

4

Dispersion of Pollutants in the Atmosphere

4-1 Introduction

The atmospheric dispersion of effluents from vents, stacks, area sources, and mobile sources depends upon many interrelated factors: for example, the physical and chemical nature of the effluents, the meteorological characteristics of the environment, the location of the stack with relation to obstructions to air motion, and the nature of the terrain downwind from the stack. Several analytical methods have been developed to relate the dispersion of effluents to a selected number of these factors; however, none accounts for all of them.

Stack effluents may consist of gases alone, or gases and particulate matter. If the particles are on the order of $20\text{ }\mu\text{m}$ or smaller in diameter, they have such a low settling velocity that they move essentially in the same manner as the gas in which they are immersed. The analytical procedures developed for gas dispersion may be applied to the dispersion of small particles. Large particles, however, cannot be treated in the same way; they have a significant settling velocity that results in a higher ground-level concentration of the solid pollutant closer to the stack than is the case for gases. The deposition of particulate matter will be discussed in Chapter 5.

To achieve maximum dispersion, the effluents should leave the stack with sufficient momentum and buoyancy that they continue to rise from the stack exit. When there is no wind speed, low-density plumes tend to reach high elevations, and ground concentrations are low. Large particles and dense gas plumes fall to the ground in the vicinity of the stack. High wind velocities increase the diluting action of the atmosphere, giving rise to lower ground-level concentrations downwind from the stack.

The rise of the majority of hot plumes is caused almost entirely by buoyancy due to the higher temperature of the gases. When the plume is deflected over in the wind, it is diluted along its axis in proportion to the average wind speed u at plume elevation. In stratified air, the plume's buoyancy is dissipated as a result of the stability of the surrounding atmosphere. When neutral atmospheric conditions exist, the plume is diffused by turbulence, the intensity of which is a function of ground roughness, height, and, most importantly, wind speed.

To prevent downwash of the plume at the stack exit, the gas exit velocity V_s must be sufficiently large. One approximation is expressed by the following ratio:

$$\frac{V_s}{u} \geq 1.5$$

That is, downwash from a stack is minimal when the stack gas velocity is at least 1.5 times as great as the wind speed at the top of the stack.

The ability to predict ambient concentrations of pollutants in urban areas on the basis of dispersion from sources within the region is essential if national ambient air quality standards are to be attained and maintained, in spite of future industrial and residential growth. Thus, mathematical models for estimating the dispersion of pollutants from ground and elevated sources, whether single or grouped, must be developed and used to simulate the atmospheric process.

4-2 The Eddy Diffusion Model

The most comprehensive approach to transport theory is based on the *eddy diffusion model*, which in turn involves the use of the "mixing length" concept. This is the usual starting point in the development of a dispersion model for the atmosphere. The basic equation for this model is mathematically quite complex, but with minor assumptions it may be reduced to the form

$$\frac{dC}{dt} = K_{xx} \left(\frac{\partial^2 C}{\partial x^2} \right) + K_{yy} \left(\frac{\partial^2 C}{\partial y^2} \right) + K_{zz} \left(\frac{\partial^2 C}{\partial z^2} \right) \quad (4-1a)$$

where C is the concentration, t is the time, and the K_{ii} quantities are the eddy diffusion coefficients in the three coordinate directions. This equation is known as the *Fickian diffusion equation*. It is difficult to apply this result, however, to the actual process in the atmosphere. Consequently, the following additional assumptions are commonly made:

1. The concentration of the pollutant emanates from a continuous point source.
2. The process is steady state, that is, $dC/dt = 0$.
3. The major transport direction due to the wind is chosen to lie along the x -axis.
4. The wind speed u is chosen to be constant at any point in the x, y, z coordinate system.
5. The transport of pollutant due to the wind in the x -direction is dominant over the downwind diffusion, that is, $u(dC/dx) \gg K_{xx}(\partial^2 C/\partial x^2)$.

As a result, the Fickian diffusion equation reduces to

$$u \frac{\partial C}{\partial x} = K_{yy} \frac{\partial^2 C}{\partial y^2} + K_{zz} \frac{\partial^2 C}{\partial z^2} \quad (4-1b)$$

where $K_{yy} \neq K_{zz}$. The solution to this equation must also fulfill the following boundary conditions:

1. $C \rightarrow \infty$ as $x \rightarrow 0$ (large concentration at the point source).
2. $C \rightarrow 0$ as $x, y, z \rightarrow \infty$ (zero concentration at a great distance from the source).
3. $K_{zz} (\partial C / \partial z) \rightarrow 0$ as $z \rightarrow 0$ (no diffusion into the surface).
4. $\int_0^\infty \int_{-\infty}^\infty u C(x, y, z) dy dz = Q, x > 0$ (rate of transport of pollutant downwind is constant and equal to the emission rate Q of the pollutant at the source).

Lowry and Boubel [1] give the following approximate solution to the above equation, as derived by Sutton.

$$C(x, y, z) = \frac{Q}{4\pi r (K_{yy} K_{zz})^{1/2}} \exp \left[\frac{-u \left(\frac{y^2}{K_{yy}} + \frac{z^2}{K_{zz}} \right)}{4x} \right] \quad (4-2)$$

where $r^2 = x^2 + y^2 + z^2$. Unfortunately, Equation (4-2) shows two serious departures when compared to experimental evidence for centerline concentrations. Along the center line at ground level Equation (4-2) reduces to

$$C(x, 0, 0) = \frac{Q}{4\pi r (K_{yy} K_{zz})^{1/2}}$$

Therefore the approximate solution to the simplified theoretical equation indicates that the ground-level value of C along the center line of the plume is inversely proportional to x and independent of the wind speed u . Experimental observations indicate that C is inversely proportional to $(ux^{1.76})$. Improved solutions to the eddy diffusion model are needed to improve its agreement with observations. Until the acceptance and ready availability of more sophisticated methods, other models must be considered. In seeking other models, the format of Equation (4-2) is helpful. This equation indicates that away from the center line the concentration decays exponentially in both the y - and z -directions. Mathematically this means the C in the cross-wind and vertical directions may be "normally" distributed. In addition, the decrease in the value of C in the x direction is largely dependent upon the values of K_{zz} and K_{yy} . Consequently, any other model should show heavy dependence upon the diffusion coefficients as well. The most widely accepted model at the present time is the Gaussian plume model. It does exhibit the "normal" distribution suggested by Equation (4-2), and it does require extensive information, in an indirect manner, on the mass diffusion coefficients in the y - and z -directions.

