Thermodynamics: An Engineering Approach, 5<sup>th</sup> Edition Yunus A. Cengel, Michael A. Boles McGraw-Hill, 2008

# GAS-VAPOR MIXTURES AND AIR-CONDITIONING

### Prof. Dr. Ali PINARBAŞI

Yildiz Technical University Mechanical Engineering Department Yildiz, ISTANBUL

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.

### GAS-VAPOR MIXTURES AND AIR-CONDITIONING

- 14–1 Dry and Atmospheric Air
- 14-2 Specific and Relative Humidity of Air
- 14-3 Dew-Point Temperature
- 14-4 Adiabatic Saturation and Wet-Bulb Temperatures
- 14-5 The Psychrometric Chart
- 14-6 Human Comfort and Air-Conditioning
- 14-7 Air-Conditioning Processes

Simple Heating and Cooling ( $\omega$  =constant) Heating with Humidification Cooling with Dehumidification Evaporative Cooling Adiabatic Mixing of Airstreams Wet Cooling Towers

# Objectives

- Explain the meaning of the terms dry air and atmospheric air.
- Define and calculate the specific and relative humidity of atmospheric air.
- Calculate the dew-point temperature of atmospheric air.
- Relate the adiabatic saturation temperature and wet-bulb temperatures of atmospheric air.
- Use the psychrometric chart as a tool to determine the properties of atmospheric air.
- Relate the desire for human comfort to air-conditioning requirements.
- Solve problems associated with the conservation of mass and energy for various air-conditioning processes.

### DRY AND ATMOSPHERIC AIR

Atmospheric air : Air is a mixture of nitrogen, oxygen, and small amounts of some other gases.

We will be concerned with the mixture of dry air and water vapor. This mixture is often called atmospheric air.

Dry air: air that contains no water vapor is called dry air.

$$h_{\rm dry\,air} = c_p T = (1.005 \text{ kJ/kg} \cdot ^{\circ}\text{C})T \qquad (\text{kJ/kg})$$

 $\Delta h_{\rm dry\,air} = c_p \Delta T = (1.005 \,\text{kJ/kg} \cdot ^\circ\text{C}) \,\Delta T \qquad (\text{kJ/kg})$ 

where T is the air temperature in °C and T is the change in temperature.

In air-conditioning processes we are concerned with the *changes* in enthalpy  $\Delta h$ , which is independent of the reference point selected.

water vapor in air behaves as if it existed alone and obeys the ideal-gas relation Pv = RT. Then the atmospheric air can be treated as an ideal-gas mixture whose pressure is the sum of the partial pressure of dry air\*  $P_a$  and that of water vapor  $P_v$ :

$$P = P_a + P_v \qquad (kPa)$$
Prof. Dr. Ali PINARBAŞI
Chapter 14 GAS-VAPOR MIXTURES AND AIR-CONDITIONING

### $P = P_a + P_v \quad \text{(kPa)}$

 $P_a$  the sum of the partial pressure of dry air  $P_v$  water vapor

The partial pressure of water vapor is usually referred to as the vapor pressure.

It is the pressure water vapor would exert if it existed alone at the temperature and volume of atmospheric air.

DRY AIR		
<i>T</i> , °C	$-c_p$ , kJ/kg · °C	
-10	1.0038	
0	1.0041	
10	1.0045	
20	1.0049	
	1.0054	
40	1.0059	
50	1.0065	

The  $c_p$  of air can be assumed to be constant at 1.005 kJ/kg  $\cdot$  °C in the temperature range 10 to 50°C with an error under 0.2 %.

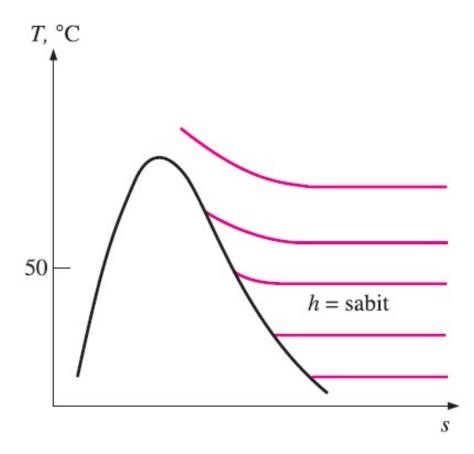
It certainly would be very convenient to also treat the water vapor in the air as an ideal gas and you would probably be willing to sacrifice some accuracy for such convenience. Well, it turns out that we can have the convenience without much sacrifice. At 50°C, the saturation pressure of water is 12.3 kPa. At pressures below this value, water vapor can be treated as an ideal gas with negligible error (under 0.2 %), even when it is a saturated vapor. For water vapor:  $h_g = 2500.9 \text{ kJ/kg at } 0^{\circ}\text{C}$  $c_{p,avg} = 1.82 \text{ kJ/kg} \cdot ^{\circ}\text{C} \text{ at } -10 \text{ to } 50^{\circ}\text{C} \text{ range}$ 



Since water vapor is an ideal gas, the enthalpy of water vapor is a function of temperature only, that is, h = h(T).

Therefore, the enthalpy of water vapor in air can be taken to be equal to the enthalpy of saturated vapor at the same temperature. That is,

 $h_{\nu}(T, \text{low } P) \cong h_g(T)$ 



C	WATE	R VAPOR	
	$h_g$ , kJ/kg		Difference.
<i>T</i> , °C	Table A-4	Eq. 14-4	kJ/kg
-10	2482.1	2482.7	-0.6
0	2500.9	2500.9	0.0
10	2519.2	2519.1	0.1
20	2537.4	2537.3	0.1
30	2555.6	2555.5	0.1
40	2573.5	2573.7	-0.2
50	2591.3	2591.9	-0.6

At temperatures below 50°C, the h = constant lines coincide with the T=constant lines in the superheated vapor region of water.

In the temperature range 10 to 50°C, the  $h_g$  of water can be determined from  $(h_g(T) \cong 2500.9 + 1.82 T)$  with negligible error.

# SPECIFIC AND RELATIVE HUMIDITY OF AIR

**Absolute or Specific humidity (humidity ratio):** The amount of water vapor in the air can be specified in various ways. Probably the most logical way is to specify directly the mass of water vapor present in a unit mass of dry air.

This is called **absolute** or **specific humidity** (also called *humidity ratio*) and is denoted by v:

$$\omega = \frac{m_{\nu}}{m_{a}} = \frac{P_{\nu} V/R_{\nu} T}{P_{a} V/R_{a} T} = \frac{P_{\nu} /R_{\nu}}{P_{a} /R_{a}} = 0.622 \frac{P_{\nu}}{P_{a}}$$

The specific humidity can also be expressed as

$$\omega = \frac{m_v}{m_a} \qquad \text{(kg water vapor/kg dry air)}$$

where *P* is the total pressure:

$$\omega = \frac{0.622P_v}{P - P_v}$$

Saturated air: The air is said to be saturated with moisture.

**Relative humidity** ( $\phi$ ) : The comfort level depends more on the amount of moisture the air holds ( $m_v$ ) relative to the maximum amount of moisture the air can hold at the same temperature ( $m_a$ ). The ratio of these two quantities is called the **relative humidity**.

$$\phi = \frac{m_v}{m_g} = \frac{P_v V/R_v T}{P_g V/R_v T} = \frac{P_v}{P_g}$$

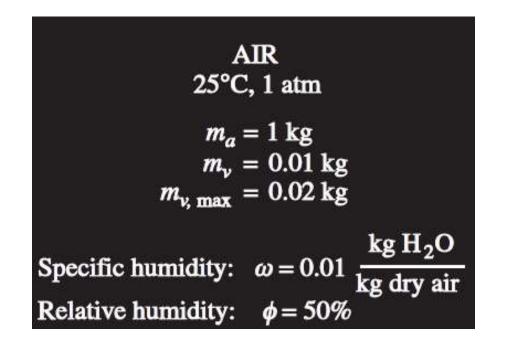
$$P_g = P_{\operatorname{doy} @ T}$$

AIR
25°C, 100 kPa
$(P_{\text{sat, H}_2\text{O} @ 25^{\circ}\text{C}} = 3.1698 \text{ kPa})$
$P_{\rm v} = 0 \rightarrow dry air$
$P_{v} < 3.1698$ kPa $\rightarrow$ unsaturated air
$P_v = 3.1698 \text{ kPa} \rightarrow \text{ saturated air}$

For saturated air, the vapor pressure is equal to the saturation pressure of water.

