ $e^{-t}dt = dv$

12

$u.v - \int v\,du = -te^{-t} - \int -e^{-t}.dt$

$\int e^{-\sqrt{x}}\,dx = 2\int e^{-t}\,t\,dt$ ① $= -2\left(t.e^{-t} + e^{-t}\right) + c$

② $\left(\sqrt{x} = t,\ \frac{1}{2\sqrt{x}}dx = dt\right)$ $\left(\begin{matrix} t = u & e^{-t}dt = dv \\ dt = du & -e^{-t} = v \end{matrix}\right)$ ②

① $I = \lim_{A\to\infty}\left[-2\left(\sqrt{x}\,e^{-\sqrt{x}} + e^{-\sqrt{x}}\right)\right]\Big|_0^A = \lim_{A\to\infty}\left[-2\left(\sqrt{A}\,e^{-\sqrt{A}} + e^{-\sqrt{A}}\right) + 2\right]$ ①

$\underbrace{e^{-\sqrt{A}}(\sqrt{A}+1)}_{0.\ \infty\ B.S.}$

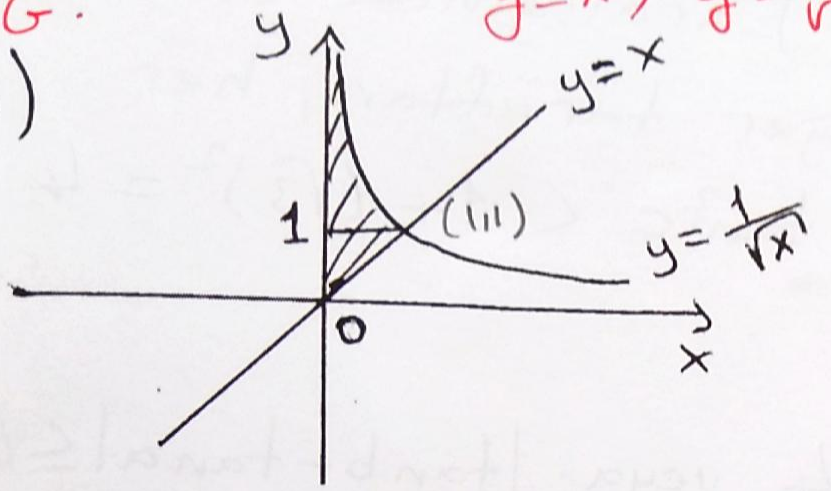
$= 2 - 2\lim_{A\to\infty}\frac{\sqrt{A}+1}{e^{\sqrt{A}}} \overset{\frac{\infty}{\infty}}{\underset{L}{=}} 2 - 2\lim \frac{\frac{1}{2\sqrt{A}}}{\frac{1}{2\sqrt{A}}.e^{\sqrt{A}}} = 2$ ①

① ② $\underbrace{\qquad}$

$x \to 0$

. G. $y = x$, $y = \frac{1}{\sqrt{x}}$ eğrinin $x=0$ doğru ile bölgenin alanı?

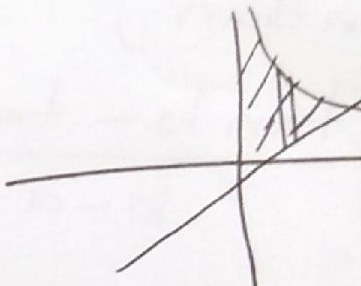
b)



(2)

$x = \frac{1}{\sqrt{x}}$

$x^2 = \frac{1}{x}$



$\int_{x=0}^{x=1} \left(\frac{1}{\sqrt{x}}\right.$

$x=0$ d

$\lim_{c \to 0^+}$

$$A = \int_0^1 y\,dy + \int_1^\infty \frac{1}{y^2}\,dy \quad (3)$$

$$= \frac{y^2}{2}\Big|_0^1 + \lim_{k\to\infty} \int_1^k \frac{dy}{y^2} \quad (2)$$

$$= \frac{1}{2} + \lim_{k\to\infty} \left(-\frac{1}{y}\right|_1^k \quad (3)$$

$$= \frac{1}{2} + \lim_{k\to\infty} \left(-\frac{1}{k} + 1\right) = \frac{1}{2} + 1 = \frac{3}{2}$$