4-3 The Gaussian or Normal Distribution

In Section 4-4 we develop a model for estimating the concentration of gaseous pollutants downwind from a source. Although several basic approaches to the problem are possible, usually a number of simplifying assumptions are necessary in any case to obtain a mathematically tractable solution. As a result, all these theories tend to lead to the same distribution function for the pollutant concentration, that is, a *Gaussian* distribution function. To understand the significance of this type of distribution function in the context of air pollution, it is useful to review some of the general characteristics of the Gaussian or normal distribution.

A variable x is said to be normally distributed if the density function $f(x)$ fulfills the relation

$$f(x) = \frac{1}{\sigma(2\pi)^{1/2}} \exp \left[\frac{-(x - x_0)^2}{2\sigma^2} \right] \quad (4-3)$$

where x is any real number that is the mean of the distribution; σ is any real number that is the standard deviation with a value greater than zero; and “exp” means e raised to the power of the expression in the brackets. The nature of this function is more easily grasped by reference to Figure 4-1. The value of $f(x)$ is the vertical height above the horizontal axis. The value of x_0 sets the location of the maximum value of $f(x)$ on the x -axis, and the curve is symmetrical with respect to the position of x_0 . When $x_0 = 0$, the curve is symmetrical around the $x = 0$ axis. Hence x_0 simply shifts the position of the overall distribution curve with respect to $x = 0$, as noted for the case where $x_0 = -2$.

The normal or Gaussian distribution function represented by Equation (4-3) is in a normalized form. That is, the area under the curve has a value of unity. The role of σ is to broaden or sharpen the shape of the curve, while still retaining a unit area under the curve. The standard deviation, σ , is a measure of the position of the point of inflection on either side of the curve. When σ increases, as shown by the case of the σ_2 versus σ_1 curves centered at $x = 0$ in Figure 4-1, the maximum value of $f(x)$ decreases but $f(x)$ retains a significant value over a wider range around the major axis. This is necessary, of course, if the area under both of the curves

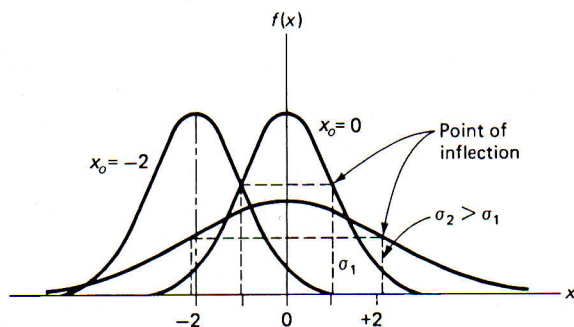


FIGURE 4-1 The Gaussian or normal distribution function for different values of x_0 and σ_0

centered at $x = 0$ is to be the same. In general, over 68 percent of the area under the curve lies between $+\sigma$ and $-\sigma$, and over 95 percent lies between $\pm 2\sigma$. This increased spread in the distribution function as σ increases has an important physical significance in the atmospheric dispersion of pollutants.

It is important to keep in mind the role of x_o and σ in determining the general position and shape of the Gaussian distribution function as we develop the atmosphere dispersion equations for various situations. In general, these dispersion equations will take on the format of a double Gaussian distribution. A double Gaussian distribution in two coordinate directions, such as y and z , is simply the product of the single Gaussian distributions in each of the coordinate directions.

Based on Equation (4-3) and the fact that $f(y, z) = f(y)f(z)$,

$$f(y, z) = \frac{1}{2\pi\sigma_y\sigma_z} \exp\left[\frac{-(y-y_o)^2}{2\sigma_y^2}\right] \exp\left[\frac{-(z-z_o)^2}{2\sigma_z^2}\right] \quad (4-4)$$

or since $\exp[A] \cdot \exp[B] = \exp[A+B]$, then an alternative expression would be

$$f(y, z) = \frac{1}{2\pi\sigma_y\sigma_z} \exp\left[\frac{-(y-y_o)^2}{2\sigma_y^2} + \frac{-(z-z_o)^2}{2\sigma_z^2}\right]$$

where σ_y , σ_z , y_o , and z_o have essentially the same interpretation as that for the single Gaussian distribution. The values, y_o , and z_o , shift the peak of the distribution away from the origin.

In a similar manner, the Gaussian equation describing a distribution which is three-dimensional (x, y, z) is

$$f(x, y, z) = \frac{1}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z} \exp\left[\frac{-(x-x_o)^2}{2\sigma_x^2}\right] \exp\left[\frac{-(y-y_o)^2}{2\sigma_y^2}\right] \exp\left[\frac{-(z-z_o)^2}{2\sigma_z^2}\right] \quad (4-5)$$

The two dimensional Gaussian distribution equation will be utilized to describe continuous emissions from point sources in which the plume disperses in the y and z directions as it moves in the downwind or x direction. The three-dimensional equation will be used to describe the movement of an instantaneous puff of emissions which disperses in the x, y , and z directions as it travels in the downwind or x direction.

4-4 The Gaussian Dispersion Model

A mathematical model of atmospheric dispersion must attempt to simulate the gross behavior of plumes emitted from ground-level or stack-height sources. For localized point sources such as a stack, the general appearance of the plume might be represented by the schematic shown in Figure 4-2. Although the plume originates at a stack height h_s it rises an additional height Δh , owing to buoyancy of the hot gases and the momentum of the gases leaving the stack vertically with a velocity V_s . Consequently, the plume appears as if it originated as a point source at an

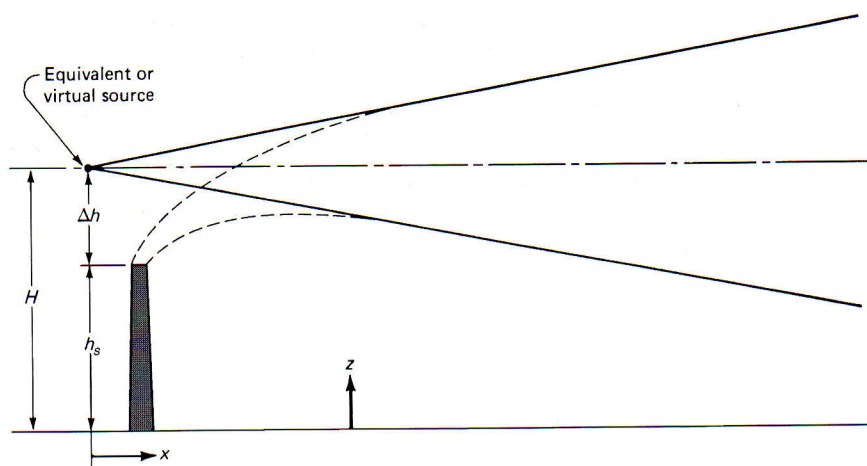


FIGURE 4-2 A dispersion model with virtual source at an effective stack height H .