The amount of moisture in the air has a definite effect on how comfortable we feel in an environment.

However, the comfort level depends more on the amount of moisture the air holds  $(m_v)$  relative to the maximum amount of moisture the air can hold at the same temperature  $(m_q)$ . The ratio of these two quantities is called the **relative humidity**  $\phi$ 



Specific humidity is the actual amount of water vapor in 1 kg of dry air, whereas relative humidity is the ratio of the actual amount of moisture in the air to the maximum amount of moisture air can hold at the temperature.

$$\phi = \frac{\omega P}{(0.622 + \omega)P_g} \quad \text{and} \quad \omega = \frac{0.622\phi P_g}{P - \phi P_g}$$

The relative humidity ranges from 0 for dry air to 1 for saturated air. Note that the amount of moisture air can hold depends on its temperature. Therefore, the relative humidity of air changes with temperature even when its specific humidity remains constant.

Atmospheric air is a mixture of dry air and water vapor, and thus the enthalpy of air is expressed in terms of the enthalpies of the dry air and the water vapor. In most practical applications, the amount of dry air in the air–water-vapor mixture remains constant, but the amount of water vapor changes.

Therefore, the enthalpy of atmospheric air is expressed *per unit mass of dry air* instead of per unit mass of the air–water-vapor mixture.

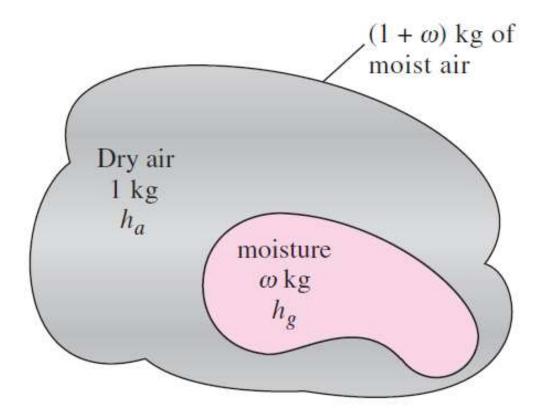
$$H = H_a + H_v = m_a h_a + m_v h_v$$

$$h = \frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$$

$$h = h_a + \omega h_g \quad (kJ/kg dry air)$$

$$h_v \simeq h_g$$

**Dry-bulb temperature:** The ordinary temperature of atmospheric air is frequently referred to as the **dry-bulb temperature** to differentiate it from other forms of temperatures.



 $h = h_a + \omega h_g$ , kJ/kg dry air

The enthalpy of moist (atmospheric) air is expressed per unit mass of dry air, not per unit mass of moist air.

Prof. Dr. Ali PINARBAŞI

Example 14-1

A 5-m 5-m 3-m room contains air at 25°C and 100 kPa at a relative humidity of 75%. Determine (*a*) the partial pressure of dry air, (*b*) the specific humidity, (*c*) the enthalpy per unit mass of the dry air, and (*d*) the masses of the dry air and water vapor in the room.

(a) the partial pressure of dry air

$$P_a = P - P_v$$

ROOM  

$$5 \text{ m} \times 5 \text{ m} \times 3 \text{ m}$$
  
 $T = 25^{\circ}\text{C}$   
 $P = 100 \text{ kPa}$   
 $\phi = 75\%$ 

$$P_v = \phi P_g = \phi P_{\text{sat @ 25°C}} = (0.75)(3.1698 \text{ kPa}) = 2.38 \text{ kPa}$$
  
 $P_a = (100 - 2.38) \text{ kPa} = 97.62 \text{ kPa}$ 

(b) the specific humidity

$$\omega = \frac{0.622P_v}{P - P_v} = \frac{(0.622)(2.38 \text{ kPa})}{(100 - 2.38) \text{ kPa}} = 0.0152 \text{ kg H}_2\text{O/kg dry air}$$

(c) the enthalpy per unit mass of the dry air

$$h = h_a + \omega h_v \cong c_p T + \omega h_g$$
  
= (1.005 kJ/kg · °C)(25°C) + (0.0152)(2546.5 kJ/kg)  
= 63.8 kJ/kg dry air

The enthalpy of water vapor (2546.5 kJ/kg) could also be determined from the approximation given  $(h_g(T) \cong 2500.9 + 1.82 T)$ 

$$h_{g@25^{\circ}C} \cong 2500.9 + 1.82(25) = 2546.4 \text{ kJ/kg}$$

(d) Both the dry air and the water vapor fill the entire room completely. Therefore, the volume of each gas is equal to the volume of the room:

$$V_a = V_v = V_{\text{room}} = (5)(5)(3) = 75 \text{ m}^3$$

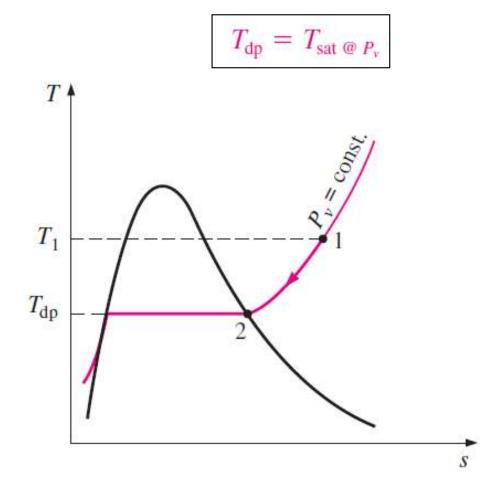
$$m_{a} = \frac{P_{a}V_{a}}{R_{a}T} = \frac{(97.62 \text{ kPa})(75 \text{ m}^{3})}{(0.287 \text{ kPa} \cdot \text{m}^{3}/\text{kg} \cdot \text{K})(298 \text{ K})} = 85.61 \text{ kg}$$
$$m_{v} = \frac{P_{v}V_{v}}{R_{v}T} = \frac{(2.38 \text{ kPa})(75 \text{ m}^{3})}{(0.4615 \text{ kPa} \cdot \text{m}^{3}/\text{kg} \cdot \text{K})(298 \text{ K})} = 1.3 \text{ kg}$$

The mass of the water vapor in the air could also be determined from  $\omega = \frac{m_v}{m_c}$ 

$$m_v = \omega m_a = (0.0152)(85.61 \text{ kg}) = 1.30 \text{ kg}$$

# DEW-POINT TEMPERATURE

**Dew-point temperature**  $T_{dp}$ :  $T_{dp}$  is defined as the temperature at which condensation begins when the air is cooled at constant pressure.



Constant-presssure cooling of moist air and the dew-point temperature on the *T*-*s* diagram of water. The vapor in the air (state 1) undergoes a constant-pressure cooling process until it strikes the saturated vapor line (state 2).

The temperature at this point is  $T_{dp}$ , and if the temperature drops any further, some vapor condenses out.

As a result, the amount of vapor in the air decreases, which results in a decrease in  $P_v$ . The air remains saturated during the condensation process and thus follows a path of 100 % relative humidity (the saturated vapor line). The ordinary temperature and the dew-point temperature of saturated air are identical.



When the temperature of a cold drink is below the dewpoint temperature of the surrounding air, it "sweats."

You have probably noticed that when you buy a cold canned drink from a vending machine on a hot and humid day, dew forms on the can.

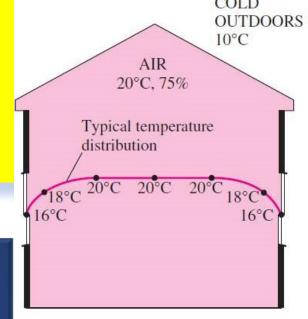
The formation of dew on the can indicates that the temperature of the drink is below the dew-point temperature of the surrounding air .

### Example 14-2

In cold weather, condensation frequently occurs on the inner surfaces of the windows due to the lower air temperatures near the window surface. Consider a house that contains air at 20°C and 75 percent relative humidity. At what window temperature will the moisture in the air start condensing on the inner surfaces of the windows?