(2)

N.Güzel

3.SORU: $f(x)=\dfrac{(x+1)^2}{x-2}$

Tanım Kümesi

a) $D(f)=\mathbb{R}\setminus\{2\}=(-\infty,2)\cup(2,+\infty)$ 2

$x=2$ de ve $x\to\mp\infty$ iken $f(x)$ in durumunu inceleyelim.

b) $\lim\limits_{x\to 2^+}\dfrac{(x+1)^2}{x-2}=\lim\limits_{h\to 0}\dfrac{(3+h)^2}{2+h-2}=\infty$ $\left(\lim\limits_{h\to 0}\dfrac{(2+h+1)^2}{\cancel{2}+h-\cancel{2}}=\dfrac{3^2}{0}=\infty\right)$

$\lim\limits_{x\to 2^-}\dfrac{(x+1)^2}{x-2}=\lim\limits_{h\to 0}\dfrac{(3-h)^2}{2-h-2}=-\infty$ $\left(\lim\limits_{h\to 0}\dfrac{(2-h+1)}{\cancel{2}-h-\cancel{2}}=\dfrac{3}{-0}=-\infty\right)$

$\boxed{x=2}$ düşey asimtot ✓ 3

c) $\lim\limits_{x\to\pm\infty}\dfrac{(x+1)^2}{x-2}=\pm\infty$ yatay asimtot yok. Eğik Asimptot olabilir.

d) $m=\lim\limits_{x\to\pm\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\pm\infty}\dfrac{(x+1)^2}{x(x-2)}=1$

$n=\lim\limits_{x\to\pm\infty}(f(x)-mx)=\lim\limits_{x\to\pm\infty}\dfrac{4x+1}{x-2}=4$

Yada

$$\begin{array}{r|l} x^2+2x+1 & x-2 \\ \hline \pm x^2\pm 2x & x+4 \\ \hline 4x+1 & \\ -4x\pm 8 & \\ \hline 9 & \end{array}$$

E.A

$\boxed{y=x+4}$ eğik asimtot 3

e) $y'=\dfrac{(x-5)(x+1)}{(x-2)^2}$ $f'(x)=0$ $x=-1$, $x=5$

$\left.\begin{array}{l} x=-1 \\ x=5 \\ x=2\notin D(f) \end{array}\right\}$ K.N.

f) $y''=\dfrac{18}{(x-2)^3}$

$y''(-1)<0$ $x=-1$ max $f(-1)=0$

$y''(5)>0$ $x=5$ min $f(5)=\dfrac{36}{3}$

Özel Noktar (Eksenleri Lentiği noktalar)

g) $x=0 \Rightarrow y=-1/2$

$y=0 \Rightarrow x=-1$ 2

	$-\infty$	-1		0		2		5		$+\infty$
f'	$+$	0	$-$		$-$	$\|$	$-$	0	$+$	$+$
f''	$-$		$-$		$-$	0	$+$		$+$	
f	$\nearrow$	max	$\searrow$		$\searrow$		$\searrow$	min	$\nearrow$	
f	$\frown$		$\smile$		$\frown$		$\smile$		$\frown$	