equivalent or effective stack height $H = h_s + \Delta h$. The virtual point source may also lie somewhat upwind of the center line of the stack position, although in most cases the point is assumed to be directly above the stack. In the case of sources which have a relatively large area of emissions, it may be necessary to use a virtual point that is some distance upwind of the actual source to account for the initial width of the source.

4-4-A Point Source at Elevation H Above the Ground, Without Reflection

Using the parameters developed in Section 4-3 for the Gaussian distribution, the most general Gaussian dispersion equation is derived in the Appendix of this chapter. The resulting general equation, used frequently as the basis for modeling emissions from continuous point sources of emissions, is

$$C(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp \left[\frac{-(y - y_o)^2}{2\sigma_y^2} \right] \exp \left[\frac{-(z - z_o)^2}{2\sigma_z^2} \right] \quad (4-6)$$

Here, Q is the emission strength of the source (mass/time) and u is the average windspeed taken through the plume. The units on the concentration C are determined by the units used to express the quantities Q , u , σ_y , and σ_z . The terms, σ_y and σ_z , are usually given in meters, u is in meters per second, and Q is in grams per second. The units on the concentration then become grams per cubic meter (g/m^3). Equation (4-6) describes the change in concentration as the plume travels in the downwind direction x and gradually disperses in the vertical (z) and perpendicular to the direction of travel (the y direction). When Equation (4-6) is rearranged so that the left side is equal to Cu/Q then the right side will have the identical format of $f(y, z)$ described in Section 4-3. For the completely generalized equation, it is assumed that there is no interference or limitation to dispersion in any direction. Interferences such as the plume dispersing in the z direction and

reflecting off of the ground or off of an elevated inversion will be discussed in the following sections. Equation (4-6) is referred to as the general Gaussian dispersion equation, *without reflection*.

The values of y_o and z_o define the location of the centerline of the plume (i.e., the Gaussian distribution equation). It is convention to locate the origin of a single stack at the base of the stack. On that basis, $y_o = 0$. Further, since H is the effective height of the emissions, then z_o (the location of the peak in the vertical direction) has a value equal to H . Substitution of these two values into Equation (4-6) allows a simplification of the Gaussian dispersion equation, *without reflection*:

$$C(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp \left[\frac{-y^2}{2\sigma_y^2} \right] \exp \left[\frac{-(z-H)^2}{2\sigma_z^2} \right] \quad (4-7)$$

By combining the two exponential terms, Equation (4-7) becomes

$$C(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp \left(-\frac{1}{2} \left[\frac{y^2}{\sigma_y^2} + \frac{(z-H)^2}{\sigma_z^2} \right] \right) \quad (4-8)$$

The restriction "without reflection" is extremely important. The above equation is an appropriate expression for the concentration in the downwind direction up to the point in the x -direction where the concentration at ground level ($z = 0$) is significant. Then appreciable "reflection" of the gaseous pollutant will occur by diffusion back into the atmosphere from ground level. Such a model assumes that the earth's surface is not a sink for a pollutant.

4-4-B Point Source at Elevation H Above the ground, With Reflection

It is a relatively simple task to modify the preceding equation to account for reflection of a gaseous pollutant back into the atmosphere, once it has reached ground level. By referral to Figure 4-3, we see that reflection at some distance x is mathematically equivalent to having a mirror image of the source at $-H$. The shaded area beyond position I on the diagram indicates the region of the atmosphere in which the concentration will increase over that normally supplied by the source at H . This increased concentration is determined mathematically by linear superposition of two Gaussian-type concentration curves, one centered at H (i.e., $z_o = H$) and the other at $-H$ (i.e., $z_o = -H$). This is equivalent to adding together two equations like Equation (4-7). However, one equation contains a $(z + H)$ term, rather than a $(z - H)$ term. As a result, the concentration equation for an elevated source *with reflection* becomes

$$C(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \left[\exp - \left(\frac{y^2}{2\sigma_y^2} \right) \right] \left\{ \exp \left[\frac{-(z-H)^2}{2\sigma_z^2} \right] + \exp \left[\frac{-(z+H)^2}{2\sigma_z^2} \right] \right\} \quad (4-8)$$

The effect of ground reflection on the pollutant concentration above ground level is shown in Figure 4-4. At position I the two Gaussian-type curves predict essen-

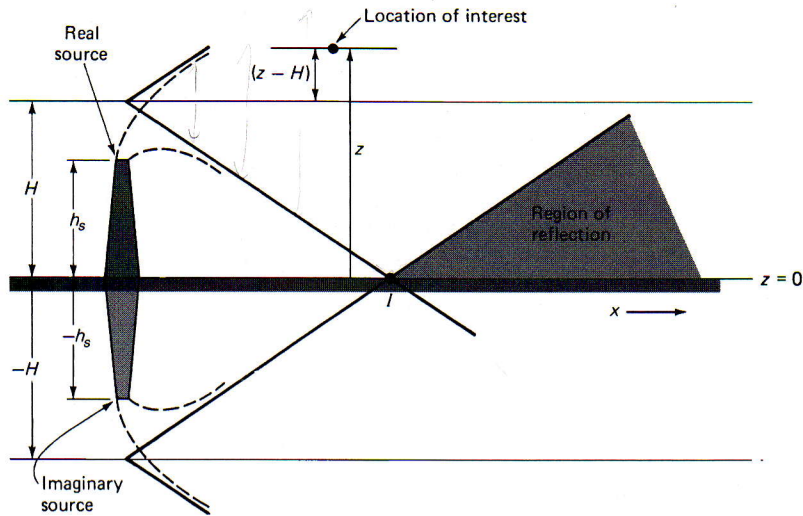


FIGURE 4-3 Use of an imaginary source to describe mathematically gaseous reflection at the surface of the earth.

