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# Chapter 4

## ENERGY ANALYSIS OF CLOSED SYSTEMS

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# ENERGY ANALYSIS OF CLOSED SYSTEMS

4-1 Moving Boundary Work

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Specific Heat Relations of Ideal Gases

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Internal Energy Changes

Enthalpy Changes

# Objectives

- Examine the moving boundary work or  $P dV$  work commonly encountered in reciprocating devices such as automotive engines and compressors.
- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.
- Develop the general energy balance applied to closed systems.
- Define the specific heat at constant volume and the specific heat at constant pressure.
- Relate the specific heats to the calculation of the changes in internal energy and enthalpy of ideal gases.
- Describe incompressible substances and determine the changes in their internal energy and enthalpy.
- Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for general pure substances, ideal gases, and incompressible substances.

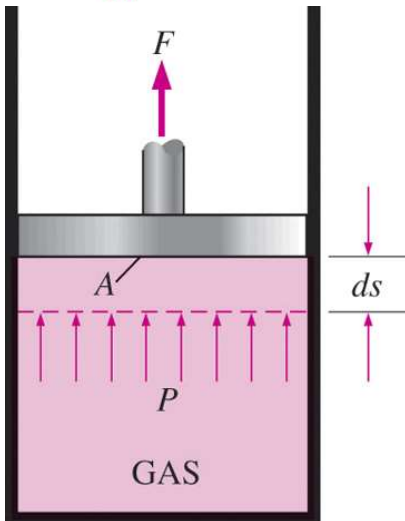
# MOVING BOUNDARY WORK

## Moving boundary work ( $P dV$ work):

The expansion and compression work in a piston-cylinder device.

$$\delta W_b = F ds = PA ds = P dV$$

$$W_b = \int_1^2 P dV \quad (\text{kJ})$$

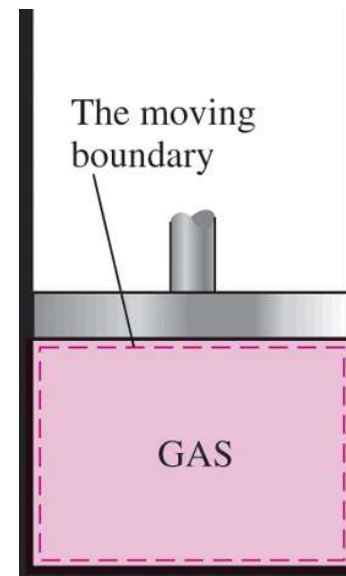


A gas does a differential amount of work  $\delta W_b$  as it forces the piston to move by a differential amount  $ds$ .

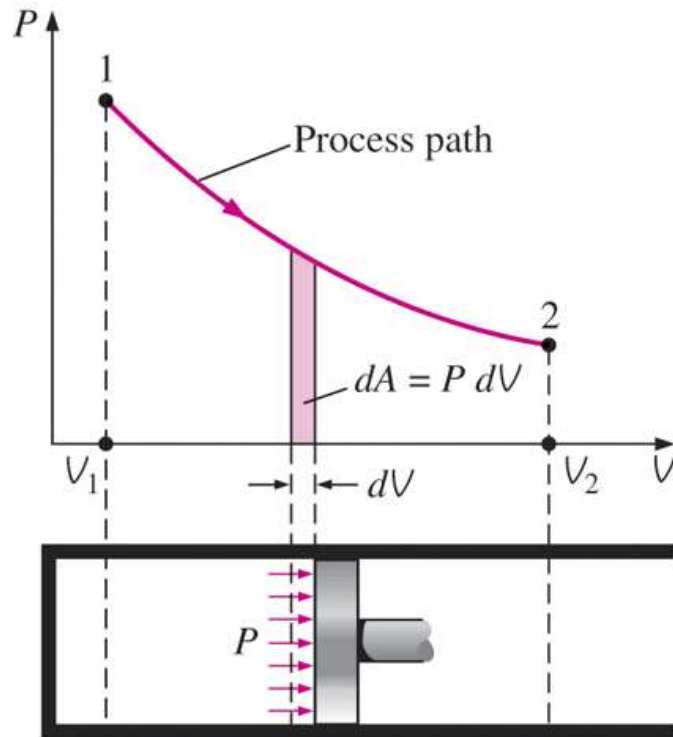
## Quasi-equilibrium process:

A process during which the system remains nearly in equilibrium at all times.

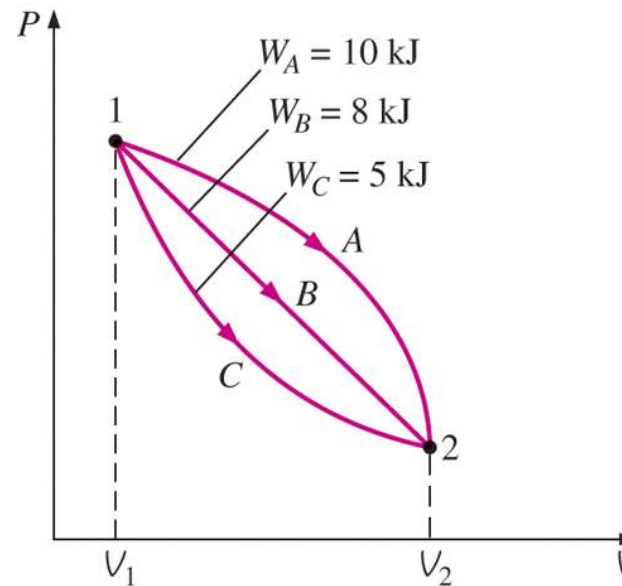
$W_b$  is positive  $\rightarrow$  for expansion  
 $W_b$  is negative  $\rightarrow$  for compression



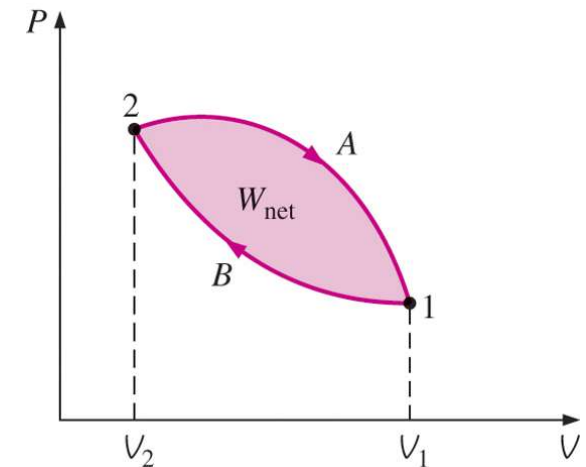
The work associated with a moving boundary is called *boundary work*.



The area under the process curve on a  $P$ - $V$  diagram represents the boundary work.



The boundary work done during a process depends on the path followed as well as the end states.



The net work done during a cycle is the difference between the work done by the system and the work done on the system.

The quasi-equilibrium expansion process described above is shown on a  $P$ - $V$  diagram.

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P dV$$

*the area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.*

*(On the P-V diagram, it represents the boundary work done per unit mass.)*

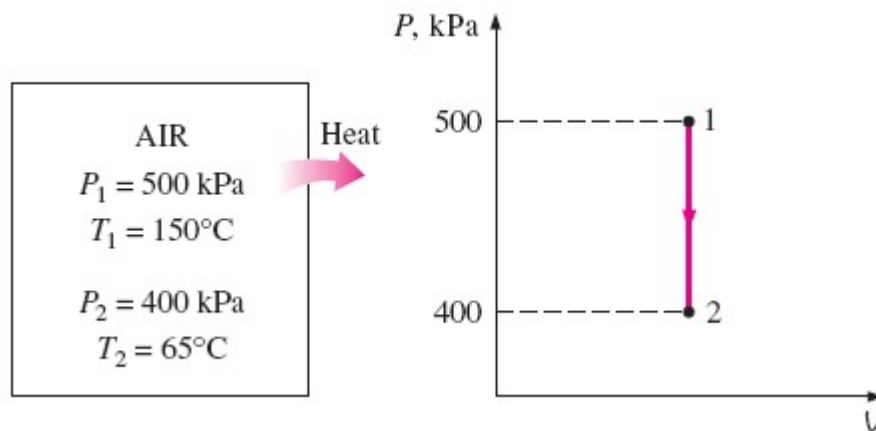
$$W_b = \int_1^2 P_i dV$$

In a car engine, for example, the boundary work done by the expanding hot gases is used to overcome friction between the piston and the cylinder, to push atmospheric air out of the way, and to rotate the crankshaft. Therefore,

$$W_b = W_{\text{friction}} + W_{\text{atm}} + W_{\text{crank}} = \int_1^2 (F_{\text{friction}} + P_{\text{atm}}A + F_{\text{crank}}) dx$$

### EXAMPLE 4-1

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.



$$W_b = \int_1^2 P dV = 0$$

**Discussion** This is expected since a rigid tank has a constant volume and  $dV = 0$  in this equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the  $P$ - $V$  diagram of the process (the area under the process curve is zero).

## EXAMPLE

Sketch a PV diagram and find the work done by the gas during the following stages.