$-\infty$ 0 $-4/2$ $-\infty$ ∞ 12 $+\infty$

$x=-1$'de yerel max

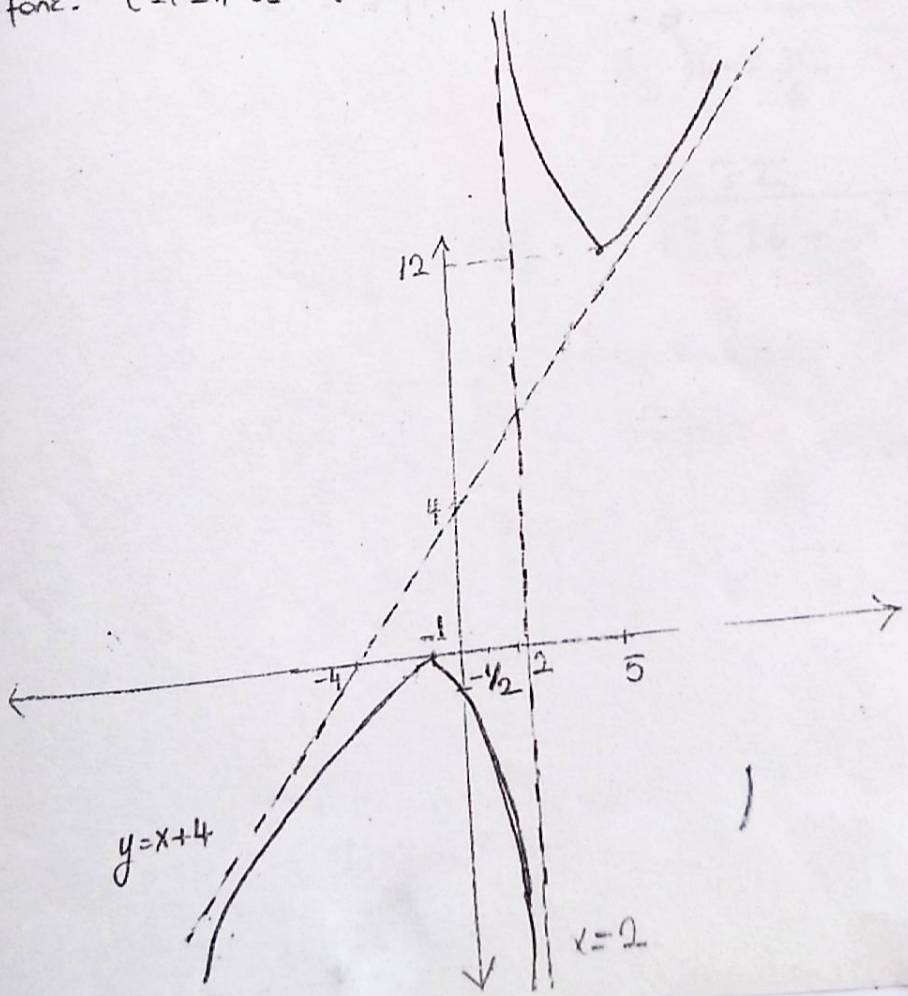
$x=5$'de yerel min

5

bükülme noktası yok (çünkü $x=2$ düşey asimtot)

$f(x)$-fonk. $(-\infty,-1)$ ve $(5,\infty)$ da artandır.

$f(x)$-fonk. $(-1,2)$ ve $(2,5)$ de azalandır



10

Mat. I (5 saat) (B Grubu) 2. Vize Cevaplar

22. 12. 2012

N. Gutel

$\int_0^{\sin x} f(t)dt = \operatorname{arccotg} x$ old. gore

$$\frac{d}{dx}\left(\int_0^{\sin x} f(t)\,dt\right) = \frac{d}{dx}\left(\operatorname{arc\,cotg} x\right) \rightarrow 5$$

$$\cos x \cdot f(\sin x) = \frac{-1}{1+x^2} \rightarrow 10$$

5

$$f(\sin x) = \frac{-1}{(1+x^2)\cos x}$$

$x \in (0, \frac{\pi}{2})$ olup, $f(\frac{1}{2})$ istendiginden $x = \frac{\pi}{6}$

$$f\left(\sin\frac{\pi}{6}\right) = f\left(\frac{1}{2}\right) = \frac{-1}{\left(1+\frac{\pi^2}{36}\right)\cos\frac{\pi}{6}}$$

$$= \frac{-1}{\frac{36+\pi^2}{36}\cdot\frac{\sqrt{3}}{2}} = \frac{-72}{\sqrt{3}(36+\pi^2)} \quad 5$$

4 f $(0, \frac{\pi}{2})$ aralıgında sürekli bir fonksiyon olmak üzere;

$$\int_0^{\sin(x)} f(t)\,dt = \cot^{-1}(x) = \operatorname{arccot}(x)$$

ise $f(1/2)$ degerini bulunuz

11

a-) $\int \frac{\sqrt{x}}{x(1+x)}\,dx = ? \int \frac{dx}{\sqrt{x}\,(1+x)} = \int \frac{2\,du}{1+u^2}$ (2)