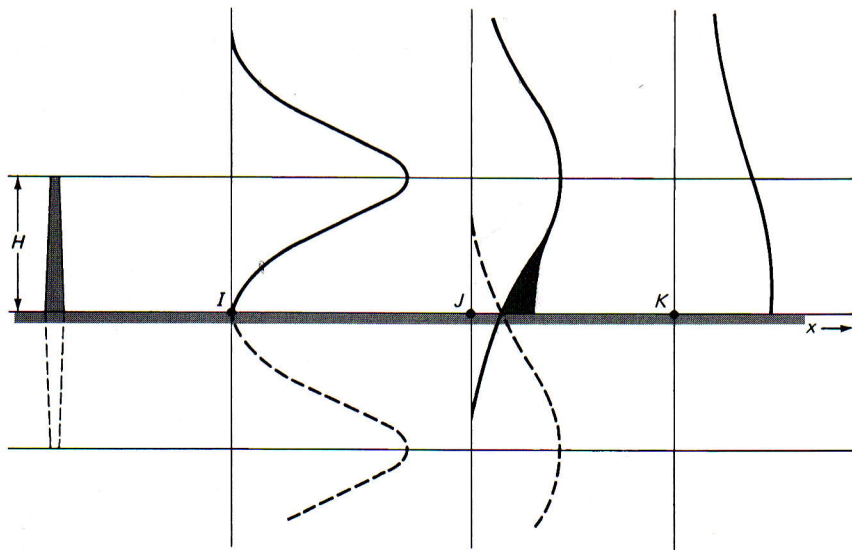


FIGURE 4-4 Effect of ground reflection on pollutant concentration downwind.

tially an overlap in concentration, but at positions downwind from I the overlap will become significant and increase as x increases. At position J downwind the overlap is appreciable. By adding that portion of the lower curve that extends above ground level ($z = 0$) to the original upper curve, we find that the upper concentration curve is altered by the addition of the shaded area shown. At some distance K further downwind from J , the shaded contribution due to reflection might lead to the profile shown at K in Figure 4-4. Obviously the effect of ground-level

reflection is to increase the ground-level concentration well above that anticipated without reflection.

Another equation of significance when considering ground-level reflection is that representing the concentration at ground level. In this case $z = 0$, and Equation (4-8) reduces to

$$C(x, y, 0) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(\frac{-H^2}{2\sigma_z^2}\right) \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \quad (4-9)$$

If the center-line, ground-level concentration estimate is desired, the last exponential term becomes unity.

A typical concentration profile in the z -direction at a given value of x and a profile in the x -direction along the center line at ground level are superimposed on a schematic of the diffusion process from an elevated tack in Figure 4-5. Note that the Gaussian distribution in the z -direction is centered at the effective stack height H . Also, the center-line concentration downwind maximizes at some value of x and then falls off at increasing values of x . A profile similar to that in the z -direction would also be valid in the y -direction. However, the sharpness of the Gaussian distributions in the y - and z -directions could be quite different, since the values of σ_y and σ_z at the given x are found to be significantly different.

Equation (4-9) can be further simplified in the case where the concentration of interest is directly downwind on centerline and the effective stack height is zero (i.e., $y = 0$ and $H = 0$). The equation reduces to

$$C(x, 0, 0) = \frac{Q}{\pi u \sigma_y \sigma_z} \quad (4-10)$$

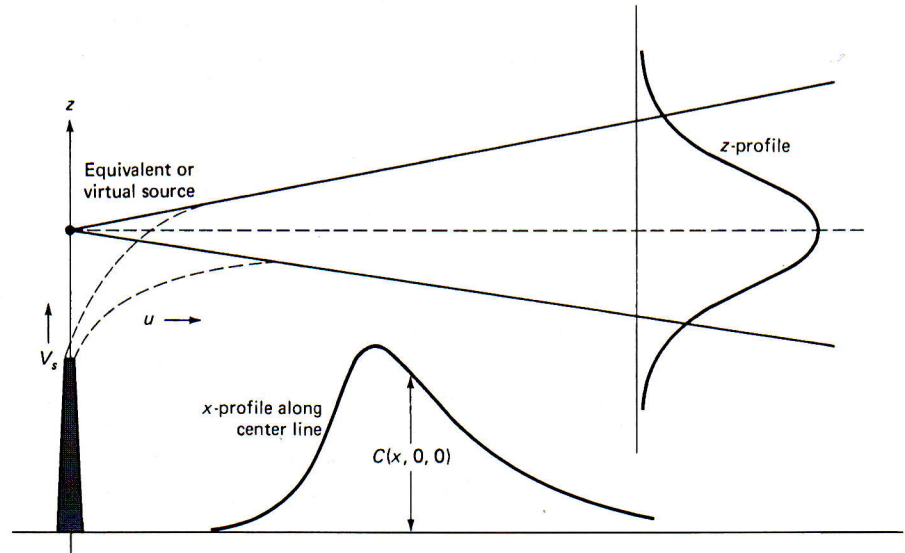


FIGURE 4-5 Concentration profiles along the center line in the x -direction and in the z -direction.

This equation applies to the ground-level, centerline concentration from a point source at ground level. In this case, the maximum concentration will occur at the source and will decrease in the downwind direction.

4-5 Evaluation of the Standard Deviations

Several equations were developed in the preceding section for estimating the concentration downwind resulting from a continuous plume. Besides physical data such as the coordinates x , y , and z , the emission strength Q , and the effective height H of the plume center line, it is necessary to have values of u , σ_y , and σ_z .

It has been pointed out in the preceding chapter that the wind speed, u , is a function of height, z . The typical variation of u with z is given by Equation (3-13), which includes a factor to adjust for various stability conditions in the atmosphere. The appropriate value of u to use in the dispersion equations is the mean value taken through the plume [4]. In most cases, it would be impossible to determine the mean, since sufficient atmospheric data would not be available. In lieu of this, the average wind speed at the top of the stack is commonly used. Since in most cases not even this value is known, the measured meteorological value at 10 m is used in conjunction with Equation (3-13) to estimate the wind speed at the top of the stack. For purposes of working problems in this text, when the windspeed is specified without mention of a height of measurement, it should be assumed to be representative of the windspeed averaged through the plume. If, on the other hand, a windspeed is provided and specified to have been measured at a specific height (i.e., 10 m), then a windspeed correction would be needed using Equation (3-13) to correct the windspeed to the top of the stack for purposes of dispersion calculations.