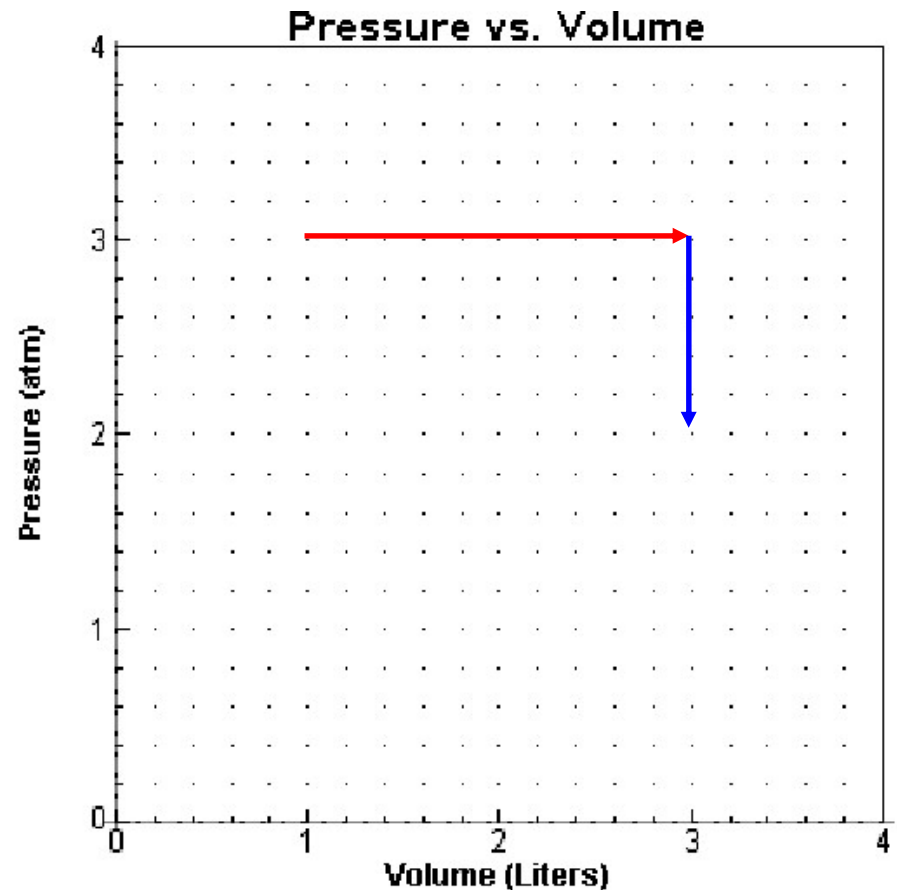
- (a) A gas is expanded from a volume of 1.0 L to 3.0 L at a constant pressure of 3.0 atm.

$$W_{BY} = P\Delta V = 3 \times 10^5 (0.003 - 0.001) = \mathbf{600 \text{ J}}$$

- (b) The gas is then cooled at a constant volume until the pressure falls to 2.0 atm

$$W = P\Delta V = 0$$

$$\text{since } \Delta V = 0$$

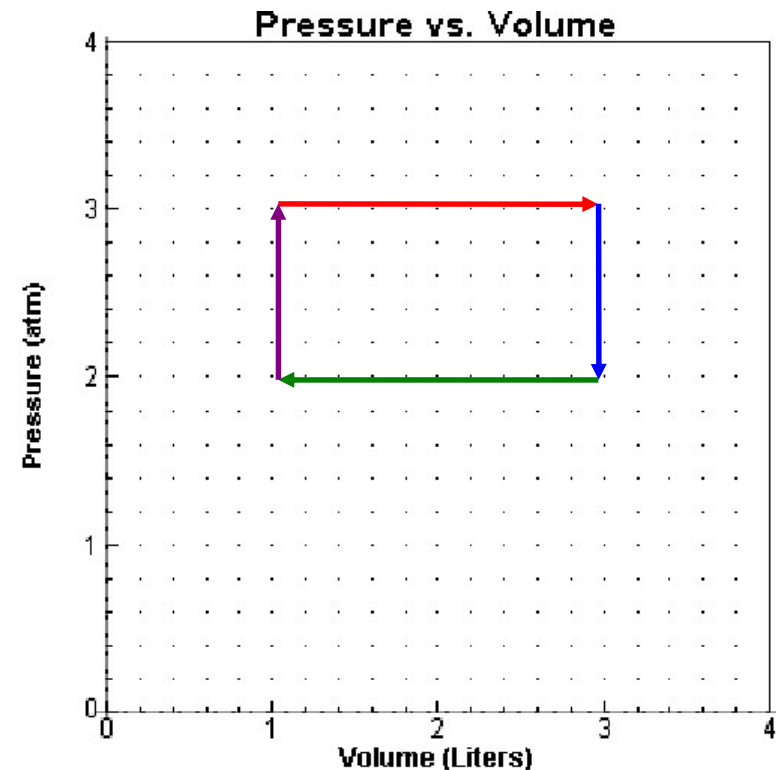


## EXAMPLE CONTINUED

- a) The gas is then compressed at a constant pressure of 2.0 atm from a volume of 3.0 L to 1.0 L.

$$W_{ON} = -P\Delta V = 2 \times 10^5 (.001 - .003) = -400 \text{ J}$$

- b) The gas is then heated until its pressure increases from 2.0 atm to 3.0 atm at a constant volume.



$$W = P\Delta V = 0$$

$$\text{since } \Delta V = 0$$

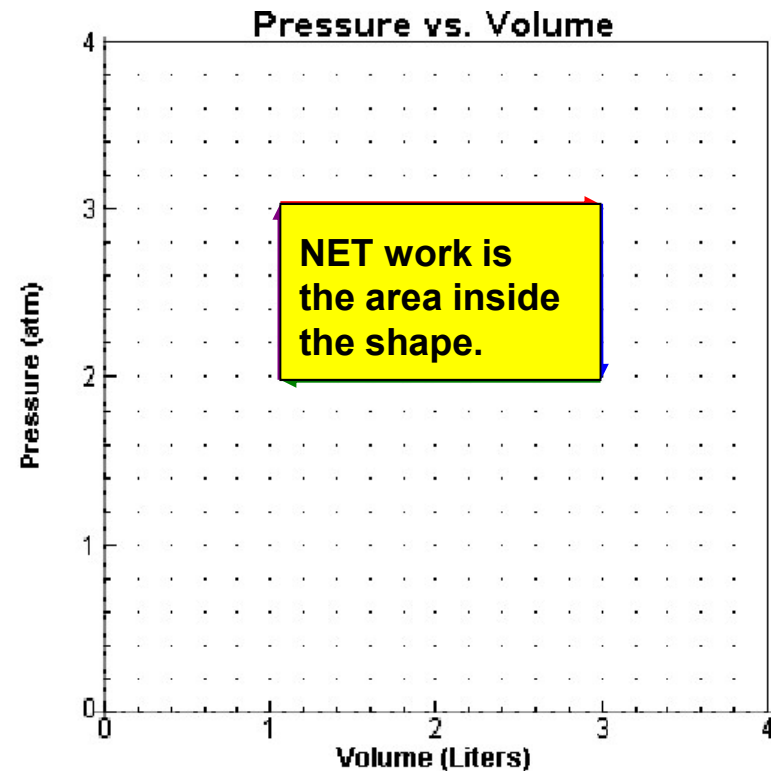
## EXAMPLE CONTINUED

What is the NET WORK?

$$600 \text{ J} + -400 \text{ J} = 200 \text{ J}$$

**Rule of thumb:** If the system rotates *CW*, the NET work is positive.

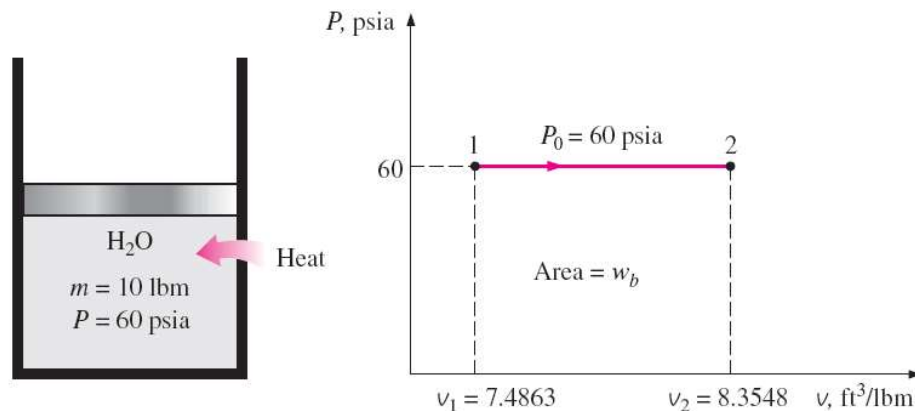
If the system rotates *CCW*, the NET work is negative.



## EXAMPLE 4-2

A frictionless piston–cylinder device contains 10 lbm of water vapor at 60 psia and 320°F. Heat is now transferred to the steam until the temperature reaches 400°F. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.

**Solution** Water vapor in a piston cylinder device is heated and the temperature rises at constant pressure. The boundary work done is to be determined.