N.GÜZ

$$\begin{cases} \sqrt{x} = u \\ \frac{dx}{2\sqrt{x}} = du \end{cases}$$

(2)

$= 2.\operatorname{arctg} u + C$

$= 2.\operatorname{arctg}\sqrt{x} + C$ (1)

(2)

7

b-) N.Güzel

$x^2 + xy + \frac{1}{4}y^2 = 0$, $-2 \le y \le 0$ aralığında yay uzunluğunu bulunuz.

$F(x,y) = x^2 + xy + \frac{1}{4}y^2$

$\frac{dx}{dy} = -\frac{F_y}{F_x} = -\frac{x + \frac{1}{2}y}{2x + y} = -\frac{1}{2}$ (6)

$S = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy = \int_{-2}^{0} \sqrt{1 + \left(-\frac{1}{2}\right)^2}\,dy$ (3)

(2)

$= \int_{-2}^{0} \sqrt{1 + \frac{1}{4}}\,dy = \sqrt{\frac{5}{4}} \cdot 2 = \sqrt{5}$ (3)

Veya : $x^2 + xy + \frac{1}{4}y^2 = \left(\frac{1}{2}y + x\right)^2 = 0$ (4)

$\frac{1}{2}y$

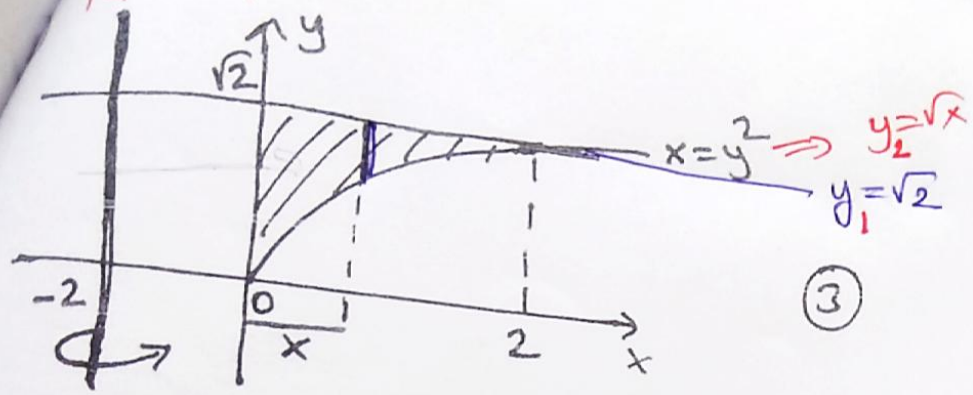
$\frac{1}{2}y$

$y = -2x \Rightarrow x = -\frac{1}{2}y \Rightarrow \frac{dx}{dy} = -\frac{1}{2}$ (2)

$S = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy = \int_{-2}^{0} \sqrt{1 + \frac{1}{4}}\,dy = \sqrt{\frac{5}{4}} \cdot 2 = \sqrt{5}$ (3)

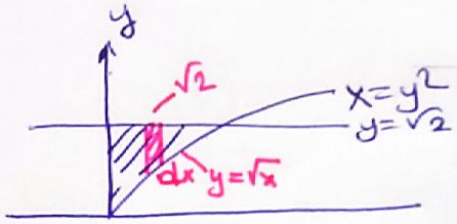
(2) (3)

N Güzel



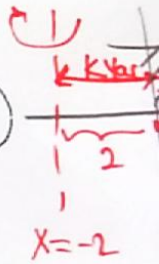
3

a-) $A = \int_0^2 (\sqrt{2} - \sqrt{x})\,dx$ 6



b-) $V_{x=-2} = 2\pi \int_0^2 (x+2)(\sqrt{2} - \sqrt{x})\,dx$ 8

Kabuk Yöntemi K. Yar. K. Yük.