As might be anticipated from the physical description of the dispersion problem, the horizontal and vertical dispersion coefficients, σ_y and σ_z , are a function of the downwind position x as well as the atmospheric stability conditions. Many experimental measurements in the atmosphere have led to an evaluation and correlation of σ_y and σ_z values with atmospheric stability and downwind distance for both rural and urban areas.

One of the earliest and most widely accepted set of charts for σ_y and σ_z is presented by the solid curves in Figures 4-6 and 4-7, as prepared by Turner [2]. The curves are commonly referred to as the Pasquill-Gifford curves. These correlations are based on the following restrictions:

1. The concentrations estimated from the use of these charts should correspond to sampling times of approximately 10 minutes. As will be discussed in Section 4-8-A, regulatory models now assume that the concentrations predicted using these values represent one-hour averages.
2. The solid curves are based on terrain representative of open country and are referred to as rural values [2], whereas the dashed curves, referred to as the McElroy-Pooler curves are representative of urban values.
3. The estimated concentrations more nearly represent only the lowest several hundred meters of the atmosphere.

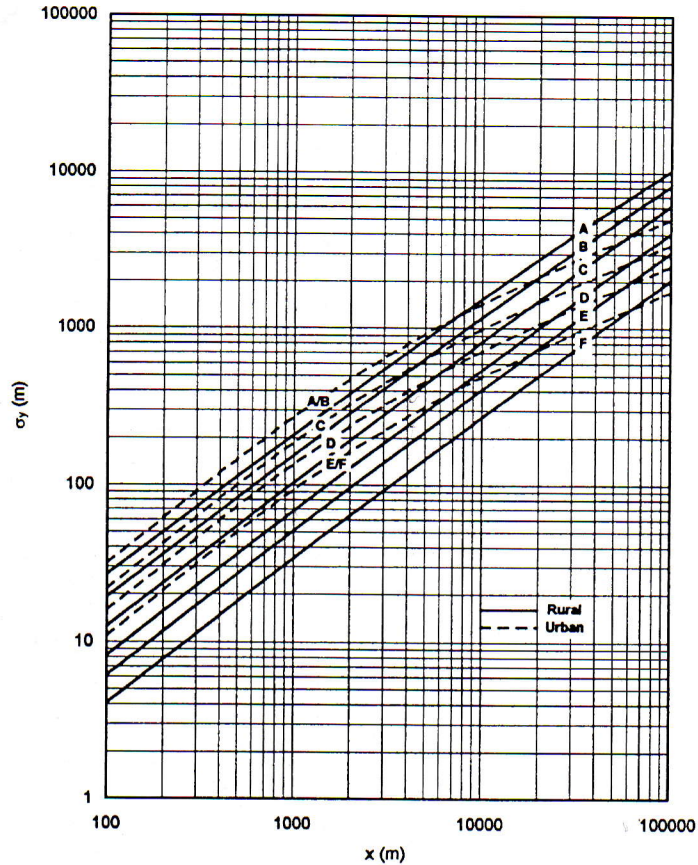


FIGURE 4-6 Rural and urban horizontal dispersion coefficients (σ_y) as a function of stability category. (Graph prepared by S.M. Claggett [20].)

As noted by Turner, the σ_z -values are more in doubt than the σ_y -values. This is especially true for downwind distances of more than 1 km. In several cases, such as for neutral to moderately unstable atmospheric conditions and distances out to a few kilometers, the center-line, ground-level concentrations predicted using these charts should be within a factor of 2 or 3 of actual values.

Turner has prepared a listing of atmospheric conditions that aids in determining which of the six stability classes (A through F) appearing on the σ charts is appropriate. The listing showing the key to the various stability categories is included in Table 3-1.

Due to the difficulty in reading the values of σ_y and σ_z from graphs such as Figures 4-6 and 4-7 and the need to computerize the calculations associated with the Gaussian dispersion equations, the σ_y and σ_z curves have been fit to empirical equations [3]. The equations are generally of the form

$$\sigma_y = cx^a \text{ and } \sigma_z = ax^b \quad (4-11)$$

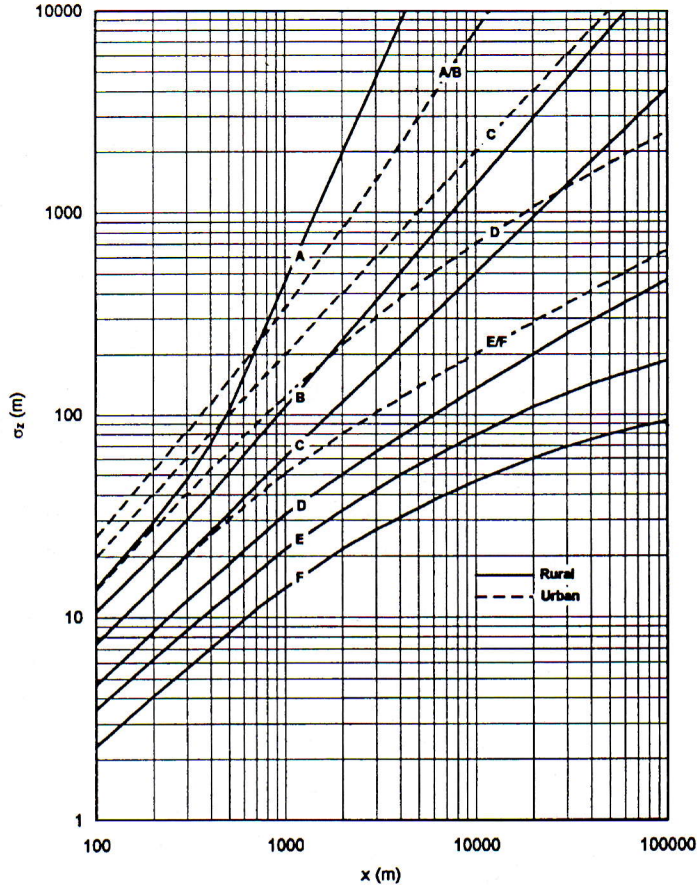


FIGURE 4-7 Rural and urban vertical dispersion coefficients (σ_z) as a function of stability category. (Graph prepared by S.M. Claggett [20].)