Even though it is not explicitly stated, the pressure of the steam within the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant. Therefore, this is a constant-pressure process,

$$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

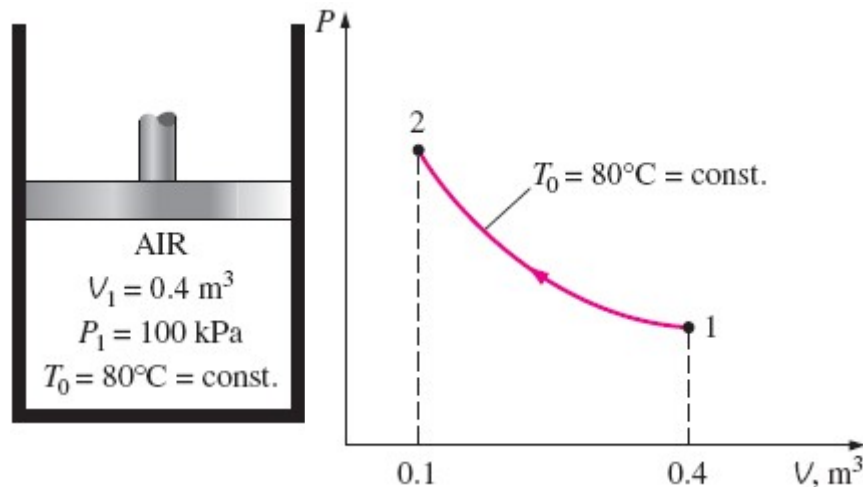
$$W_b = mP_0(v_2 - v_1)$$

$$W_b = (10 \text{ lbm})(60 \text{ psia})[(8.3458 - 7.4863) \text{ ft}^3/\text{lbm}]\left(\frac{1 \text{ Btu}}{5.404 \text{ psia} \cdot \text{ft}^3}\right) = \mathbf{96.4 \text{ Btu}}$$

**Discussion** The positive sign indicates that the work is done by the system. That is, the steam used 96.4 Btu of its energy to do this work. The magnitude of this work could also be determined by calculating the area under the process curve on the P-V diagram, which is simply  $P_0 \Delta V$  for this case.

### EXAMPLE 4-3

A piston–cylinder device initially contains  $0.4 \text{ m}^3$  of air at  $100 \text{ kPa}$  and  $80^\circ\text{C}$ . The air is now compressed to  $0.1 \text{ m}^3$  in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process..



**Solution** Air in a piston–cylinder device is compressed isothermally. The boundary work done is to be determined.

**Assumptions** **1** The compression process is quasi-equilibrium. **2** At the specified conditions air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

$$W_b = \int_1^2 P dV = \int_1^2 \frac{C}{V} dV = C \int_1^2 \frac{dV}{V} = C \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{V_2}{V_1}$$

$$W_b = (100 \text{ kPa})(0.4 \text{ m}^3) \left( \ln \frac{0.1}{0.4} \right) \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) = -55.45 \text{ kJ}$$

**Discussion** The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.

# Polytropic, Isothermal, and Isobaric processes

$$P = CV^{-n}$$

Polytropic process:  $C, n$  (polytropic exponent) constants

$$W_b = \int_1^2 P dV = \int_1^2 CV^{-n} dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

Polytropic process

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \quad n \neq 1$$

Polytropic and for ideal gas

$$W_b = \int_1^2 P dV = \int_1^2 CV^{-1} dV = PV \ln\left(\frac{V_2}{V_1}\right)$$

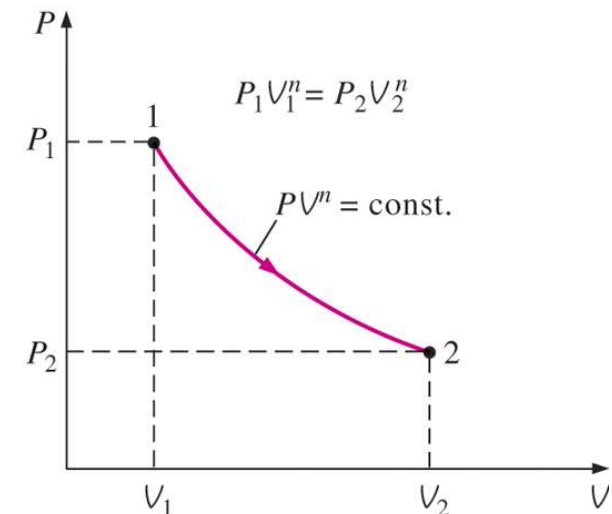
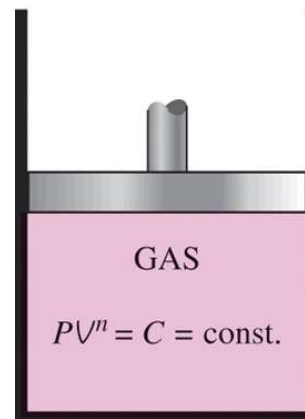
When  $n = 1$  (isothermal process)

$$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

Constant pressure process

What is the boundary work for a constant-volume process?

Schematic and  $P$ - $V$  diagram for a polytropic process.

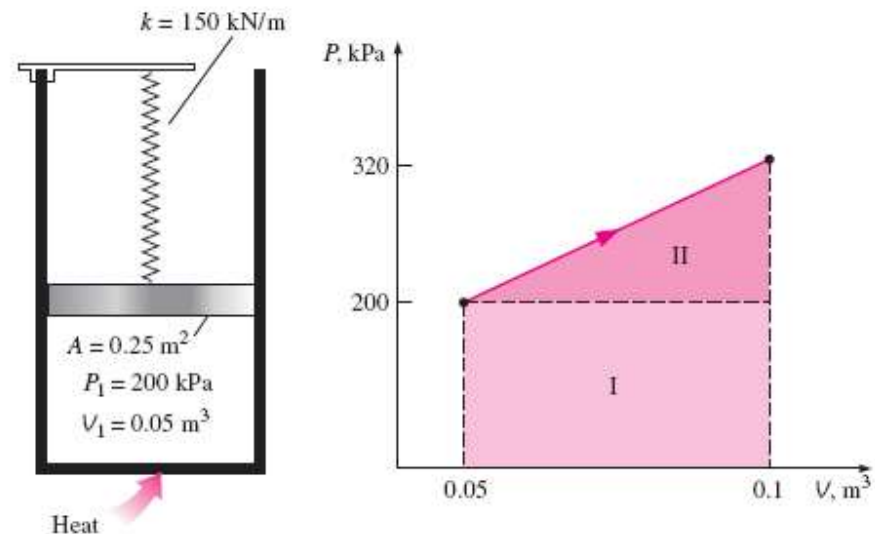


## EXAMPLE 4-4

A piston–cylinder device contains  $0.05 \text{ m}^3$  of a gas initially at  $200 \text{ kPa}$ . At this state, a linear spring that has a spring constant of  $150 \text{ kN/m}$  is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is  $0.25 \text{ m}^2$ , determine (a) the final pressure inside the cylinder, (b) the total work done by the gas, and (c) the fraction of this work done against the spring to compress it.

**Solution** A gas in a piston–cylinder device equipped with a linear spring expands as a result of heating. The final gas pressure, the total work done, and the fraction of the work done to compress the spring are to be determined.

**Assumptions** 1 The expansion process is quasi-equilibrium. 2 The spring is linear in the range of interest.



(a) The enclosed volume at the final state is

Then the displacement of the piston (and of the spring) becomes

The force applied by the linear spring at the final state is

$$V_2 = 2V_1 = (2)(0.05 \text{ m}^3) = 0.1 \text{ m}^3$$

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05) \text{ m}^3}{0.25 \text{ m}^2} = 0.2 \text{ m}$$

$$F = kx = (150 \text{ kN/m})(0.2 \text{ m}) = 30 \text{ kN}$$

The additional pressure applied by the spring on the gas at this state is

$$P = \frac{F}{A} = \frac{30 \text{ kN}}{0.25 \text{ m}^2} = 120 \text{ kPa}$$

Without the spring, the pressure of the gas would remain constant at 200 kPa while the piston is rising. But under the effect of the spring, the pressure rises linearly from 200 kPa to

$$200 + 120 = \mathbf{320 \text{ kPa}}$$

(b) An easy way of finding the work done is to plot the process on a  $P$ - $V$  diagram and find the area under the process curve.

$$W = \text{area} = \frac{(200 + 320) \text{ kPa}}{2} [(0.1 - 0.05) \text{ m}^3] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}} \right) = \mathbf{13 \text{ kJ}}$$

(c) The work represented by the rectangular area (region I) is done against the piston and the atmosphere, and the work represented by the triangular area (region II) is done against the spring. Thus,

$$W_{\text{spring}} = \frac{1}{2} [(320 - 200) \text{ kPa}] (0.05 \text{ m}^3) \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}} \right) = \mathbf{3 \text{ kJ}}$$

**Discussion** This result could also be obtained from

$$W_{\text{spring}} = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1}{2} (150 \text{ kN/m}) [(0.2 \text{ m})^2 - 0^2] \left( \frac{1 \text{ kJ}}{1 \text{ kN} \cdot \text{m}} \right) = 3 \text{ kJ}$$

# ENERGY BALANCE FOR CLOSED SYSTEMS

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}} \quad (\text{kJ})$$

Energy balance for any system undergoing any process

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \quad (\text{kW})$$

Energy balance in the rate form

The total quantities are related to the quantities per unit time is

$$Q = \dot{Q} \Delta t, \quad W = \dot{W} \Delta t, \quad \text{and} \quad \Delta E = \Delta \dot{E} \Delta t$$

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}}$$

Energy balance per unit mass basis

Energy balance in differential form

$$\delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}} \quad \text{or} \quad \delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}}$$

$$W_{\text{net, out}} = Q_{\text{net, in}} \quad \text{or} \quad \dot{W}_{\text{net, out}} = \dot{Q}_{\text{net, in}}$$