#### The saturation pressure of water at 20°C is P<sub>sat</sub> = 2.3392 kPa

The temperature distribution in a house, in general, is not uniform. When the outdoor temperature drops in winter, so does the indoor temperature near the walls and the windows. Therefore, the air near the walls and the windows remains at a lower temperature than at the inner parts of a house even though the total pressure and the vapor pressure remain constant throughout the house. As a result, the air near the walls and the windows will undergo a  $P_v$  constant cooling process until the moisture in the air starts condensing. This will happen when the air reaches its dew-point temperature  $T_{dp}$ . The dew point is determined from;



$$T_{dp} = T_{sat @ P_v} \qquad P_v = \phi P_{g @ 20^{\circ}C} = (0.75)(2.3392 \text{ kPa}) = 1.754 \text{ kPa}$$
$$T_{dp} = T_{sat @ 1.754 \text{ kPa}} = \mathbf{15.4 ^{\circ}C}$$

Therefore, the inner surface of the window should be maintained above 15.4°C if condensation on the window surfaces is to be avoided.

### ADIABATIC SATURATION AND WET-BULB TEMPERATURES

Relative humidity and specific humidity are frequently used in engineering and atmospheric sciences, and it is desirable to relate them to easily measurable quantities such as temperature and pressure.

One way of determining the relative humidity is to determine the dew-point temperature of air, as discussed in the last section. Knowing the dew-point temperature, we can determine the vapor pressure  $P_v$  and thus the relative humidity. This approach is simple, but not quite practical.

Mass balance:		
	$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$	(The mass flow rate of dry air remains constant)
	$\dot{m}_{w_1} + \dot{m}_f = \dot{m}_{w_2}$	(The mass flow rate of vapor in the air increases by an amount equal to the rate of evaporation $\dot{m}_f$ )

$$\dot{m}_a \omega_1 + \dot{m}_f = \dot{m}_a \omega_2$$

$$\dot{m}_f = \dot{m}_a(\omega_2 - \omega_1)$$

Energy balance:  

$$(\dot{Q} = 0 \text{ ve } \dot{W} = 0)$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_a h_1 + \dot{m}_f h_{f_2} = \dot{m}_a h_2$$

$$(\dot{m}_a h_1 + \dot{m}_a (\omega_2 - \omega_1) h_{f_2} = \dot{m}_a h_2$$

$$(\dot{m}_a h_1 + \dot{m}_a (\omega_2 - \omega_1) h_{f_2} = \dot{m}_a h_2$$

$$(\dot{m}_a h_1 + \dot{m}_a (\omega_2 - \omega_1) h_{f_2} = \dot{m}_a h_2$$

$$(c_p T_1 + \omega_1 h_{g_1}) + (\omega_2 - \omega_1) h_{f_2} = (c_p T_2 + \omega_2 h_{g_2})$$

$$(\omega_1 = \frac{c_p (T_2 - T_1) + \omega_2 h_{f_{g_2}}}{h_{g_1} - h_{f_2}}$$

$$(\omega_2 = \frac{0.622P_{g_2}}{P_2 - P_{g_2}}$$

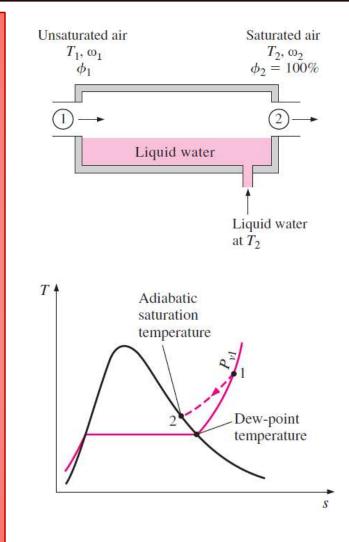
Chapter 14 GAS-VAPOR MIXTURES AND AIR-CONDITIONING

Prof. Dr. Ali PINARBAŞI

Another way of determining the absolute or relative humidity is related to an *adiabatic saturation process*, shown schematically and on a *T-s* diagram in Figure.

The system consists of a long insulated channel that contains a pool of water. A steady stream of unsaturated air that has a specific humidity of  $\omega_1$  (unknown) and a temperature of  $T_1$  is passed through this channel.

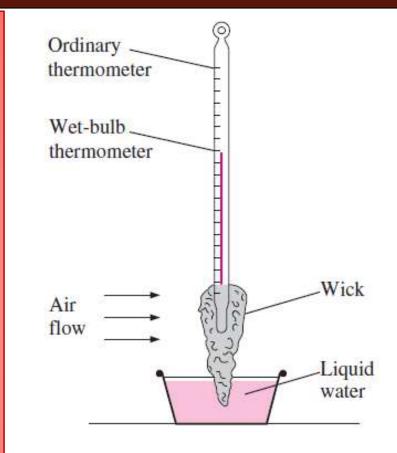
As the air flows over the water, some water will evaporate and mix with the airstream. The moisture content of air will increase during this process, and its temperature will decrease, since part of the latent heat of vaporization of the water that evaporates will come from the air. If the channel is long enough, the airstream will exit as saturated air ( $\phi$ = 100 percent) at temperature  $T_2$ , which is called the **adiabatic saturation temperature**.



The adiabatic saturation process discussed above provides a means of determining the absolute or relative humidity of air, but it requires a long channel or a spray mechanism to achieve saturation conditions at the exit.

A more practical approach is to use a thermometer whose bulb is covered with a cotton wick saturated with water and to blow air over the wick, as shown in Figure. The temperature measured in this manner is called the **wetbulb temperature**  $T_{wb}$ , and it is commonly used in airconditioning applications.

The basic principle involved is similar to that in adiabatic saturation. When unsaturated air passes over the wet wick, some of the water in the wick evaporates. As a result, the temperature of the water drops, creating a temperature difference (which is the driving force for heat transfer) between the air and the water. After a while, the heat loss from the water by evaporation equals the heat gain from the air, and the water temperature stabilizes. The thermometer reading at this point is the wet-bulb temperature.

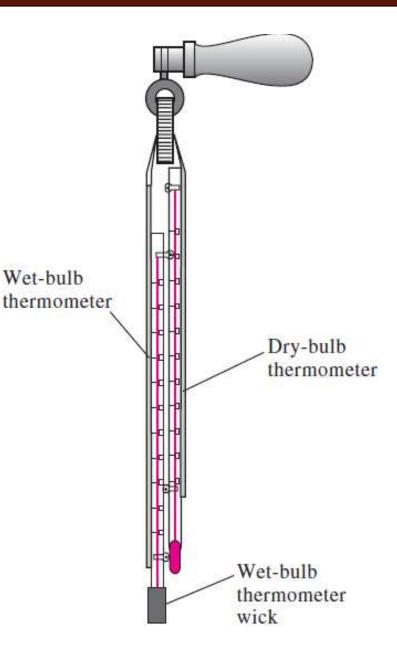


A simple arrangement to measure the wet-bulb temperature.

The wet bulb temperature can also be measured by placing the wet-wicked thermometer in a holder attached to a handle and rotating the holder rapidly that is, by moving the thermometer instead of the air. that is, by moving the thermometer instead of the air.

Advances in electronics made it possible to measure humidity directly in a fast and reliable way. It appears that sling psychrometers and wet-wicked thermometers are about to become things of the past.

Today, hand-held electronic humidity measurement devices based on the capacitance change in a thin polymer film as it absorbs water vapor are capable of sensing and digitally displaying the relative humidity within 1 percent accuracy in a matter of seconds.



### Example 14-3

The dry- and the wet-bulb temperatures of atmospheric air at 1 atm (101.325-kPa) pressure are measured with a sling psychrometer and determined to be 25 and 15°C, respectively. Determine (*a*) the specific humidity, (*b*) the relative humidity, and (*c*) the enthalpy of the air.