These equations produce a linear curve on a log/log graph and therefore are an attempt to linearize the curves in Figures 4-6 and 4-7. The equations presented below are those utilized in the Industrial Source Complex (ISC3) Dispersion Models [4] developed under U.S. EPA guidance. The equation used to fit the Pasquill-Gifford curves for the rural mode for σ_y in meters is

$$\sigma_y = 465.11628 (x) \tan (TH) \quad (4-12)$$

where

$$TH = 0.01745(c - (d) \ln(x)) \quad (4-13)$$

The downwind distance, x , is in kilometers, the coefficients c and d are listed in Table 4-1, and TH is in radians. The equation for σ_z in meters is

$$\sigma_z = ax^b \quad (4-14)$$

TABLE 4-1 Parameters Used to Calculate Pasquill-Gifford σ_y

Pasquill Stability Category	<i>c</i>	<i>d</i>
A	24.1670	2.5334
B	18.3330	1.8096
C	12.5000	1.0857
D	8.3330	0.72382
E	6.2500	0.54287
F	4.1667	0.36191

where the distance, x , is in kilometers, and the coefficients a and b are given in Table 4-2.

Tables 4-3 and 4-4 can be used to determine the dispersion coefficients for an urban environment (dashed curves in Figures 4-6 and 4-7). The equations for the urban coefficients were reported by Gifford [5] to have been developed by Briggs based on diffusion data developed by McElroy and Pooler [6]. While other σ -data appear in the literature [7-12], many of the existing Gaussian dispersion models, particularly those used in modeling for compliance purposes, use the combination of the Pasquill-Gifford curves for rural areas, and the McElroy-Pooler data for urban areas.

EXAMPLE 4-1

Sulfur dioxide is emitted at a rate of 160 g/s from a stack with an effective height H of 60 m. The wind speed at stack height is 6 m/s, and the atmospheric stability class is D for the overcast day. Determine the ground-level concentration along the center line at a distance of 500 m from the stack, in micrograms per cubic meter for a rural area.

Solution

From Figures 4-6 and 4-7 the horizontal and vertical standard deviations, σ_y and σ_z , at 500 m for stability class D are 36 m and 18.5 m, respectively. Substitution of these values and other given data into Equation (4-9) yields, for $y = 0$,

$$\begin{aligned}
 C(500, 0, 0) &= \frac{160 \times 10^6}{\pi(6)(36)(18.5)} \exp \left[-0.5 \left(\frac{60}{18.5} \right)^2 \right] \\
 &= 12.7 \times 10^3 (5.25 \times 10^{-3}) \\
 &= 66.0 \mu\text{g}/\text{m}^3 \text{ of SO}_2
 \end{aligned}$$

It is interesting to note that this value is just within the primary air quality standards of $80 \mu\text{g}/\text{m}^3$ listed in Table 2-1. Equation (4-9) has been used since considerable reflection occurs at 500 m downwind. ◆

TABLE 4-2 Parameters Used to Calculate Pasquill-Gifford σ_z

<i>Pasquill Stability Category</i>	<i>x (km)</i>	<i>a</i>	<i>b</i>
A*	<.10	122.800	0.94470
	0.10 – 0.15	158.080	1.05420
	0.16 – 0.20	170.220	1.09320
	0.21 – 0.25	179.520	1.12620
	0.26 – 0.30	217.410	1.26440
	0.31 – 0.40	258.890	1.40940
	0.41 – 0.50	346.750	1.72830
	0.51 – 3.11	453.850	2.11660
	>3.11	**	**
B*	<.20	90.673	0.93198
	0.21 – 0.40	98.483	0.98332
	>0.40	109.300	1.09710
C*	All	61.141	0.91465
D	<.30	34.459	0.86974
	0.31 – 1.00	32.093	0.81066
	1.01 – 3.00	32.093	0.64403
	3.01 – 10.00	33.504	0.60486
	10.01 – 30.00	36.650	0.56589
	>30.00	44.053	0.51179
E	<.10	24.260	0.83660
	.010 – 0.30	23.331	0.81956
	0.31 – 1.00	21.628	0.75660
	1.01 – 2.00	21.628	0.63077
	2.01 – 4.00	22.534	0.57154
	4.01 – 10.00	24.703	0.50527
	10.01 – 20.00	26.970	0.46713
	20.01 – 40.00	35.420	0.37615
	>40.00	47.618	0.29592
F	<.20	15.209	0.81558
	0.21 – 0.70	14.457	0.78407
	0.71 – 1.00	13.953	0.68465
	1.01 – 2.00	13.953	0.63227
	2.01 – 3.00	14.823	0.54503
	3.01 – 7.00	16.187	0.46490
	7.01 – 15.00	17.836	0.41507
	15.01 – 30.00	22.651	0.32681
	30.01 – 60.00	27.074	0.27436
	>60.00	34.219	0.21716

*If the calculated value of σ_z exceeds 5000 m, σ_z is set to 5000 m.** σ_z is equal to 5000 m.

TABLE 4-3 Briggs Formulas Used to Calculate McElroy-Pooler σ_y

Pasquill Stability Category	σ_y (meters) *
A	$0.32 \times (1.0 + 0.0004 x)^{-1/2}$
B	$0.32 \times (1.0 + 0.0004 x)^{-1/2}$
C	$0.22 \times (1.0 + 0.0004 x)^{-1/2}$
D	$0.16 \times (1.0 + 0.0004 x)^{-1/2}$
E	$0.11 \times (1.0 + 0.0004 x)^{-1/2}$
F	$0.11 \times (1.0 + 0.0004 x)^{-1/2}$

*Where x is in meters**TABLE 4-4** Briggs Formulas Used to Calculate McElroy-Pooler σ_z

Pasquill Stability Category	σ_z (meters) *
A	$0.24 \times (1.0 + 0.001 x)^{1/2}$
B	$0.24 \times (1.0 + 0.001 x)^{1/2}$
C	$0.20 x$
D	$0.14 \times (1.0 + 0.003 x)^{-1/2}$
E	$0.08 \times (1.0 + 0.015 x)^{-1/2}$
F	$0.08 \times (1.0 + 0.015 x)^{-1/2}$

*Where x is in meters.**EXAMPLE 4-2**

For the data given in Example 4-1, determine the concentration crosswind at 50 m from the center line for the downwind distance of 500 m.