c-) $V_x = \pi \int_0^2 [(\sqrt{2})^2 - (\sqrt{x})^2]\,dx$ 8

$y_1^2 - y_2^2$

Sorusu $x=y^2$, $y=\sqrt{2}$ ve y-ekseni ile sınırlanan bölgenin (şeklini çizerek)

a) Alanın x'e bağlı belirli integral ile ifade ediniz.

b) $x=-2$ doğrusu etrafında döndürülmesi ile elde edilen cismin hacmini silindirik kabuk yöntemini kullanarak belirli integrali ifade edinin

c) x ekseni etrafında döndürülmesi ile oluşan dönel cismin hacmini pul (disk) yöntemiyle ifade ediniz

Soru 2 $f(x) = e^{g(x)}$ ve $g(x) = \int_{x^2}^{x^3} \frac{t^2 dt}{1+t^2}$ ise, $f'(1)$ değerini bulunuz.

$$f'(x) = g'(x) e^{g(x)}$$ 5

$$g'(x) = 3x^2 \left[\frac{x^6}{1+x^6}\right] - 2x \left[\frac{x^4}{1+x^4}\right]$$ 10

$$g'(1) = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$$

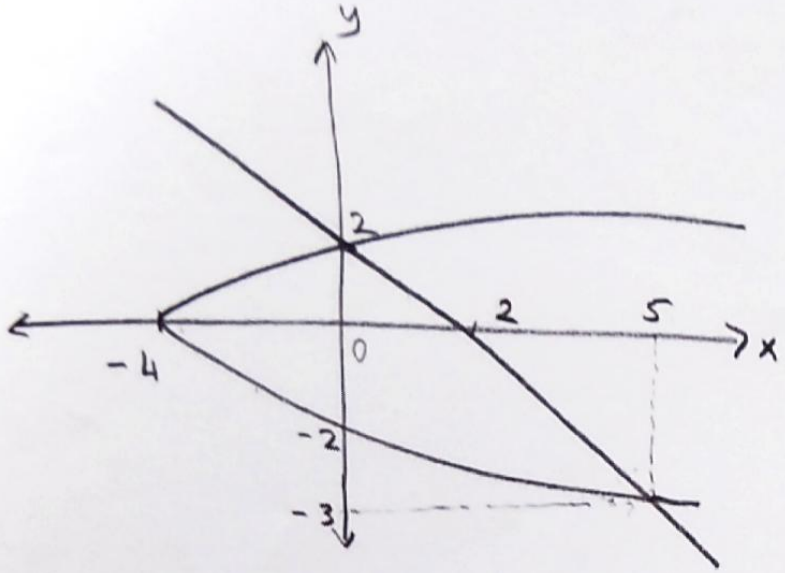
$$g(1) = \int_1^1 \frac{t^2 dt}{1+t^2} = 0$$ 5

$$f'(1) = g'(1) e^{g(1)} = \frac{1}{2} e^0 = \frac{1}{2}$$ 5

Soru 4 $x = y^2 - 4$ parabolü ile $x + y = 2$ doğrusu arasında kalan bölgenin alanını bulunuz.

(şekil çizilecek)

N. Güzel



5

$y^2 - 4 = 2 - y \Rightarrow y^2 + y - 6 = 0$

$(y+3)(y-2) = 0$

$(0,2)$ ve $(5,-3)$ kesişen noktalar

5

I. yol

$$A = \int_{-3}^{2} \left[(2-y) - (y^2-4)\right] dy = \frac{125}{6}$$

15

II. yol

$$A = \int_{-4}^{0} \left[\sqrt{x+4} - \left(-\sqrt{x+4}\right)\right] dx + \int_{0}^{5} \left[(2-x) - \left(-\sqrt{x+4}\right)\right] dx$$

$$= \frac{4}{3}(x+4)^{\frac{3}{2}} \Big|_{-4}^{0} + \left[2x - \frac{x^2}{2} + \frac{2}{3}(x+4)^{\frac{3}{2}}\right]\Big|_{0}^{5}$$