(a) the specific humidity 
$$\omega_1$$
  

$$\omega_1 = \frac{c_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}} = \frac{0.622 P_{g_2}}{P_2 - P_{g_2}} = \frac{(0.622)(1.7057 \text{ kPa})}{(101.325 - 1.7057) \text{ kPa}}$$

$$= 0.01065 \text{ kg H}_2\text{O/kg dry air}$$

$$\omega_1 = \frac{(1.005 \text{ kJ/kg} \cdot ^\circ\text{C})[(15 - 25)^\circ\text{C}] + (0.01065)(2465.4 \text{ kJ/kg})}{(2546.5 - 62.982) \text{ kJ/kg}}$$

 $= 0.00653 \text{ kg H}_2\text{O/kg dry air}$ 

#### (b) the relative humidity $\phi_1$

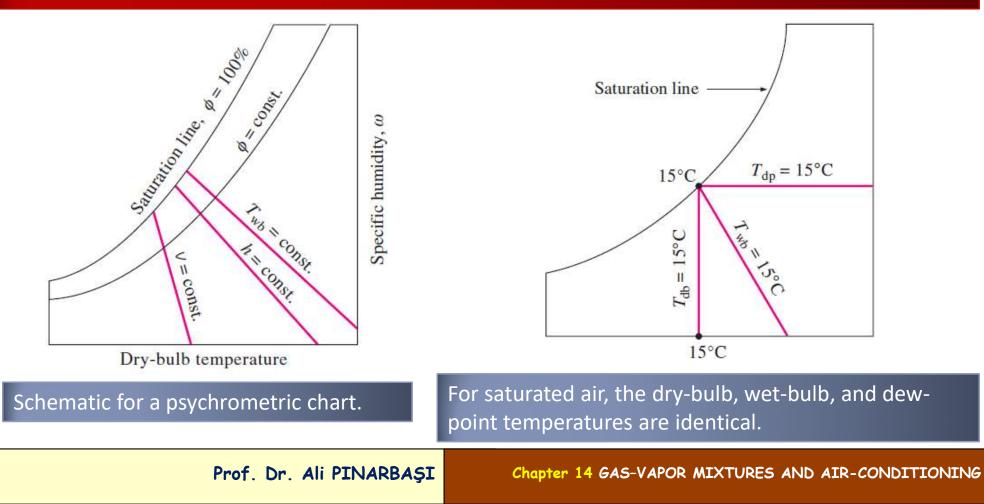
$$\phi_1 = \frac{\omega_1 P_2}{(0.622 + \omega_1) P_{g_1}} = \frac{(0.00653)(101.325 \text{ kPa})}{(0.622 + 0.00653)(3.1698 \text{ kPa})} = 0.332 \text{ or } 33.2\%$$

(c) the enthalpy of the air

$$h_1 = h_{a_1} + \omega_1 h_{\nu_1} \cong c_p T_1 + \omega_1 h_{g_1}$$
  
= (1.005 kJ/kg · °C)(25°C) + (0.00653)(2546.5 kJ/kg)  
= **41.8 kJ/kg dry air**

# THE PSYCHROMETRIC CHART

The state of the atmospheric air at a specified pressure is completely specified by two independent intensive properties. The rest of the properties can be calculated easily from the relations above. The sizing of a typical air conditioning system involves numerous such calculations, which may eventually get on the nerves of even the most patient engineers. Therefore, there is clear motivation to computerize calculations or to do these calculations once and to present the data in the form of easily readable charts. Such charts are called **psychrometric charts**, and they are used extensively in air-conditioning applications.



#### Example 14-4

Consider a room that contains air at 1 atm,  $35^{\circ}$ C, and 40 percent relative humidity. Using the psychrometric chart, determine (*a*) the specific humidity, (*b*) the enthalpy, (*c*) the wet-bulb temperature, (*d*) the dew-point temperature, and (*e*) the specific volume of the air.

(a) The specific humidity is determined by drawing a horizontal line from the specified state to the right until it intersects with the  $\omega$  axis;

 $\omega = 0.0142 \text{ kg H}_2\text{O/kg dry air}$ 

b) The enthalpy of air per unit mass of dry air is determined by drawing a line parallel to the h=constant lines from the specific state until it intersects the enthalpy scale;

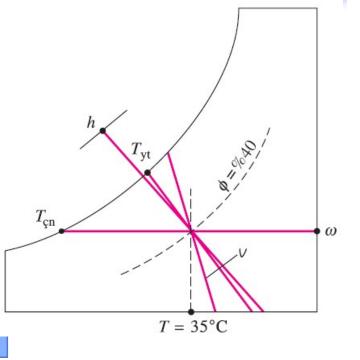
h = 71.5 kJ/kg dry air

(c) The wet-bulb temperature is determined by drawing a line parallel to the  $T_{wb}$ =cnst lines from the specified state until it intersects the saturation line.

 $T_{\rm wb} = 24^{\circ}{\rm C}$ 

(d) The dew-point temperature is determined by drawing a horizontal line from the specified state to the left until it intersects the saturation line.

(e) The specific volume per unit mass of dry air is determined by noting the distances between the specified state and the *v* constant lines on both sides of the point.,

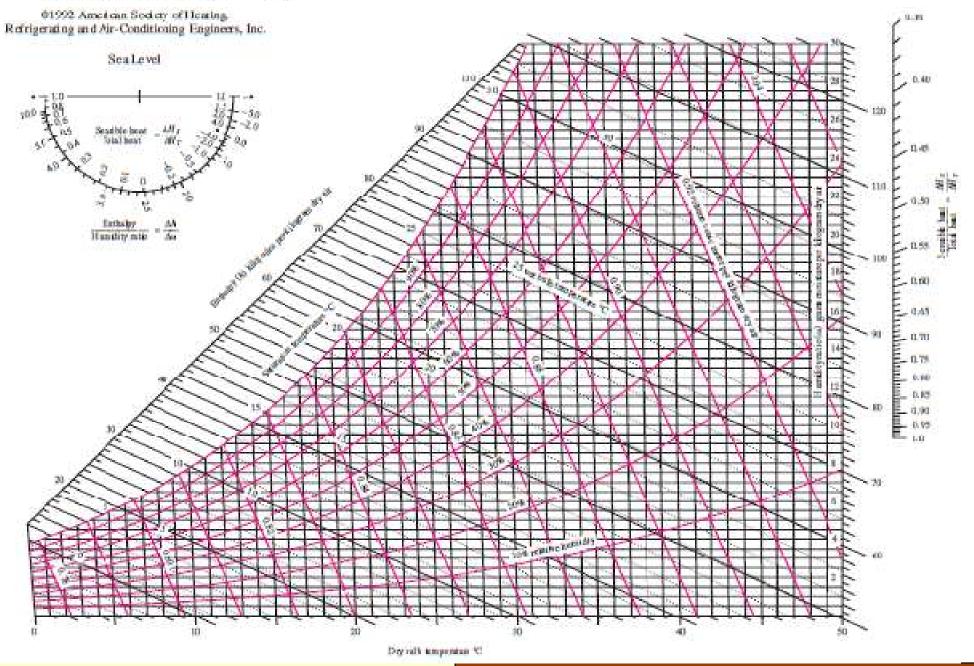


 $T_{\rm dp} = 19.4^{\circ}{\rm C}$ 

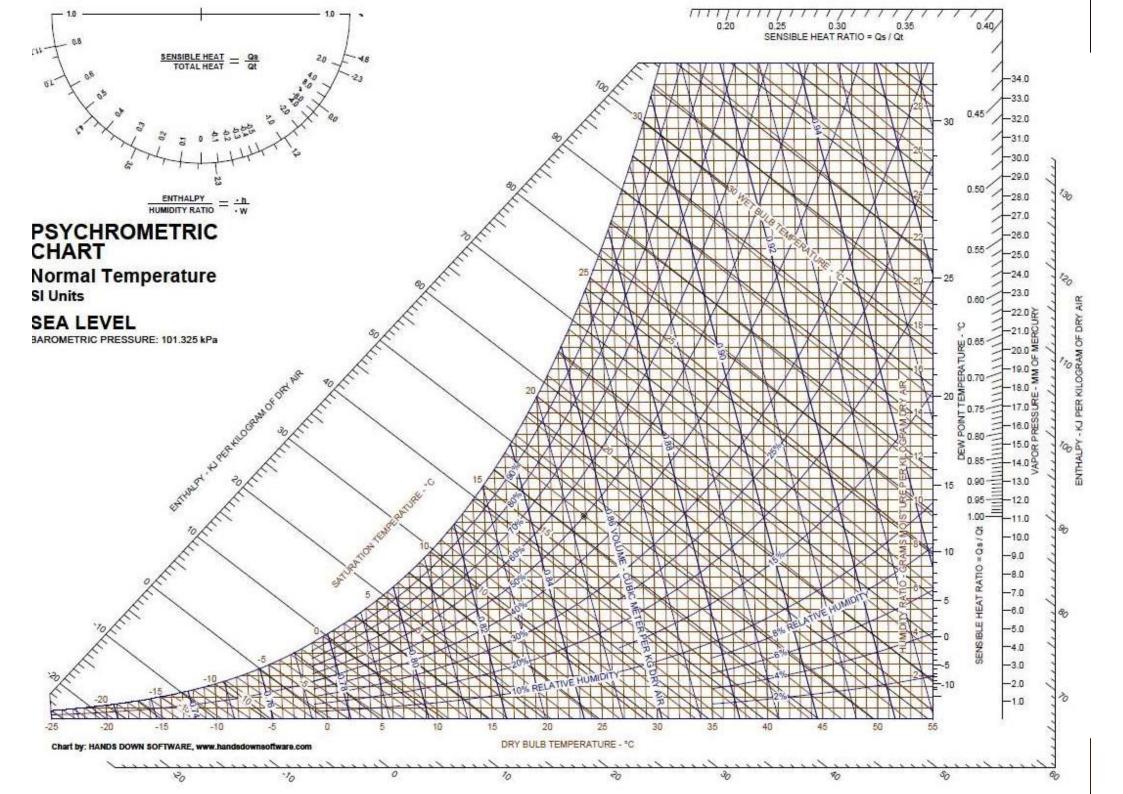
$$v = 0.893 \text{ m}^3/\text{kg} \text{ dry air}$$

ASH RAE Psychrometric Chart No. 1 Normal Temperature Barometric Pressure: 101.325 kPa





Prof. Dr. Ali PINARBAŞI



# HUMAN COMFORT AND AIR-CONDITIONING

Human beings have an inherent weakness—they want to feel comfortable. They want to live in an environment that is neither hot nor cold, neither humid nor dry. However, comfort does not come easily since the desires of the human body and the weather usually are not quite compatible.