Solution

To account for the concentration in the crosswind direction at ground level one must modify the above solution by the term $\exp [-0.5(y/\sigma_y)^2]$, found in Equation (4-9). Hence,

$$\begin{aligned}
 C(500, 50, 0) &= 66.0 \exp \left[-0.5 \left(\frac{50}{36} \right)^2 \right] \\
 &= 66.0(0.38) = 23 \mu\text{g}/\text{m}^3 \text{ of } \text{SO}_2
 \end{aligned}$$

Thus, at a crosswind distance that is 10 percent of the downwind distance, the estimated concentration has fallen off by nearly 60 percent. \blacklozenge

EXAMPLE 4-3

For the data of Example 4-1, determine sufficient values of C as a function of x on the ground-level center line so that the variation on either side of the maximum value is established.

Solution

The general solution to this problem, in terms of Equation (4-9), is

$$C(x, 0, 0, 60) = \frac{160 \times 10^6}{\pi(6)\sigma_y\sigma_z} \exp \left[-0.5 \left(\frac{60}{\sigma_z} \right)^2 \right]$$

The table below summarizes the computations in a convenient manner, where the column headed "exp" represents the exponential term in the equation. Note that the preexponential factor decreases rapidly with increasing distance, owing to the steadily increasing values of σ_y and σ_z . The exponential factor, however, rapidly increases from an extremely small value toward a value of unity as x becomes large. Since these two terms are multiplied together to obtain C , there must be a maximum value of C at some distance x . In this case the maximum concentration occurs around 1.5 km. The data indicate that the concentration at ground level along the center line builds up rapidly as x increases, but the falloff in concentration is rather slow after the maximum point. This is fairly typical of the Gaussian-type solution for the atmospheric dispersion.

x (km)	σ_y	σ_z	$Q/\pi\sigma_y\sigma_z u$	$-\frac{1}{2}(H/\sigma_z)^2$	exp	C ($\mu\text{g}/\text{m}^3$)
0.5	36	18	13,090	5.55	0.0039	50
0.8	60	27	5,240	2.47	0.085	445
1.0	76	32	3,490	1.76	0.172	600
1.5	110	45	1,710	0.89	0.411	700
1.7	140	50	1,210	0.72	0.487	590
2.0	160	55	960	0.595	0.552	530
3.0	220	71	540	0.357	0.700	380
5.0	350	100	240	0.180	0.835	200
10.0	620	150	90	0.080	0.923	83

4-6 The Maximum Ground-Level In-Line Concentration

The effect of ground reflection, as noted in Section 4-4-B, is to increase the ground-level concentrations of gaseous pollutants as x increases, to a point well above the level expected without reflection. However, such an increase in C in the x -direction cannot continue indefinitely. Eventually diffusion outward (crosswind) in the y -direction and upward in the z -direction will diminish the concentration at ground level ($z = 0$) and along the center line ($y = 0$). Thus, as noted in Figure 4-5 and Example 4-3, the curve of C versus x has a maximum point before falling off toward zero at large x -values.

One method of determining the downwind distance for the maximum concentration, and the maximum concentration at that point, has been developed by Turner [2] in a graphical format. Figure 4-8 is based upon this work, which originally was developed on the basis of Equation (4-9) and Figures 4-6 and 4-7 for rural conditions. In Figure 4-8 the maximum value of the parameter Cu/Q is plotted versus the distance to the maximum concentration, x_{\max} , with information on the stability class and the effective stack height appearing within the diagram. In the typical problem, the known data are the stability class and the effective height. These data determine a particular point on the figure. From this point we read downward and to the left to ascertain x_{\max} and C_{\max} , respectively.

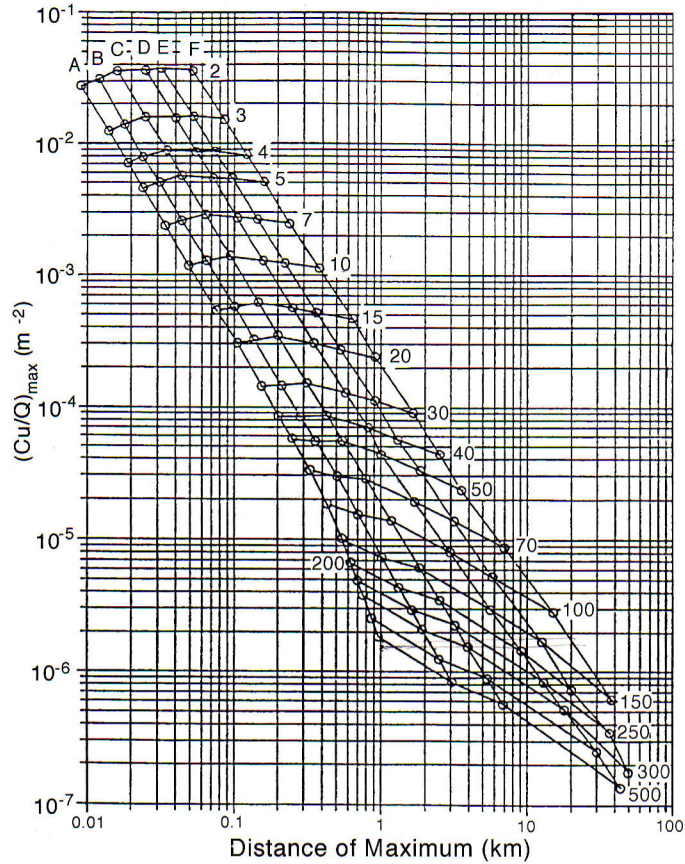


FIGURE 4-8 Maximum Cu/Q value as a function of stability class and effective stack height in meters. (Source: D. B. Turner. *Workbook of Atmospheric Dispersion Estimates*, 2nd Ed., Lewis Publishers, Boca Raton, Florida, 1994.)

Rather than rely on reading values from Figure 4-8 to obtain the maximum concentration downwind, the data can be fit to a general equation and the solution found algebraically. A general equation developed by Ranchoux [13] is of the form

$$\left(\frac{Cu}{Q}\right)_{\max} = \exp[a + b(\ln H) + c(\ln H)^2 + d(\ln H)^3] \quad (4-15)$$

where H is in meters and Cu/Q is in m^{-2} . The values of the coefficients a , b , c , and d for each stability class are shown in Table 4-5. The error between the equation and the actual curves is less than 2 percent for curves A, B, and C, and is less than 4.5 percent for the D, E, and F curves.