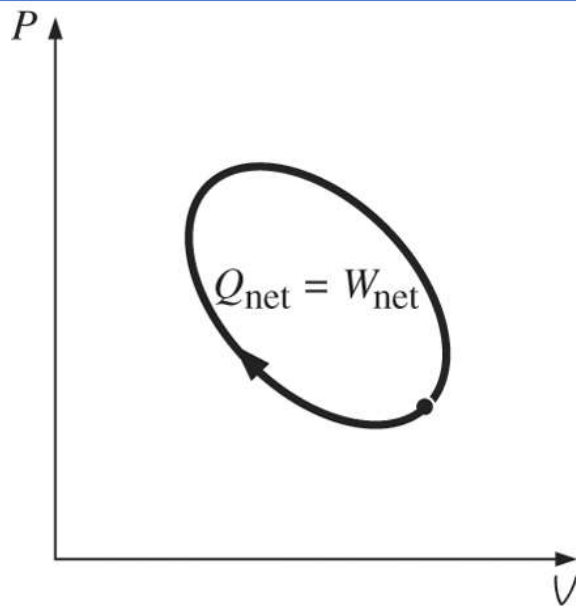
Energy balance for a cycle

$$Q_{\text{net in}} - W_{\text{net, out}} = \Delta E_{\text{system}} \quad \text{or} \quad Q - W = \Delta E$$

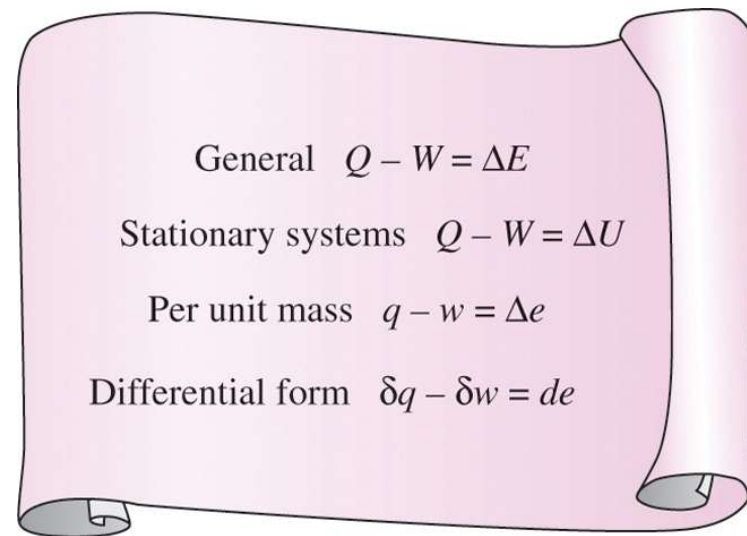
$$Q = Q_{\text{net, in}} = Q_{\text{in}} - Q_{\text{out}}$$

$$W = W_{\text{net, out}} = W_{\text{out}} - W_{\text{in}}$$

Energy balance when sign convention is used (i.e., heat input and work output are positive; heat output and work input are negative).



For a cycle  $\Delta E = 0$ , thus  $Q = W$ .



Various forms of the first-law relation for closed systems when sign convention is used.

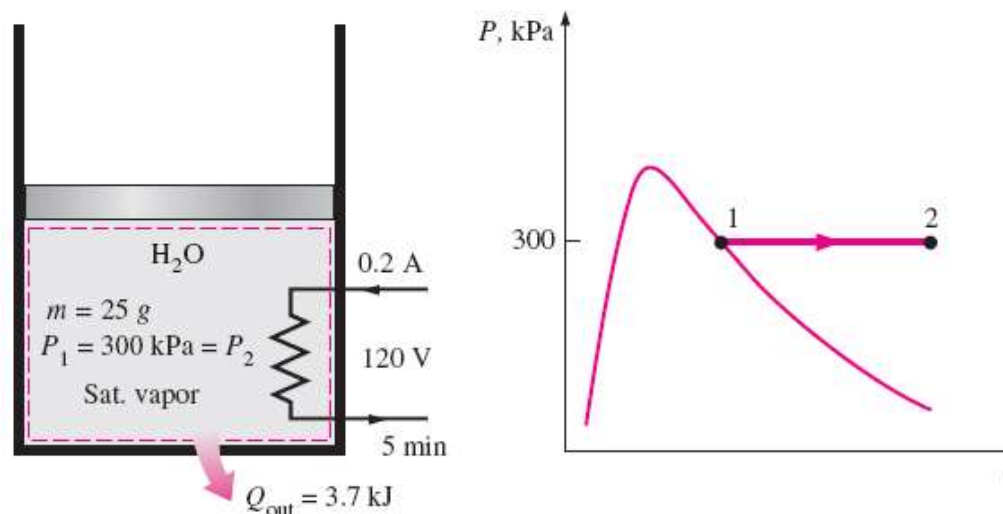
The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

### EXAMPLE 4-5

A piston–cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source. At the same time, a heat loss of 3.7 kJ occurs. (a) Show that for a closed system the boundary work  $W_b$  and the change in internal energy  $\Delta U$  in the first-law relation can be combined into one term,  $\Delta H$ , for a constant pressure process. (b) Determine the final temperature of the steam.

**Solution** Saturated water vapor in a piston–cylinder device expands at constant pressure as a result of electric resistance heating. It is to be shown that  $\Delta U + W_b = \Delta H$ , and the final temperature is to be determined.

**Assumptions 1** The tank is stationary and thus the kinetic and potential energy changes are zero,  $\Delta KE = \Delta PE = 0$ . Therefore,  $\Delta E = \Delta U$  and internal energy is the only form of energy of the system that may change during this process. **2** Electrical wires constitute a very small part of the system, and thus the energy change of the wires can be neglected.



(a) for a closed system the boundary work  $W_b$  and the change in internal energy  $\Delta U$  in the first-law relation can be combined into one term,  $\Delta H$ , for a constant pressure process.

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$Q - W = \Delta U + \cancel{\Delta KE}^0 + \cancel{\Delta PE}^0$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

$$P_0 = P_2 = P_1 \rightarrow Q - W_{\text{other}} = (U_2 + P_2 V_2) - (U_1 + P_1 V_1)$$

$$H = U + PV,$$

$$Q - W_{\text{other}} = H_2 - H_1 \quad (\text{kJ})$$

(b) The only other form of work in this case is the electrical work, which can be determined from

$$W_e = VI \Delta t = (120 \text{ V})(0.2 \text{ A})(300 \text{ s}) \left( \frac{1 \text{ kJ/s}}{1000 \text{ VA}} \right) = 7.2 \text{ kJ}$$

$$\text{State 1: } \left. \begin{array}{l} P_1 = 300 \text{ kPa} \\ \text{sat, vapor} \end{array} \right\} h_1 = h_g @ 300 \text{ kPa} = 2724.9 \text{ kJ/kg}$$

$$W_{e, \text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e, \text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1) \quad (\text{since } P = \text{constant})$$

$$7.2 \text{ kJ} - 3.7 \text{ kJ} = (0.025 \text{ kg})(h_2 - 2724.9) \text{ kJ/kg}$$

$$h_2 = 2864.9 \text{ kJ/kg}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 300 \text{ kPa} \\ h_2 = 2864.9 \text{ kJ/kg} \end{array} \right\} T_2 = 200^\circ\text{C}$$

**Discussion** Strictly speaking, the potential energy change of the steam is not zero for this process since the center of gravity of the steam rose somewhat. Assuming an elevation change of 1 m (which is rather unlikely), the change in the potential energy of the steam would be 0.0002 kJ, which is very small compared to the other terms in the first-law relation. Therefore, in problems of this kind, the potential energy term is always neglected.

## EXAMPLE 4-6

A rigid tank is divided into two equal parts by a partition. Initially, one side of the tank contains 5 kg of water at 200 kPa and 25°C, and the other side is evacuated. The partition is then removed, and the water expands into the entire tank. The water is allowed to exchange heat with its surroundings until the temperature in the tank returns to the initial value of 25°C. Determine (a) the volume of the tank, (b) the final pressure, and (c) the heat transfer for this process.

**Solution** One half of a rigid tank is filled with liquid water while the other side is evacuated. The partition between the two parts is removed and water is allowed to expand and fill the entire tank while the temperature is maintained constant. The volume of tank, the final pressure, and the heat transfer are to be determined.

**Assumptions** **1** The system is stationary and thus the kinetic and potential energy changes are zero,  $\Delta KE = \Delta PE = 0$  and  $\Delta E = \Delta U$ . **2** The direction of heat transfer is to the system (heat gain,  $Q_{in}$ ). A negative result for  $Q_{in}$  will indicate the assumed direction is wrong and thus it is heat loss. **3** The volume of the rigid tank is constant, and thus there is no energy transfer as boundary work. **4** There is no electrical, shaft, or any other kind of work involved.