Achieving comfort requires a constant struggle against the factors that cause discomfort, such as high or low temperatures and high or low humidity. As engineers, it is our duty to help people feel comfortable.

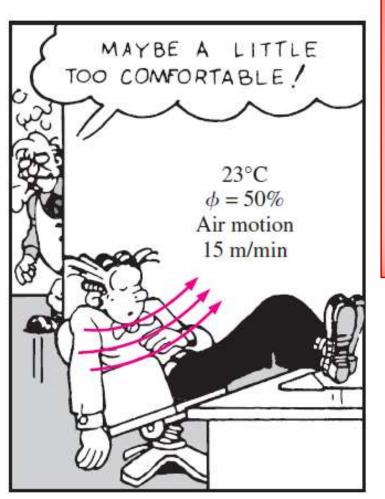


The human body can be viewed as a heat engine whose energy input is food. As with any other heat engine, the human body generates waste heat that must be rejected to the environment if the body is to continue operating. The rate of heat generation depends on the level of the activity. For an average adult male, it is about 87 W when sleeping, 115 W when resting or doing office work, 230 W when bowling, and 440 W when doing heavy physical work.



A body feels comfortable when it can freely dissipate its waste heat, and no more..

Prof. Dr. Ali PINARBAŞI



In hot environments, we have the opposite problem—we do not seem to be dissipating enough heat from our bodies, and we feel as if we are going to burst. We dress lightly to make it easier for heat to get away from our bodies, and we reduce the level of activity to minimize the rate of waste heat generation in the body.

We also turn on the fan to continuously replace the warmer air layer that forms around our bodies as a result of body heat by the cooler air in other parts of the room.

The corresponding numbers for an adult female are about 15 percent less. (This difference is due to the body size, not the body temperature. The deep-body temperature of a healthy person is maintained constant at 37°C.)

A body will feel comfortable in environments in which it can dissipate this waste heat comfortably.

The comfort of the human body depends primarily on three factors: The (dry-bulb) temperature, relative humidity, and air motion.

The temperature of the environment is the single most important index of comfort. Most people feel comfortable when the environment temperature is between 22-27°C.

The relative humidity also has a considerable effect on comfort since it affects the amount of heat a body can dissipate through evaporation.

Relative humidity is a measure of air's ability to absorb more moisture.

High relative humidity slows down heat rejection by evaporation, and low relative humidity speeds it up.

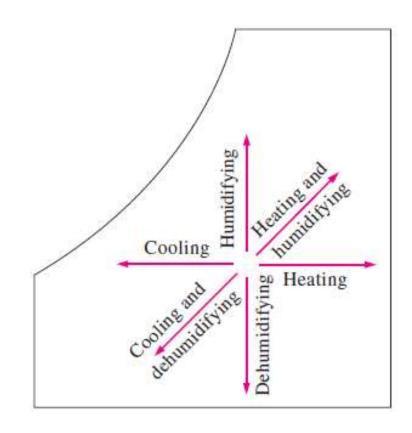
Most people prefer a relative humidity of 40 to 60 percent.

## AIR-CONDITIONING PROCESSES

Maintaining a living space or an industrial facility at the desired temperature and humidity requires some processes called air-conditioning processes. These processes include

simple heating (raising the temperature), simple cooling (lowering the temperature), humidifying (adding moisture), and dehumidifying (removing moisture).

Sometimes two or more of these processes are needed to bring the air to a desired temperature and humidity level.



Various air-conditioning processes are illustrated on the psychrometric chart. Notice that simple heating and cooling processes appear as horizontal lines on this chart since the moisture content of the air remains constant ( $\omega = \text{constant}$ ) during these processes. Air is commonly heated and humidified in winter and cooled and dehumidified in summer.

Notice how these processes appear on the psychrometric chart.

Most air-conditioning processes can be modeled as steady-flow processes, and thus the mass balance relation can be expressed for *dry air* and *water* as;

$$\dot{m}_{\rm in} = \dot{m}_{\rm out}$$

Mass balance for dry air: 
$$\sum \dot{m}_{a,i} = \sum \dot{m}_{a,e}$$
 (kg/s)  
Mass balance for water:  $\sum \dot{m}_{w,i} = \sum \dot{m}_{w,e}$  or  $\sum \dot{m}_{a,i} \omega_i = \sum \dot{m}_{a,e} \omega_e$ 

where the subscripts *i* and *e* denote the inlet and the exit states, respectively. Disregarding the kinetic and potential energy changes, the *steady-flow energy balance* relation can be expressed in this case as;

#### Energy balance

Mass balance

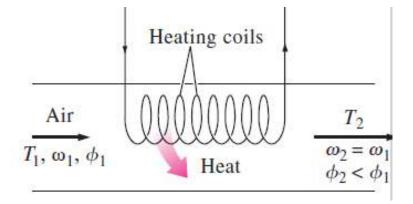
$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$

$$\dot{Q}_{\rm in} + \dot{W}_{\rm in} + \sum \dot{m}_i h_i = \dot{Q}_{\rm out} + \dot{W}_{\rm out} + \sum \dot{m}_e h_e$$

The work term usually consists of the *fan work input,* which is small relative to the other terms in the energy balance relation.

### Simple Heating and Cooling ( $\omega$ = constant)

Many residential heating systems consist of a stove, a heat pump, or an electric resistance heater. The air in these systems is heated by circulating it through a duct that contains the tubing for the hot gases or the electric resistance wires.



The amount of moisture in the air remains constant during this process since no moisture is added to or removed from the air. That is, the specific humidity of the air remains constant ( $\omega$  = constant) during a heating (or cooling) process with no humidification or dehumidification. Such a heating process will proceed in the direction of increasing dry-bulb temperature following a line of constant specific humidity on the psychrometric chart, which appears as a horizontal line.

Notice that the relative humidity of air decreases during a heating process even if the specific humidity v remains constant. This is because the relative humidity is the ratio of the moisture content to the moisture capacity of air at the same temperature, and moisture capacity increases with temperature. Therefore, the relative humidity of heated air may be well below comfortable levels, causing dry skin, respiratory difficulties, and an increase in static electricity.

The conservation of mass equations for a heating or cooling process that involves no humidification or dehumidification reduce to;

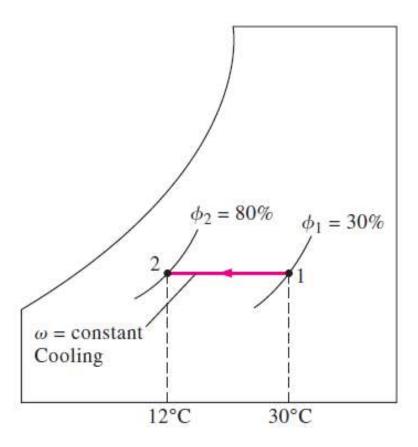
for dry air  
for water  
Energy balance  

$$\dot{Q} = \dot{m}_a(h_2 - h_1)$$
 or  $q = h_2 - h_1$ 

The conservation of mass equations for a heating or cooling process that involves no humidification or dehumidification reduce to  $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$  for dry air and v1 v2 for water. Neglecting any fan work that may be present, the conservation of energy equation in this case reduces to Q veya q ;

$$\dot{Q} = \dot{m}_a(h_2 - h_1)$$
 or  $q = h_2 - h_1$ 

where  $h_1$  and  $h_2$  are enthalpies per unit mass of dry air at the inlet and the exit of the heating or cooling section, respectively.