$$= \frac{32}{3} + \frac{61}{6} = \frac{125}{6}$$

4. b) $a \in \mathbb{R}$ olmak üzere, $\forall x > 0$ için $6 + \int_a^x \frac{f(t)}{t^2} dt = 2\sqrt{x}$ eşitliğini sağlayan f fonksiyo

a sayısını bulunuz. *(10 puan)*

İntegral hesabın temel teoreminden,

$$\frac{f(x)}{x^2} = \frac{1}{\sqrt{x}} \Rightarrow \boxed{f(x) = x^{\frac{3}{2}}}$$

$$6 + \int_a^x \frac{t^{3/2}}{t^2} dt = 2\sqrt{x} \Rightarrow 6 + \int_a^x \frac{dt}{\sqrt{t}} = 2\sqrt{x}$$

$$6 + 2\int_a^x \frac{dt}{2\sqrt{t}} = 2\sqrt{x}$$

$$6 + 2\sqrt{t}\Big|_a^x = 2\sqrt{x}$$

$$6 + \cancel{2\sqrt{x}} - 2\sqrt{a} = \cancel{2\sqrt{x}} \Rightarrow \sqrt{a} = 3 \Rightarrow \boxed{a = 9}$$

$$-2\sqrt{a} = -6$$

$$\sqrt{a} = 3 \quad a = 9$$

3. a) $\int_1^e \frac{dx}{\ln x^x}$ integralini hesaplayınız, sonucu yorumlayınız. *(12 puan)*

$$\int_1^e \frac{dx}{\ln x^x} = \int_1^e \frac{dx}{x\ln x} = \lim_{c\to 1^+} \int_c^e \frac{dx}{x\ln x}$$

$\ln x = u \Rightarrow \frac{dx}{x} = du$

$x=e \to u=1$

$x=c \to u=\ln c$

$x=1$ de tanımsız

$$= \lim_{c\to 1^+} \int_{\ln c}^{1} \frac{du}{u} = \lim_{c\to 1^+} \ln u \Big|_{\ln c}^{1}$$

$$= \lim_{c\to 1^+} \left[\underbrace{\ln 1}_{0} - \ln(\ln c)\right] = \infty$$

Limit sonsuz olduğundan genelleştirilmiş integral ıraksaktır.

3.b) $f(x) = x\sqrt{16-x^2}$ fonksiyonunun yerel ve mutlak ekstremum **değerlerini** bulunuz. *(13 puan)*

$D(f): [-4, 4]$ $\quad \left.\begin{matrix} x=-4 \\ x=4 \end{matrix}\right\}$ uç noktalar

$$f'(x) = \sqrt{16-x^2} + x \cdot \frac{-\cancel{2}x}{\cancel{2}\sqrt{16-x^2}} = \frac{16-x^2-x^2}{\sqrt{16-x^2}}$$

$\to f'=0 \Rightarrow x^2=8 \Rightarrow x=\mp 2\sqrt{2}$

$\to f'$ tanımsız $\to x=\mp 4$

x		-4		$-2\sqrt{2}$		$2\sqrt{2}$		4	
f'			$-$	0	$+$	0	$-$		
f			$\searrow$		$\nearrow$		$\searrow$		
		yerel max		yerel min.		yerel max.		yerel min.	

$f(-4) = f(4) = 0$

$f(2\sqrt{2}) = 8$

$f(-2\sqrt{2}) = -8$

$f(2\sqrt{2}) = 8 \to$ mutlak max. değer

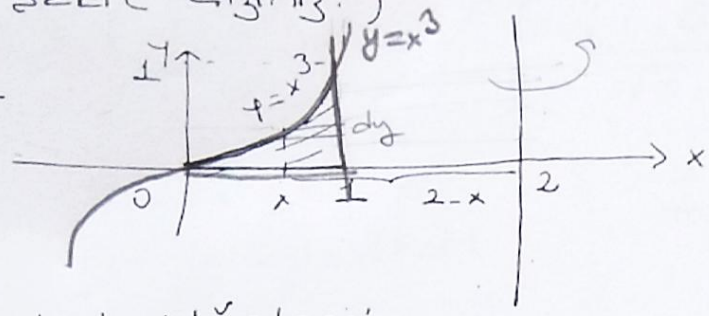
$f(-2\sqrt{2}) = -8 \to$ mutlak min. değer

$f(-4) = 0 \to$ yerel max. değer

$f(4) = 0 \to$ yerel min. değer

$y = x^3$ eğrisi, $x = 1$, $y = 0$ doğruları tarafından sınırlanan bölgenin $x = 2$ doğrusu etrafında döndürülmesiyle oluşan dönel cismin hacmini bulunuz (Şekil çiziniz.)