(a) the volume of the tank

$$v_1 \cong v_{f@25^\circ\text{C}} = 0.001003 \text{ m}^3/\text{kg} \cong 0.001 \text{ m}^3/\text{kg}$$

$$V_1 = m v_1 = (5 \text{ kg})(0.001 \text{ m}^3/\text{kg}) = 0.005 \text{ m}^3$$

The total volume of the tank is twice this amount:

$$V_{\text{tank}} = (2)(0.005 \text{ m}^3) = \mathbf{0.01 \text{ m}^3}$$

(b) At the final state, the specific volume of the water is

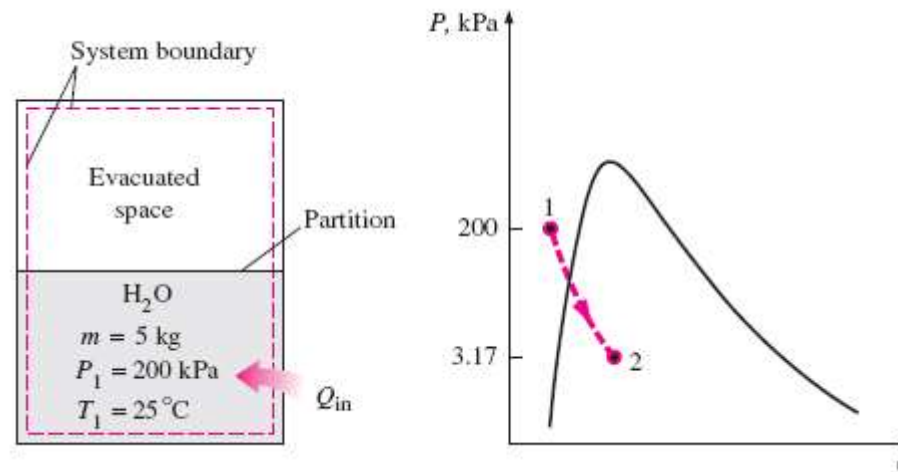
$$v_2 = \frac{V_2}{m} = \frac{0.01 \text{ m}^3}{5 \text{ kg}} = 0.002 \text{ m}^3/\text{kg}$$

which is twice the initial value of the specific volume. This result is expected since the volume doubles while the amount of mass remains constant.

$$\text{At } 25^\circ\text{C: } v_f = 0.001003 \text{ m}^3/\text{kg} \quad \text{and} \quad v_g = 43.340 \text{ m}^3/\text{kg}$$

Since  $v_f < v_2 < v_g$ , the water is a saturated liquid–vapor mixture at the final state, and thus the pressure is the saturation pressure at  $25^\circ\text{C}$ :

$$P_2 = P_{\text{sat @ } 25^\circ\text{C}} = \mathbf{3.1698 \text{ kPa}}$$



(c) Under stated assumptions and observations, the energy balance on the system can be expressed as

$$Q_{\text{in}} = \Delta U = m(u_2 - u_1)$$

$$u_1 \cong u_f @ 25^\circ\text{C} = 104.83 \text{ kJ/kg}$$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.002 - 0.001}{43.34 - 0.001} = 2.3 \times 10^{-5}$$

$$\begin{aligned} u_2 &= u_f + x_2 u_{fg} \\ &= 104.83 \text{ kJ/kg} + (2.3 \times 10^{-5})(2304.3 \text{ kJ/kg}) \\ &= 104.88 \text{ kJ/kg} \end{aligned}$$

$$Q_{\text{in}} = (5 \text{ kg})[(104.88 - 104.83) \text{ kJ/kg}] = \mathbf{0.25 \text{ kJ}}$$

**Discussion** The positive sign indicates that the assumed direction is correct, and heat is transferred to the water.

# ENERGY BALANCE FOR A CONSTANT-PRESSURE EXPANSION OR COMPRESSION PROCESS

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process.  $Q$  is *to* the system and  $W$  is *from* the system.

For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$Q - W = \Delta U + \cancel{\Delta KE}^0 + \cancel{\Delta PE}^0$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{\text{other}} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

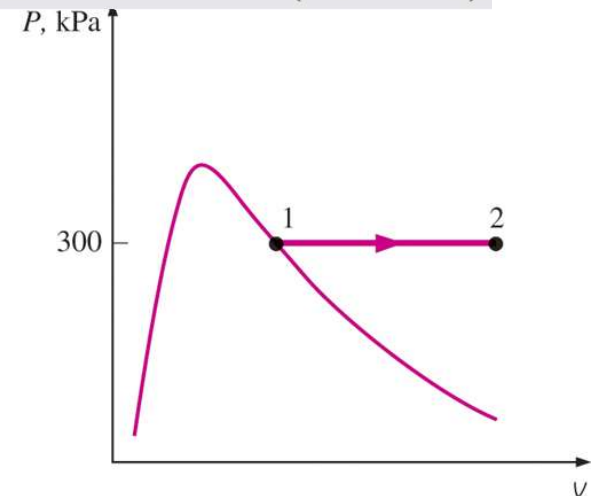
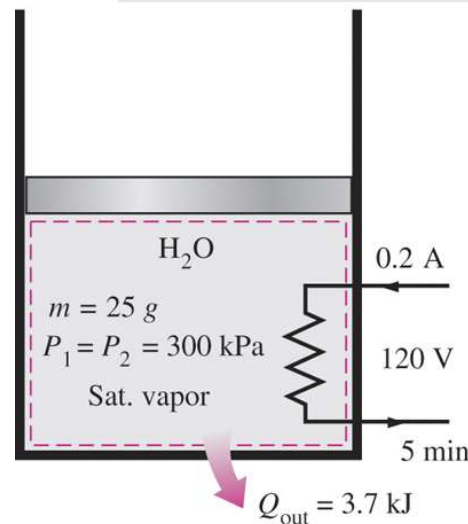
$$H = U + PV$$

$$Q - W_{\text{other}} = H_2 - H_1$$

An example of constant-pressure process

$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$

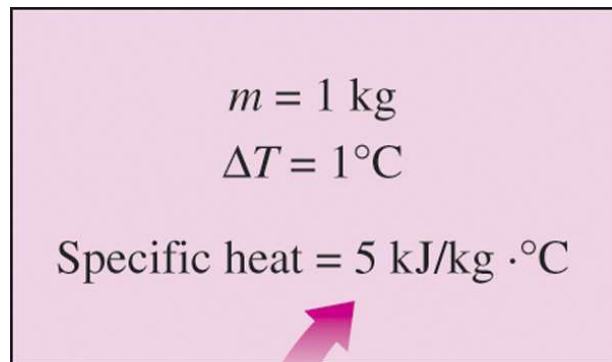
$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1)$$



# SPECIFIC HEATS

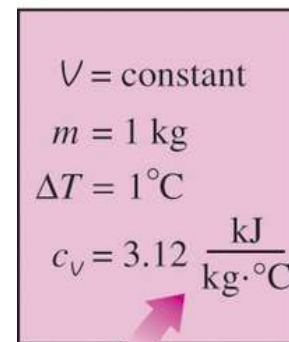
**Specific heat at constant volume,  $c_v$ :** The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

**Specific heat at constant pressure,  $c_p$ :** The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.

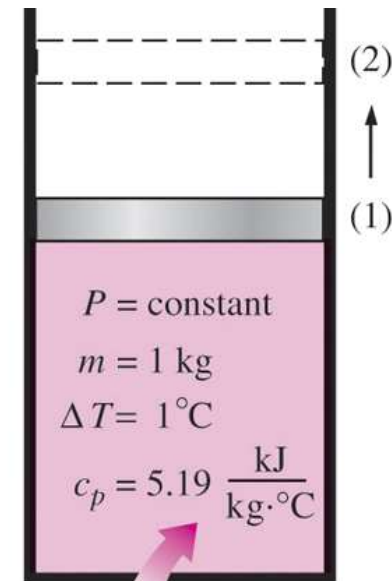


5 kJ

Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.



3.12 kJ

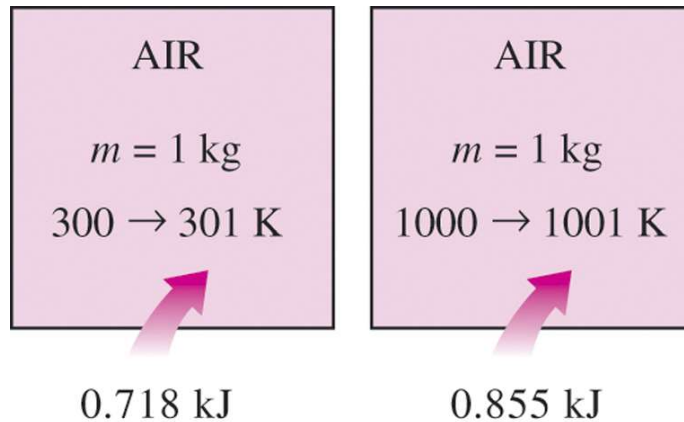


5.19 kJ

Constant-volume and constant-pressure specific heats  $c_v$  and  $c_p$  (values are for helium gas).

- The equations in the figure are valid for *any* substance undergoing *any* process
- $c_v$  and  $c_p$  are properties.
- $c_v$  is related to the changes in *internal energy* and  $c_p$  to the changes in *enthalpy*
- A common unit for specific heats is kJ/kg · °C or kJ/kg · K.

**Are these units identical?**



The specific heat of a substance changes with temperature.

**True or False?**

$c_p$  is always greater than  $c_v$ .