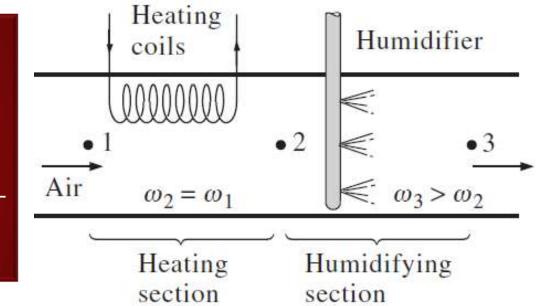


During simple cooling, specific humidity remains constant, but relative humidity increases.

# Heating with Humidification

The low relative humidity resulting from simple heating can be eliminated by humidifying the heated air.

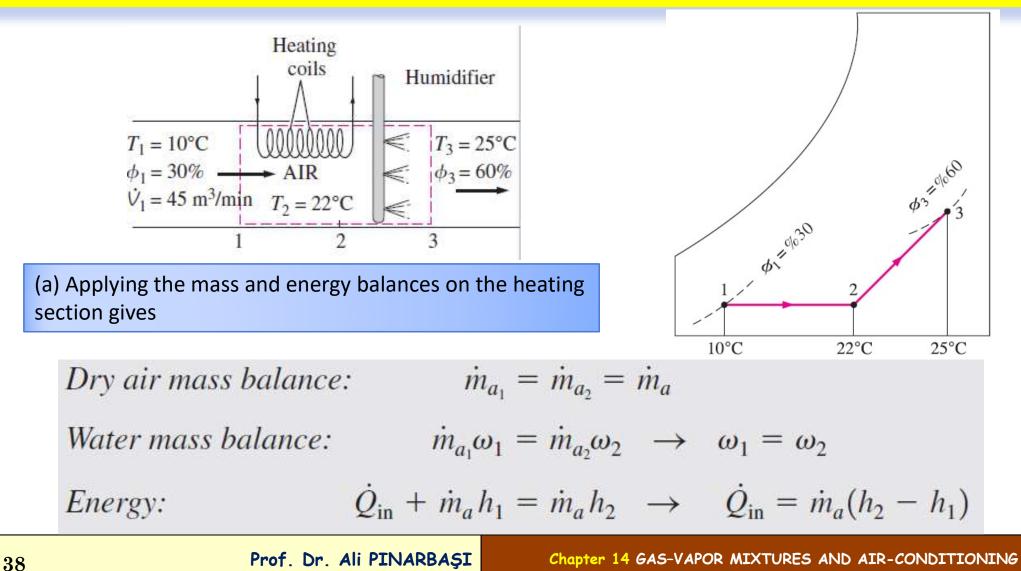
This is accomplished by passing the air first through a heating section (process 1-2) and then through a humidifying section (process 2-3)



The location of state 3 depends on how the humidification is accomplished.

If steam is introduced in the humidification section, this will result in humidification with additional heating ( $T_3 > T_2$ ). If humidification is accomplished by spraying water into the airstream instead, part of the latent heat of vaporization will come from the air, which will result in the cooling of the heated airstream ( $T_3 < T_2$ ). Air should be heated to a higher temperature in the heating section in this case to make up for the cooling effect during the humidification process.

An air-conditioning system is to take in outdoor air at 10°C and 30 percent relative humidity at a steady rate of 45 m3/min and to condition it to 25°C and 60 percent relative humidity. The outdoor air is first heated to 22°C in the heating section and then humidified by the injection of hot steam in the humidifying section. Assuming the entire process takes place at a pressure of 100 kPa, determine (a) the rate of heat supply in the heating section and (b) the mass flow rate of the steam required in the humidifying section.



$$\begin{aligned} P_{v_1} &= \phi_1 P_{g_1} = \phi P_{\text{sat } \circledast \ 10^{\circ}\text{C}} = (0.3) (1.2281 \text{ kPa}) = 0.368 \text{ kPa} \\ P_{a_1} &= P_1 - P_{v_1} = (100 - 0.368) \text{ kPa} = 99.632 \text{ kPa} \\ v_1 &= \frac{R_a T_1}{P_a} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}) (283 \text{ K})}{99.632 \text{ kPa}} = 0.815 \text{ m}^3/\text{kg} \text{ dry air} \\ \dot{m}_a &= \frac{\dot{V}_1}{v_1} = \frac{45 \text{ m}^3/\text{min}}{0.815 \text{ m}^3/\text{kg}} = 55.2 \text{ kg/min} \\ \\ \omega_1 &= \frac{0.622 P_{v_1}}{P_1 - P_{v_1}} = \frac{0.622 (0.368 \text{ kPa})}{(100 - 0.368) \text{ kPa}} = 0.0023 \text{ kg H}_2\text{O/kg dry air} \\ h_1 &= c_p T_1 + \omega_1 h_{g_1} = (1.005 \text{ kJ/kg} \cdot ^{\circ}\text{C}) (10^{\circ}\text{C}) + (0.0023) (2519.2 \text{ kJ/kg}) \\ &= 15.8 \text{ kJ/kg dry air} \\ h_2 &= c_p T_2 + \omega_2 h_{g_2} = (1.005 \text{ kJ/kg} \cdot ^{\circ}\text{C}) (22^{\circ}\text{C}) + (0.0023) (2541.0 \text{ kJ/kg}) \end{aligned}$$

= 28.0 kJ/kg dry air

K

h

since  $\omega_2 = \omega_1$ . Then the rate of heat transfer to air in the heating section becomes

$$\dot{Q}_{in} = \dot{m}_a (h_2 - h_1) = (55.2 \text{ kg/min})[(28.0 - 15.8) \text{ kJ/kg}]$$
  
= 673 kJ/min

(b) The mass balance for water in the humidifying section

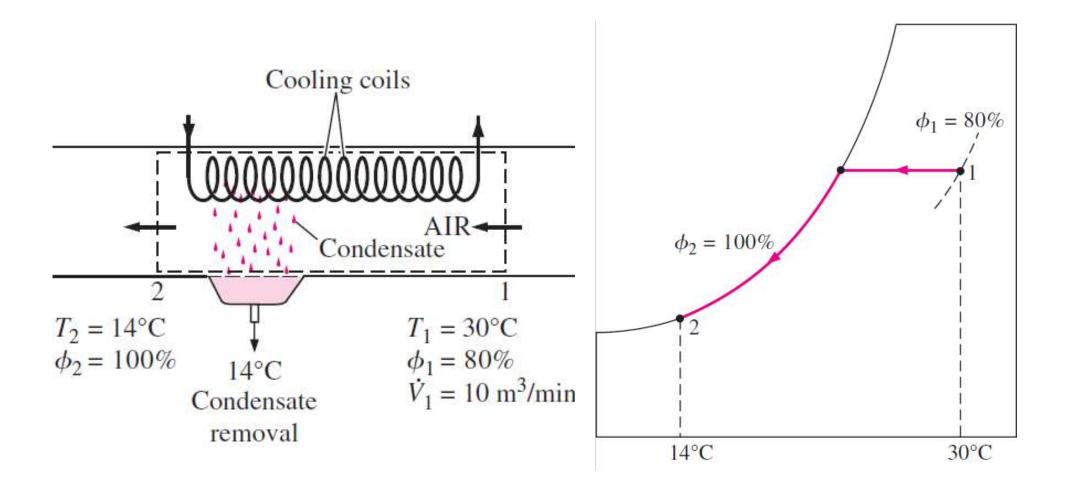
$$\dot{m}_{a_2}\omega_2 + \dot{m}_w = \dot{m}_{a_3}\omega_3 \qquad \qquad \dot{m}_w = \dot{m}_a(\omega_3 - \omega_2)$$

$$\omega_3 = \frac{0.622\phi_3 P_{g_3}}{P_3 - \phi_3 P_{g_3}} = \frac{0.622(0.60)(3.1698 \text{ kPa})}{[100 - (0.60)(3.1698)] \text{ kPa}}$$
$$= 0.01206 \text{ kg H}_2\text{O/kg dry air}$$

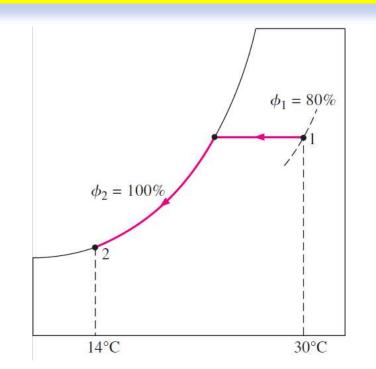
$$\dot{m}_w = (55.2 \text{ kg/min})(0.01206 - 0.0023)$$
  
= 0.539 kg/min

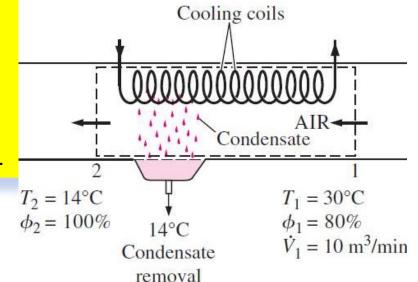
## **Cooling with Dehumidification**

The specific humidity of air remains constant during a simple cooling process, but its relative humidity increases. If the relative humidity reaches undesirably high levels, it may be necessary to remove some moisture from the air, that is, to dehumidify it. This requires cooling the air below its dewpoint temperature.