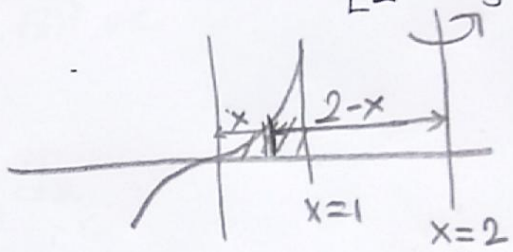
Ç:



Kabuk Yöntemi

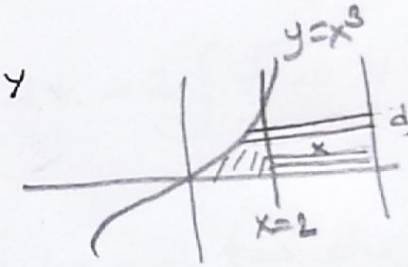
$$V = 2\pi \int_{x=0}^{x=1} \underbrace{(2-x)}_{K.Yar.} \cdot \underbrace{x^3}_{K.Yük} dx = 2\pi \int_0^1 (2x^3 - x^4)\, dx = 2\pi \left[\frac{1}{2}x^4 - \frac{x^5}{5}\right]$$

$$= 2\pi \cdot \left[\frac{1}{2} - \frac{1}{5}\right] = \boxed{\frac{3\pi}{5}} \text{ br}^3$$



Disk Yöntemi

$$V_y = \pi \int_0^1 (\underbrace{2}_{x=2} - \underbrace{\sqrt[3]{y}}_{x=\sqrt[3]{y}})^2 dy - \pi \int_0^1 (2-1)\, dy$$



$$= \pi \int_0^1 (4 - 4\sqrt[3]{y} + y^{2/3})\, dy - \pi$$

$$= \pi \left[4y - \cancel{4} \cdot \frac{y^{4/3}}{\frac{\cancel{4}}{3}} + \frac{y^{5/3}}{\frac{5}{3}}\right]_0^1 - \pi = \pi\left[\underbrace{4-3}_{1} + \frac{3}{5}\right] - \pi$$

$$= \frac{8\pi}{5} - \pi = \boxed{\frac{3\pi}{5}} \text{ br}^3$$

$$\int \sqrt{\frac{1-\sqrt{x}}{x}}\,dx$$ int. hesaplayınız.

Ç: $I = \int \sqrt{\frac{1-\sqrt{x}}{x}}\,dx = \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{x}}\,dx$

$1-\sqrt{x} = t^2$

$-\frac{1}{2\sqrt{x}}\,dx = 2t\,dt$

$I = \int \sqrt{t^2}\cdot -4t\,dt$

$= -4\int t^2\,dt = \frac{-4}{3}t^3 + c$

$= \frac{-4}{3}\sqrt{(1-\sqrt{x})^3} + c$

veya

$I = \int \sqrt{\frac{1-\sqrt{x}}{x}}\,dx = \int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{x}}\,dx$

$x = t^2$

$dx = 2t\,dt$

$I = \int \frac{\sqrt{1-t}}{\cancel{t}}\cdot 2\cancel{t}\,dt = 2\int \sqrt{1-t}\,dt$

$1-t = u^2$

$-dt = 2u\,du$

$= -2\int \sqrt{u^2}\cdot 2u\,du = -4\int u^2\,du$

$= \frac{-4}{3}u^3 + c = \frac{-4}{3}\sqrt{1-t} + c$

$= \frac{-4}{3}\sqrt{1-\sqrt{x}} + c$ //