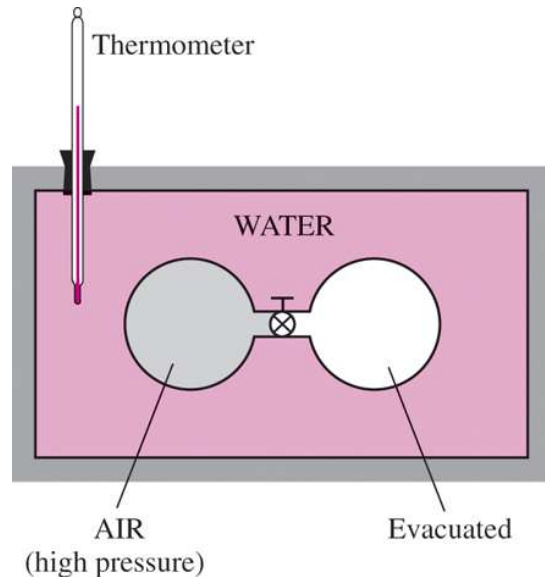
$$c_v = \left( \frac{\partial u}{\partial T} \right)_v$$

= the change in internal energy with temperature at constant volume

$$c_p = \left( \frac{\partial h}{\partial T} \right)_p$$

= the change in enthalpy with temperature at constant pressure

# INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES



Joule showed using this experimental apparatus that  $u=u(T)$

$$\begin{aligned} u &= u(T) \\ h &= h(T) \\ c_v &= c_v(T) \\ c_p &= c_p(T) \end{aligned}$$

For ideal gases,  $u$ ,  $h$ ,  $c_v$ , and  $c_p$  vary with temperature only.

$$\left. \begin{aligned} h &= u + Pv \\ Pv &= RT \end{aligned} \right\} h = u + RT$$

$$u = u(T)$$

$$h = h(T)$$

$$du = c_v(T) dT$$

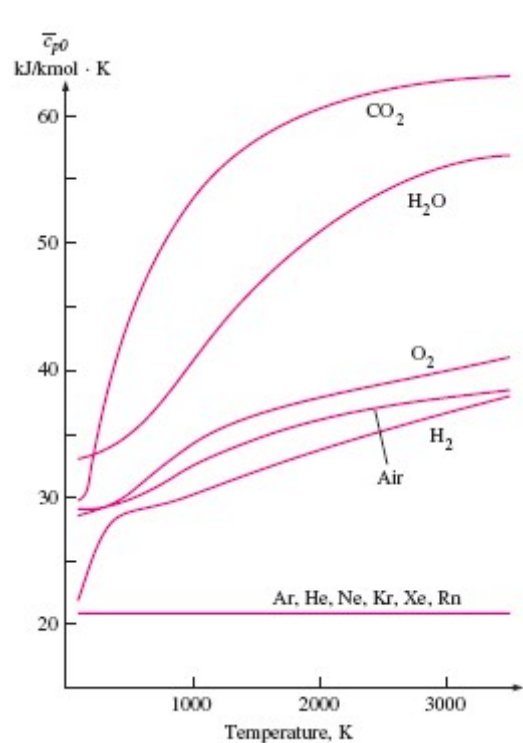
$$dh = c_p(T) dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T) dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT$$

Internal energy and enthalpy change of an ideal gas

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted  $c_{p0}$  and  $c_{v0}$ .
- $u$  and  $h$  data for a number of gases have been tabulated.
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.



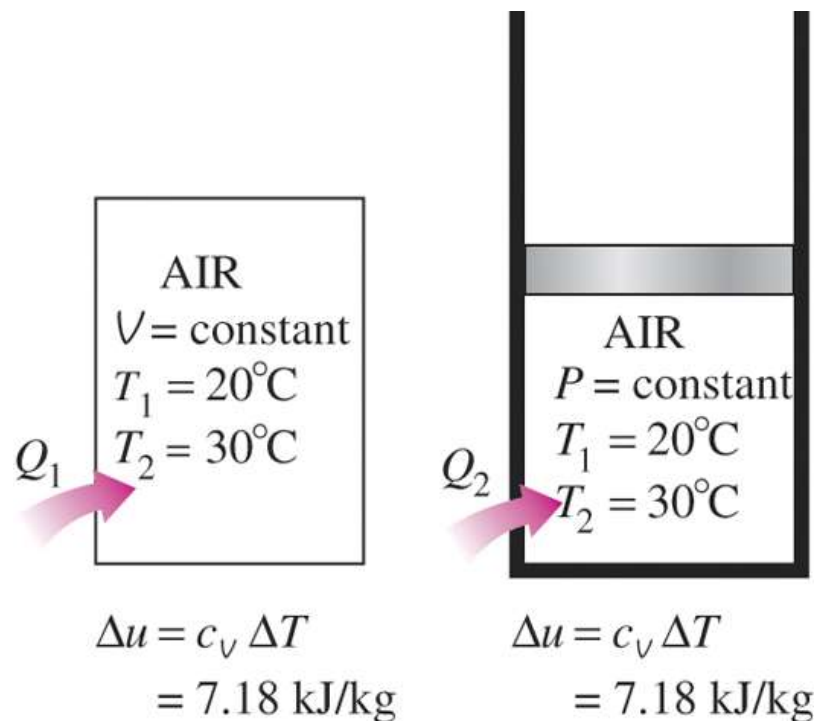
Ideal-gas constant-pressure specific heats for some gases (see Table A-2c for  $c_p$  equations).

AIR		
$T, K$	$u, kJ/kg$	$h, kJ/kg$
0	0	0
.	.	.
.	.	.
300	214.07	300.19
310	221.25	310.24
.	.	.
.	.	.

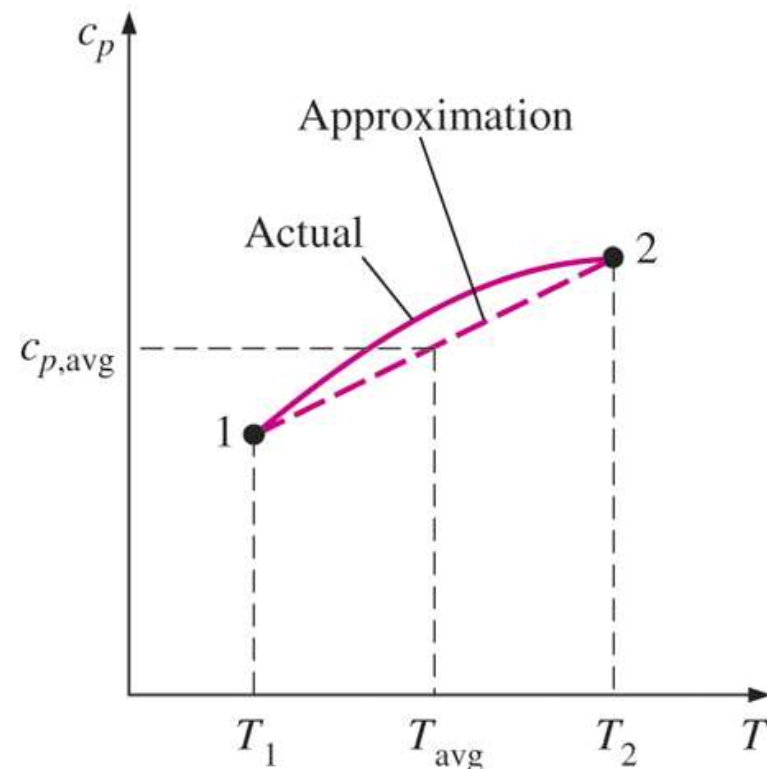
In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

Internal energy and enthalpy change when specific heat is taken constant at an average value

$$u_2 - u_1 = c_{v,av}(T_2 - T_1) \quad (\text{kJ/kg}) \quad h_2 - h_1 = c_{p,av}(T_2 - T_1) \quad (\text{kJ/kg})$$



The relation  $\Delta u = c_v \Delta T$  is valid for *any* kind of process, constant-volume or not.



For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

## Three ways of calculating $\Delta u$ and $\Delta h$

1. By using the tabulated  $u$  and  $h$  data. This is the easiest and **most accurate** way when tables are readily available.
2. By using the  $c_v$  or  $c_p$  relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are **very accurate**.
3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.

$$\Delta u = u_2 - u_1 \text{ (table)}$$

$$\Delta u = \int_1^2 c_v(T) dT$$

$$\Delta u \cong c_{v,av} \Delta T$$

Three ways of calculating  $\Delta u$ .

# SPECIFIC HEAT RELATIONS OF IDEAL GASES

$$\left. \begin{aligned} dh &= du + R dT \\ dh &= c_p dT \text{ and } du = c_v dT \end{aligned} \right\} \longrightarrow$$

The relationship between  $c_p$ ,  $c_v$  and  $R$

$$c_p = c_v + R \quad (\text{kJ/kg} \cdot \text{K})$$

On a molar basis

$$\bar{c}_p = \bar{c}_v + R_u \quad (\text{kJ/kmol} \cdot \text{K})$$

$$dh = c_p dT \text{ ve } du = c_v dT$$

Specific heat ratio

$$k = \frac{c_p}{c_v}$$

AIR at 300 K

$$\left. \begin{aligned} c_v &= 0.718 \text{ kJ/kg} \cdot \text{K} \\ R &= 0.287 \text{ kJ/kg} \cdot \text{K} \end{aligned} \right\} c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$$

or

$$\left. \begin{aligned} \bar{c}_v &= 20.80 \text{ kJ/kmol} \cdot \text{K} \\ R_u &= 8.314 \text{ kJ/kmol} \cdot \text{K} \end{aligned} \right\} \bar{c}_p = 29.114 \text{ kJ/kmol} \cdot \text{K}$$

The  $c_p$  of an ideal gas can be determined from a knowledge of  $c_v$  and  $R$ .