Air enters a window air conditioner at 1 atm, 30°C, and 80 percent relative humidity at a rate of 10 m3/min, and it leaves as saturated air at 14°C. Part of the moisture in the air that condenses during the process is also removed at 14°C. Determine the rates of heat and moisture removal from the air.





The enthalpy of saturated liquid water at 14°C is 58.8 kJ/kg (Table A–4). Also, the inlet and the exit states of the air are completely specified, and the total pressure is 1 atm. Therefore, we can determine the properties of the air at both states from the psychrometric chart to be

 $h_1 = 85.4 \text{ kJ/kg dry air} \qquad h_2 = 39.3 \text{ kJ/kg dry air}$   $\omega_1 = 0.0216 \text{ kg H}_2\text{O/kg dry air} \text{ and } \omega_2 = 0.0100 \text{ kg H}_2\text{O/kg dry air}$  $v_1 = 0.889 \text{ m}^3\text{/kg dry air}$  Dry air mass balance:

Water mass balance:

Energy balance:

$$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$

$$\dot{m}_{a_1}\omega_1 = \dot{m}_{a_2}\omega_2 + \dot{m}_w \rightarrow \dot{m}_w = \dot{m}_a(\omega_1 - \omega_2)$$

$$\sum \dot{m}_i h_i = \dot{Q}_{out} + \sum \dot{m}_e h_e \rightarrow$$

$$\dot{Q}_{out} = \dot{m}(h_1 - h_2) - \dot{m}_w h_w$$

$$h_w = h_{f@~14^\circ\text{C}} = 58.8 \text{ kJ/kg}$$

$$\dot{m}_{a} = \frac{\dot{V}_{1}}{v_{1}} = \frac{10 \text{ m}^{3}/\text{min}}{0.889 \text{ m}^{3}/\text{kg dry air}} = 11.25 \text{ kg/min}$$
$$\dot{m}_{w} = (11.25 \text{ kg/min})(0.0216 - 0.0100) = 0.131 \text{ kg/min}$$
$$\dot{Q}_{out} = (11.25 \text{ kg/min})[(85.4 - 39.3) \text{ kJ/kg}] - (0.131 \text{ kg/min})(58.8 \text{ kJ/kg})$$
$$= 511 \text{ kJ/min}$$

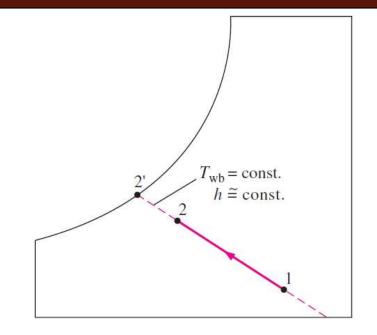
Therefore, this air-conditioning unit removes moisture and heat from the air at rates of 0.131 kg/min and 511 kJ/min, respectively.

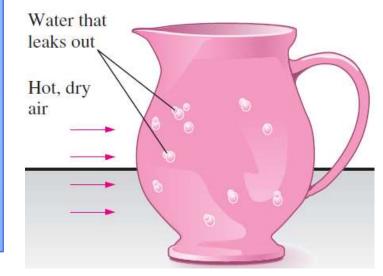
## **Evaporative** Cooling

In desert (hot and dry) climates, we can avoid the high cost of cooling by using evaporative coolers, also known as swamp coolers.

**Evaporative cooling is based on a simple principle:** As water evaporates, the latent heat of vaporization is absorbed from the water body and the surrounding air. As a result, both the water and the air are cooled during the process.

This approach has been used for thousands of years to cool water. A porous jug or pitcher filled with water is left in an open, shaded area. A small amount of water leaks out through the porous holes, and the pitcher "sweats." In a dry environment, this water evaporates and cools the remaining water in the pitcher.





Water in a porous jug left in an open, breezy area cools as a result of evaporative cooling.

# Adiabatic Mixing of Airstreams

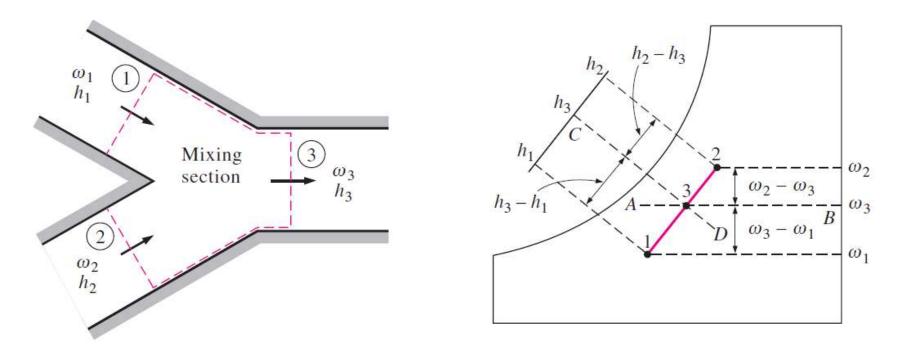
Many air-conditioning applications require the mixing of two airstreams. This is particularly true for large buildings, most production and process plants, and hospitals, which require that the conditioned air be mixed with a certain fraction of fresh outside air before it is routed into the living space. The mixing is accomplished by simply merging the two airstreams.

The heat transfer with the surroundings is usually small, and thus the mixing processes can be assumed to be adiabatic. Mixing processes normally involve no work interactions, and the changes in kinetic and potential energies, if any, are negligible.

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1}$$

Mass of dry air: $\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$ Mass of water vapor: $\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \dot{m}_{a_3}$ Energy: $\dot{m}_{a_1}h_1 + \dot{m}_{a_2}h_2 = \dot{m}_{a_3}h_3$ 

when two airstreams at two different states (states 1 and 2) are mixed adiabatically, the state of the mixture (state 3) will lie on the straight line connecting states 1 and 2 on the psychrometric chart, and the ratio of the distances 2-3 and 3-1 is equal to the ratio of mass flow rates  $\dot{m}_{a1}$  and  $\dot{m}_{a2}$ .



The concave nature of the saturation curve and the conclusion above lead to an interesting possibility. When states 1 and 2 are located close to the saturation curve, the straight line connecting the two states will cross the saturation curve, and state 3 may lie to the left of the saturation curve. In this case, some water will inevitably condense during the mixing process.

Saturated air leaving the cooling section of an air-conditioning system at 14°C at a rate of 50 m<sup>3</sup>/min is mixed adiabatically with the outside air at 32°C and 60 percent relative humidity at a rate of 20 m3/min. Assuming that the mixing process occurs at a pressure of 1 atm, determine the specific humidity, the relative humidity, the dry-bulb temperature, and the volume flow rate of the mixture.

The properties of each inlet stream are determined from the psychrometric chart to be

$h_1 = 39.4 \text{ kJ/kg} \text{ dry air}$	$h_2 = 79.0 \text{ kJ/kg dry air}$
$\omega_1 = 0.010 \text{ kg H}_2\text{O/kg dry air}$	$\omega_2 = 0.0182 \text{ kg H}_2\text{O/kg dry air}$
$v_1 = 0.826 \text{ m}^3/\text{kg} \text{ dry air}$	$v_2 = 0.889 \text{ m}^3/\text{kg} \text{ dry air}$

The mass flow rates of dry air in each stream are

$$\dot{m}_{a_1} = \frac{\dot{V}_1}{v_1} = \frac{50 \text{ m}^3/\text{min}}{0.826 \text{ m}^3/\text{kg dry air}} = 60.5 \text{ kg/min}$$
$$\dot{m}_{a_2} = \frac{\dot{V}_2}{v_2} = \frac{20 \text{ m}^3/\text{min}}{0.889 \text{ m}^3/\text{kg dry air}} = 22.5 \text{ kg/min}$$

From the mass balance of dry air,

$$\dot{m}_{a_3} = \dot{m}_{a_1} + \dot{m}_{a_2} = (60.5 + 22.5) \text{ kg/min} = 83 \text{ kg/min}$$

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1}$$
$$\frac{60.5}{22.5} = \frac{0.0182 - \omega_3}{\omega_3 - 0.010} = \frac{79.0 - h_3}{h_3 - 39.4}$$

 $\omega_3 = 0.0122 \text{ kg H}_2\text{O/kg dry air}$  $h_3 = 50.1 \text{ kJ/kg dry air}$ 

These two properties fix the state of the mixture. Other properties of the mixture are determined from the psychrometric chart:

$$T_3 = 19.0^{\circ} \text{C}$$
  
 $\phi_3 = 89\%$   
 $v_3 = 0.844 \text{ m}^3/\text{kg dry air}$ 

$$\dot{V}_3 = \dot{m}_{a_3} V_3 = (83 \text{ kg/min})(0.844 \text{ m}^3/\text{kg}) = 70.1 \text{ m}^3/\text{min}$$

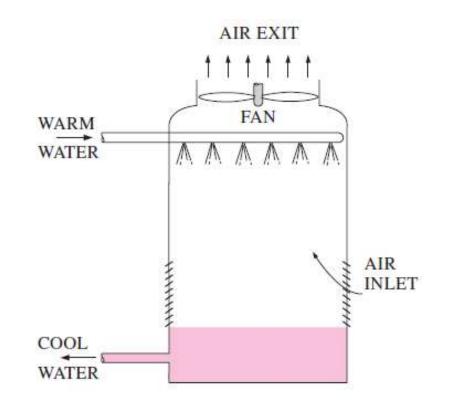
Notice that the volume flow rate of the mixture is approximately equal to the sum of the volume flow rates of the two incoming streams. This is typical in air-conditioning applications.

Chapter 14 GAS-VAPOR MIXTURES AND AIR-CONDITIONING

## Wet Cooling Towers

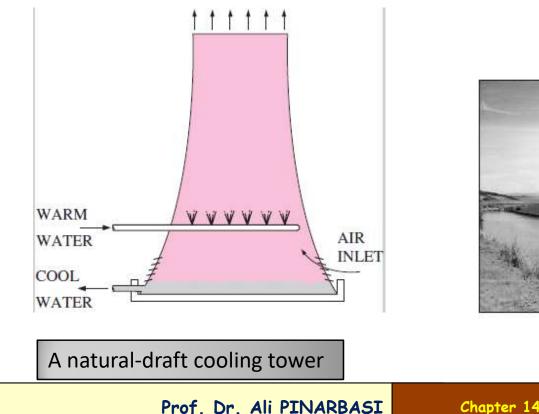
Power plants, large air-conditioning systems, and some industries generate large quantities of waste heat that is often rejected to cooling water from nearby lakes or rivers. In some cases, however, the water supply is limited or thermal pollution is a serious concern. In such cases, the waste heat must be rejected to the atmosphere, with cooling water recirculating and serving as a transport medium for heat transfer between the source and the sink (the atmosphere). One way of achieving this is through the use of wet cooling towers.

A **wet cooling tower** is essentially a semienclosed evaporative cooler.



An induced-draft counterflow cooling tower.

Another popular type of cooling tower is the **natural-draft cooling tower**, which looks like a large chimney and works like an ordinary chimney. The air in the tower has a high water-vapor content, and thus it is lighter than the outside air. Consequently, the light air in the tower rises, and the heavier outside air fills the vacant space, creating an airflow from the bottom of the tower to the top. The flow rate of air is controlled by the conditions of the atmospheric air. Natural-draft cooling towers do not require any external power to induce the air, but they cost a lot more to build than forced-draft cooling towers. The natural-draft cooling towers are hyperbolic in profile, and some are over 100 m high. The hyperbolic profile is for greater structural strength, not for any thermodynamic reason.





A spray pond.

Chapter 14 GAS-VAPOR MIXTURES AND AIR-CONDITIONING

Cooling water leaves the condenser of a power plant and enters a wet cooling tower at 35°C at a rate of 100 kg/s. The water is cooled to 22°C in the cooling tower by air that enters the tower at 1 atm, 20°C, and 60 percent relative humidity and leaves saturated at 30°C. Neglecting the power input to the fan, determine (a) the volume flow rate of air into the cooling tower and (b) the mass flow rate of the required makeup water.

$$h_1 = 42.2 \text{ kJ/kg dry air}$$
  $h_2 = 100.0 \text{ kJ/kg dry}$ 

$$\omega_1 = 0.0087 \text{ kg H}_2\text{O/kg dry air}$$
 and  $\omega_2 = 0.0273 \text{ kg H}_2\text{O/kg dry air}$ 

 $v_1 = 0.842 \text{ m}^3/\text{kg} \text{ dry air}$ 

(a) Applying the mass and energy balances on the cooling tower gives

Water mass balance:

$$m_{a_1} = m_{a_2} = m_a$$

$$\dot{m}_3 + \dot{m}_{a_1}\omega_1 = \dot{m}_4 + \dot{m}_{a_2}\omega_2$$

air

$$\dot{m}_3 - \dot{m}_4 = \dot{m}_a(\omega_2 - \omega_1) = \dot{m}_{\text{makeup}}$$

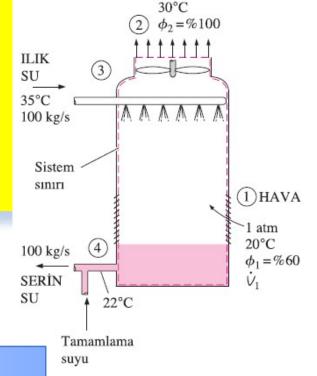
Energy balance:  $\sum \dot{m}_i h_i = \sum \dot{m}_e h_e \rightarrow \dot{m}_{a_1} h_1 + \dot{m}_3 h_3 = \dot{m}_{a_2} h_2 + \dot{m}_4 h_4$ 

or

51

or

$$\dot{m}_3 h_3 = \dot{m}_a (h_2 - h_1) + (\dot{m}_3 - \dot{m}_{\text{makeup}}) h_4$$



Solving for ma gives

$$\dot{m}_{a} = \frac{\dot{m}_{3}(h_{3} - h_{4})}{(h_{2} - h_{1}) - (\omega_{2} - \omega_{1})h_{4}}$$
$$h_{3} \cong h_{f@35^{\circ}C} = 146.64 \text{ kJ/kg H}_{2}O$$
$$h_{4} \cong h_{f@22^{\circ}C} = 92.28 \text{kJ/kg H}_{2}O$$

$$\dot{n}_a = \frac{(100 \text{ kg/s})[(146.64 - 92.28) \text{ kJ/kg}]}{[(100.0 - 42.2) \text{ kJ/kg}] - [(0.0273 - 0.0087)(92.28) \text{ kJ/kg}]} = 96.9 \text{ kg/s}$$

Then the volume flow rate of air into the cooling tower becomes

$$\dot{V}_1 = \dot{m}_a V_1 = (96.9 \text{ kg/s})(0.842 \text{ m}^3/\text{kg}) = 81.6 \text{ m}^3/\text{s}$$

(b) The mass flow rate of the required makeup water is determined from

$$\dot{m}_{\text{makeup}} = \dot{m}_a(\omega_2 - \omega_1) = (96.9 \text{ kg/s})(0.0273 - 0.0087) = 1.80 \text{ kg/s}$$

Note that over 98 percent of the cooling water is saved and recirculated in this case.

### SUMMARY

- In this chapter we discussed the air—water-vapor mixture, which is the most commonly encountered gas—vapor mixture in practice.
- The air in the atmosphere normally contains some water vapor, and it is referred to as atmospheric air.
- By contrast, air that contains no water vapor is called dry air.
- The needs of the human body and the conditions of the environment are not quite compatible. Therefore, it often becomes necessary to change the conditions of a living space to make it more comfortable.
- During a simple heating or cooling process, the specific humidity remains constant, but the temperature and the relative humidity change.