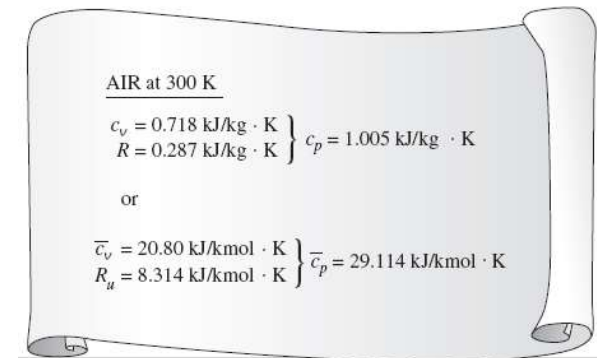
- The specific ratio varies with temperature, but this variation is very mild.
- For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

## EXAMPLE 4-7

Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass, using (a) data from the air table (Table A–17), (b) the functional form of the specific heat (Table A–2c), and (c) the average specific heat value (Table A–2b).

**Solution** The internal energy change of air is to be determine in three different ways.

**Assumptions** At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical point values.



AIR at 300 K	
$c_v = 0.718 \text{ kJ/kg} \cdot \text{K}$	$c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$
$R = 0.287 \text{ kJ/kg} \cdot \text{K}$	
or	
$\bar{c}_v = 20.80 \text{ kJ/kmol} \cdot \text{K}$	$\bar{c}_p = 29.114 \text{ kJ/kmol} \cdot \text{K}$
$R_u = 8.314 \text{ kJ/kmol} \cdot \text{K}$	

(a) One way of determining the change in internal energy of air is to read the  $u$  values at  $T_1$  and  $T_2$  from Table A–21 and take the difference:

$$u_1 = u @ 300 \text{ K} = 214.07 \text{ kJ/kg}$$

$$u_2 = u @ 600 \text{ K} = 434.78 \text{ kJ/kg}$$

$$\Delta u = u_2 - u_1 = (434.78 - 214.07) \text{ kJ/kg} = \mathbf{220.71 \text{ kJ/kg}}$$

(b) The  $\bar{c}_p(T)$  of air is given in Table A–2c in the form of a third-degree polynomial expressed as

$$\bar{c}_p(T) = a + bT + cT^2 + dT^3$$

where  $a = 28.11$ ,  $b = 0.1967 \times 10^{-2}$ ,  $c = 0.4802 \times 10^{-5}$ , and  $d = -1.966 \times 10^{-9}$ . From Eq. 4–30,

$$\bar{c}_v(T) = \bar{c}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

$$\Delta \bar{u} = \int_1^2 \bar{c}_v(T) dT = \int_{T_1}^{T_2} [(a - R_u) + bT + cT^2 + dT^3] dT$$

$$\Delta \bar{u} = 6447 \text{ kJ/kmol}$$

$$\Delta u = \frac{\Delta \bar{u}}{M} = \frac{6447 \text{ kJ/kmol}}{28.97 \text{ kg/kmol}} = \mathbf{222.5 \text{ kJ/kg}}$$

(c) The average value of the constant-volume specific heat  $c_{v, \text{av}}$  is determined from Table A-2b at the average temperature of  $(T_1 + T_2)/2 = 450 \text{ K}$  to be

$$c_{v, \text{av}} = c_v @ 450 \text{ K} = 0.733 \text{ kJ/kg} \cdot \text{K}$$

$$\begin{aligned} \Delta u &= c_{v, \text{av}}(T_2 - T_1) = (0.733 \text{ kJ/kg} \cdot \text{K})[(600 - 300)\text{K}] \\ &= 220 \text{ kJ/kg} \end{aligned}$$

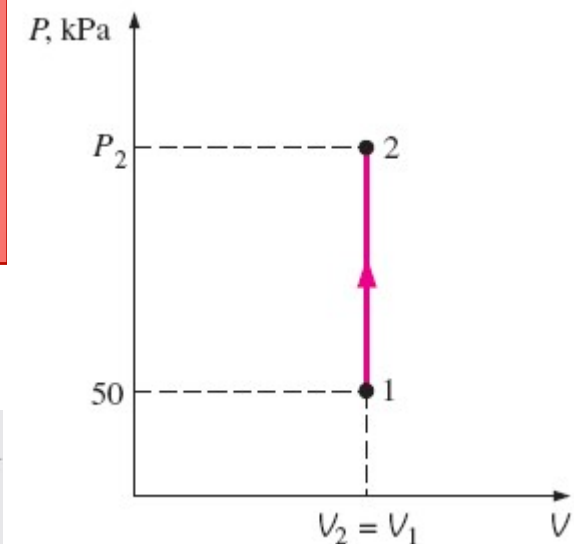
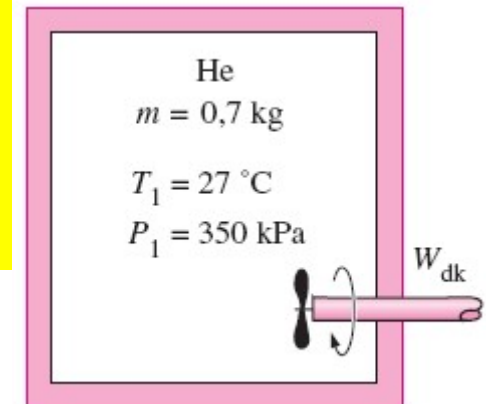
**Discussion** This answer differs from the exact result (220.71 kJ/kg) by only 0.4 percent. This close agreement is not surprising since the assumption that  $c_v$  varies linearly with temperature is a reasonable one at temperature intervals of only a few hundred degrees. If we had used the  $c_v$  value at  $T_1 = 300 \text{ K}$  instead of at  $T_{\text{av}}$ , the result would be 215 kJ/kg, which is in error by about 2 percent. Errors of this magnitude are acceptable for most engineering purposes.

### EXAMPLE 4-8

An insulated rigid tank initially contains 0.7 kg of helium at 27°C and 350 kPa. A paddle wheel with a power rating of 0.015 kW is operated within the tank for 30 min. Determine (a) the final temperature and (b) the final pressure of the helium gas.

**Solution** Helium gas in an insulated rigid tank is stirred by a paddle wheel. The final temperature and pressure of helium are to be determined.

**Assumptions** 1 Helium is an ideal gas since it is at a very high temperature relative to its critical-point value of -268°C. 2 Constant specific heats can be used for helium. 3  $\Delta KE = \Delta PE = 0$  and  $\Delta E = \Delta U$ . 4 The volume of the tank is constant,  $V_2 = V_1$ . 5 The system is adiabatic and thus there is no heat transfer.



(a) The amount of paddle-wheel work done on the system is

$$W_{dk} = \dot{W}_{dk} \Delta t = (0.015 \text{ kW})(30 \text{ dak}) \left( \frac{60 \text{ s}}{1 \text{ dak}} \right) = 27 \text{ kJ}$$

$$W_{dk, \text{giren}} = \Delta U = m(u_2 - u_1) = mc_{v, \text{ort}} (T_2 - T_1)$$

$$27 \text{ kJ} = (0.7 \text{ kg})(3.1156 \text{ kJ/kg} \cdot ^\circ\text{C})(T_2 - 27^\circ\text{C})$$

$$T_2 = 39.4^\circ\text{C}$$

(b) The final pressure is determined from the ideal-gas relation

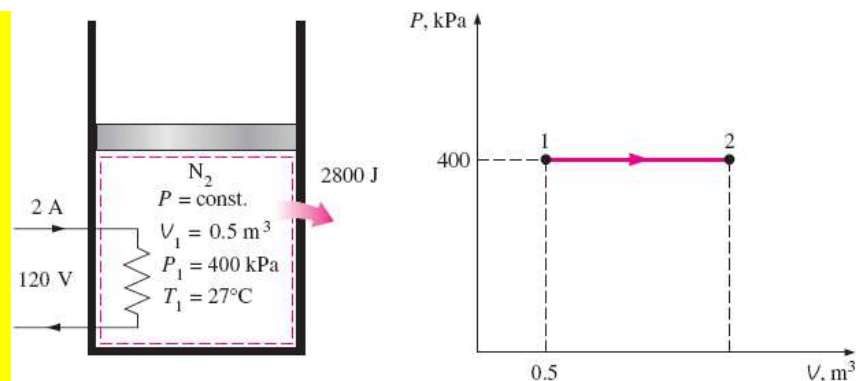
$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{350 \text{ kPa}}{(27 + 273) \text{ K}} = \frac{P_2}{(39.4 + 273) \text{ K}}$$
$$P_2 = \mathbf{364.5 \text{ kPa}}$$

where  $V_1$  and  $V_2$  are identical and cancel out. Then the final pressure becomes

### EXAMPLE 4-9

A piston–cylinder device initially contains  $0.5 \text{ m}^3$  of nitrogen gas at  $400 \text{ kPa}$  and  $27^\circ\text{C}$ . An electric heater within the device is turned on and is allowed to pass a current of  $2 \text{ A}$  for  $5 \text{ min}$  from a  $120\text{-V}$  source. Nitrogen expands at constant pressure, and a heat loss of  $2800 \text{ J}$  occurs during the process. Determine the final temperature of nitrogen.



**Solution** Nitrogen gas in a piston–cylinder device is heated by an electric resistance heater. Nitrogen expands at constant pressure while some heat is lost. The final temperature of nitrogen is to be determined.

**Assumptions** **1** Nitrogen is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values of  $147^\circ\text{C}$ , and  $3.39 \text{ MPa}$ . **2** The system is stationary and thus the kinetic and potential energy changes are zero,  $\Delta\text{KE} = \Delta\text{PE} = 0$  and  $\Delta E = \Delta U$ . **3** The pressure remains constant during the process and thus  $P_2 = P_1$ . **4** Nitrogen has constant specific heats at room temperature.

First, let us determine the electrical work done on the nitrogen:

$$W_e = VI \Delta t = (120 \text{ V})(2 \text{ A})(5 \times 60 \text{ s}) \left( \frac{1 \text{ kJ/s}}{1000 \text{ VA}} \right) = 72 \text{ kJ}$$

The mass of nitrogen is determined from the ideal-gas relation:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(400 \text{ kPa})(0.5 \text{ m}^3)}{(0.297 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(300 \text{ K})} = 2.245 \text{ kg}$$

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$W_{e, \text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e, \text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1) = mc_p(T_2 - T_1)$$

since  $\Delta U + W_b = \Delta H$  for a closed system undergoing a quasi-equilibrium expansion or compression process at constant pressure. From Table A-2a,  $c_p = 1.039 \text{ kJ/kg} \cdot \text{K}$  for *nitrogen at room temperature*. The only unknown quantity in the above equation is  $T_2$ , and it is found to be

$$72 \text{ kJ} - 2.8 \text{ kJ} = (2.245 \text{ kg})(1.039 \text{ kJ/kg} \cdot \text{K})(T_2 - 27^\circ\text{C})$$

$$T_2 = \mathbf{56.7^\circ\text{C}}$$

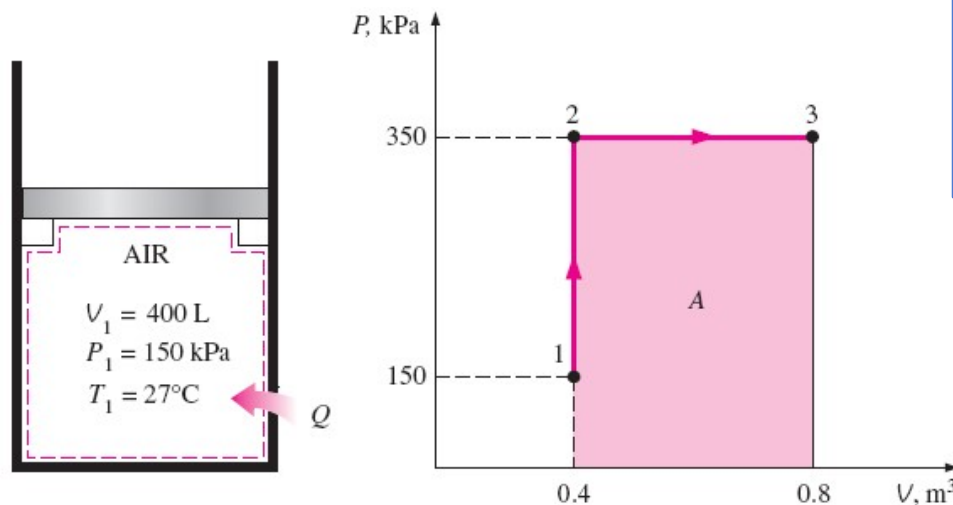
**Discussion** Note that we could also solve this problem by determining the boundary work and the internal energy change rather than the enthalpy change.

## EXAMPLE 4-10

A piston–cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops, and the enclosed volume is 400 L. The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine (a) the final temperature, (b) the work done by the air, and (c) the total heat transferred to the air.

**Solution** Air in a piston–cylinder device with a set of stops is heated until its volume is doubled. The final temperature, work done, and the total heat transfer are to be determined.

**Assumptions** 1 Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. 2 The system is stationary and thus the kinetic and potential energy changes are zero,  $\Delta KE = \Delta PE = 0$  and  $\Delta E = \Delta U$ . 3 The volume remains constant until the piston starts moving, and the pressure remains constant afterwards. 4 There are no electrical, shaft, or other forms of work involved.



(a) The final temperature can be determined easily by using the ideal-gas relation between states 1 and 3 in the following form:

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3}$$

$$\frac{(150 \text{ kPa})(V_1)}{300 \text{ K}} = \frac{(350 \text{ kPa})(2V_1)}{T_3}$$

$$T_3 = 1400 \text{ K}$$

(b) The work done could be determined by integration, but for this case it is much easier to find it from the area under the process curve on a  $P$ - $V$  diagram,

$$A = (V_2 - V_1)(P_2) = (0.4 \text{ m}^3)(350 \text{ kPa}) = 140 \text{ m}^3 \text{ kPa}$$

$$W_{13} = \mathbf{140 \text{ kJ}}$$

The work is done by the system (to raise the piston and to push the atmospheric air out of the way), and thus it is work output.

(c) Under the stated assumptions and observations, the energy balance on the system between the initial and final states (process 1–3) can be expressed as

$$Q_{\text{in}} - W_{b, \text{out}} = \Delta U = m(u_3 - u_1)$$

$$u_1 = u @ 300 \text{ K} = 214.07 \text{ kJ/kg}$$

$$u_3 = u @ 1400 \text{ K} = 1113.52 \text{ kJ/kg}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{(150 \text{ kPa})(0.4 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(300 \text{ K})} = 0.697 \text{ kg}$$

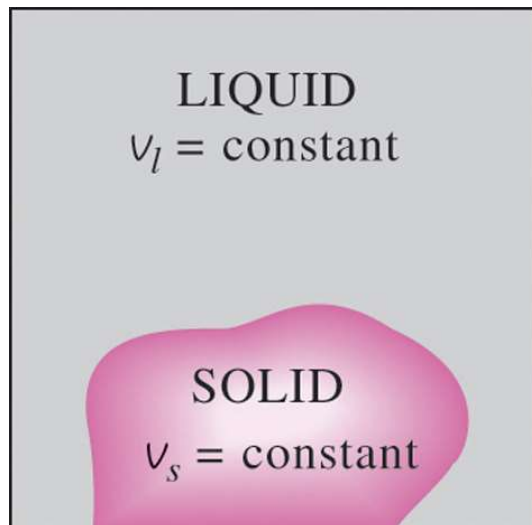
$$Q_{\text{in}} - 140 \text{ kJ} = (0.697 \text{ kg})[(1113.52 - 214.07) \text{ kJ/kg}]$$

$$Q_{\text{in}} = \mathbf{766.9 \text{ kJ}}$$

**Discussion** The positive sign verifies that heat is transferred to the system.

# INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

**Incompressible substance:** A substance whose specific volume (or density) is constant. **Solids and liquids** are incompressible substances.



The specific volumes of incompressible substances remain constant during a process.



The  $c_v$  and  $c_p$  values of incompressible substances are identical and are denoted by  $c$ .

# INTERNAL ENERGY CHANGES

$$du = c_v dT = c(T) dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T) dT \quad (\text{kJ/kg})$$

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1) \quad (\text{kJ/kg})$$

## Enthalpy Changes

$$h = u + Pv$$
$$dh = du + v dP + P d\overset{\nearrow 0}{v} = du + v dP$$

$$\Delta h = \Delta u + v \Delta P \cong c_{\text{avg}} \Delta T + v \Delta P \quad (\text{kJ/kg})$$

For *solids*, the term  $v \Delta P$  is insignificant and thus  $\Delta h = \Delta u \cong c_{\text{av}} \Delta T$ . For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ( $\Delta P = 0$ ):  $\Delta h = \Delta u \cong c_{\text{av}} \Delta T$
2. *Constant-temperature processes*, as in pumps ( $\Delta T = 0$ ):  $\Delta h = v \Delta P$

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\text{sat}})$$

A more accurate relation than

$$h_{@P,T} \cong h_{f@T}$$

The enthalpy of a compressed liquid

## EXAMPLE 4-11

Determine the enthalpy of liquid water at 100°C and 15 MPa (a) by using compressed liquid tables, (b) by approximating it as a saturated liquid, and (c) by using the correction given by Eq. 4–38.

**Solution** The enthalpy of liquid water is to be determined exactly and approximately.

**Analysis** At 100°C, the saturation pressure of water is 101.42 kPa, and since  $P > P_{\text{sat}}$ , the water exists as a compressed liquid at the specified state.

(a) From compressed liquid tables, we read

$$\left. \begin{array}{l} P = 15 \text{ MPa} \\ T = 100^\circ\text{C} \end{array} \right\} h = \mathbf{430.39 \text{ kJ/kg}}$$

(b) Approximating the compressed liquid as a saturated liquid at 100°C, as is commonly done, we obtain

$$h \cong h_{f@100^\circ\text{C}} = \mathbf{419.17 \text{ kJ/kg}}$$

This value is in error by about 2.6 %.

(c) From Eq. 4–38,

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\text{sat}})$$

$$\begin{aligned} &= (419.14 \text{ kJ/kg}) + (0.001 \text{ m}^3/\text{kg})[(15,000 - 101.42) \text{ kPa}]\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right) \\ &= \mathbf{434.07 \text{ kJ/kg}} \end{aligned}$$

**Discussion** Note that the correction term reduced the error from 2.6 to about 1 %. However, this improvement in accuracy is often not worth the extra effort involved.

# SUMMARY

- Moving boundary work
  - $W_b$  for an isothermal process
  - $W_b$  for a constant-pressure process
  - $W_b$  for a polytropic process
- Energy balance for closed systems
  - Energy balance for a constant-pressure expansion or compression process
- Specific heats
  - Constant-pressure specific heat,  $c_p$
  - Constant-volume specific heat,  $c_v$
- Internal energy, enthalpy, and specific heats of ideal gases
  - Specific heat relations of ideal gases
- Internal energy, enthalpy, and specific heats of incompressible substances (solids and liquids)