An alternate method of ascertaining the position and the values of the maximum concentration is based on a characteristic of the σ_y and σ_z charts. Under moderately unstable to near neutral conditions, the ratio σ_y/σ_z is nearly independent of the distance x . If this ratio is taken to be constant and y is set equal to

TABLE 4-5 Values of Constants to be Used in Equation (4-18) as a Function of Stability Classes

Stability Class	Coefficients			
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
A	-1.0563	-2.7153	0.1261	0
B	-1.8060	-2.1912	0.0389	0
C	-1.9748	-1.9980	0	0
D	-2.5302	-1.5610	-0.0934	0
E	-1.4496	-2.5910	0.2181	-0.0343
F	-1.0488	-3.2252	0.4977	-0.0765

Source: R. Ranchoux, *J. Air. Pollu. Control Assoc.* **26**, no. 11 (1976): 1089.

zero, then Equation (4-9) can be written so that C is solely a function of σ_z (which, in turn, is solely a function of x for a given stability class). By the maximization technique of differential calculus, one can obtain analytical information concerning the maximum concentration along the center line. The position of maximum concentration can be found only implicitly by this method, not explicitly. The result of the differentiation of Equation (4-9) in this modified form results in the following expression:

$$\sigma_z = \frac{H}{(2)^{1/2}} = 0.707H$$

The value of σ_z is first determined from the best estimate of the corrected stack height, H . Then from a chart of σ_z versus x , for various stability conditions, the value of x can be read off, which gives the position of maximum C . The value of x determined by this method is only approximate, owing to the nature of the $\sigma_z - x$ chart.

If the condition $H^2 = 2\sigma_z^2$ is substituted into Equation (4-9), and y is set equal to zero, then the maximum concentration downwind on the center line and at ground level is approximately given by

$$C_{\max, \text{reflec}} = \frac{0.1171Q}{u\sigma_y\sigma_z} \quad (4-16)$$

Again, this expression is only applicable for slightly unstable to neutral atmospheric conditions, for the reasons noted above. Example 4-4 illustrates both Turner's graphical format and the approximation equation developed above for the determination of the position and the value of the maximum concentration expected on the center line at ground level for an elevated point source. Turner's graphical technique (Figure 4-8) is not limited to neutral stabilities since it is based on Equation (4-9).

EXAMPLE 4-4

For the data of Example 4-1, determine the position downwind on the center line at ground level where the maximum concentration will occur, and determine the maximum value in micrograms per cubic meter.

Solution

For an effective stack height of 60 m, the value of σ_z that leads to the position of maximum concentration is

$$\sigma_z = 0.707H = 0.707(60) = 42.4 \text{ m}$$

From Figure 4-7 we determine that the corresponding x -value is 1.55 km, which is the estimated point of maximum concentration. The value of C at this distance is given by Equation (4-16), namely,

$$C = \frac{0.117Q}{u \sigma_y \sigma_z} = \frac{0.117(160)(10^6)}{6(105)(42.4)} = 700 \mu\text{g}/\text{m}^3$$

This value is in excellent agreement with the general results of Example 4-3 since the stability class D is approximately neutral.

The values of C_{\max} and x_{\max} may also be estimated through the use of Figure 4-8. For class D stability and an H -value of 60 m, it is found from the figure that

$$x_{\max} \approx 1.5 \text{ km and } \left(\frac{Cu}{Q}\right)_{\max} \approx 2.7 \times 10^{-5} \text{ m}^{-2}$$

Hence,

$$C_{\max} = (2.7 \times 10^{-5} \text{ m}^{-2})[(160 \times 10^6 \mu\text{g})/\text{s}](\text{s}/6 \text{ m}) = 720 \mu\text{g}/\text{m}^3$$

In this case the two methods are substantially in agreement. They do not always agree, however, since agreement depends upon the stability class involved. This concentration, as discussed earlier, is used to represent a one-hour average. This value is nearly a factor of two greater than the 24-hour primary NAAQS of $365 \mu\text{g}/\text{m}^3$ (Table 2-1). In order to directly compare the predicted value to the standard, however, it is necessary to convert the predicted one-hour average concentration to an equivalent 24-hour average concentration. This could be done by utilizing 24 hours of meteorological data. A "rough" estimate of the 24-hour concentration can be made by multiplying the one-hour value by the factor 0.4 (see Section 4-8-A). The predicted 24-hour average would then be roughly 0.4×720 , or $288 \mu\text{g}/\text{m}^3$. When comparing a predicted concentration to an air quality standard, it is important to be sure that the predicted value is for the same averaging period as the standard.

As a third method, Equation (4-15) may be used. In this case, for class D stability the equation becomes

$$\begin{aligned} \left(\frac{Cu}{Q}\right)_{\max} &= \exp[-2.5302 - 1.5610(\ln 60) - 0.0934(\ln 60)^2] \\ &= \exp(-10.487) = 2.8 \times 10^{-5} \text{ m}^{-2} \end{aligned}$$

For the given values of u and Q we find that C_{\max} is $740 \mu\text{g}/\text{m}^3$, which is reasonably close to the two preceding answers.

4-7 Calculation of the Effective Stack Height

Most of the analytical methods for predicting the concentrations of stack effluents involve the location of a virtual or equivalent origin, as shown in Figure 4-2. The elevation H of the virtual origin is obtained by adding a term Δh , the plume rise, to the actual height of the stack, h_s . There are numerous methods for calculating Δh , and these are discussed at some length by Stern [8]. Basically, three sets of parameters control the phenomenon of a gaseous plume injected into the atmosphere from a stack. These are stack characteristics, meteorological conditions, and the physical and chemical nature of the effluent. A large number of analytical expressions have been proposed to relate these factors to plume-rise predictions. It is not surprising that no one expression has proven superior for all stack geometries and atmospheric conditions.

One of the first considerations is to determine whether a plume actually downwashes around the stack due to a low exit velocity. As stated earlier, if the stack velocity is greater than or equal to 1.5 times the windspeed, stack-tip downwash is considered to be negligible. For conditions in which the velocity of the stack is less than 1.5 times the windspeed [17], the reduced stack height, h'_s , can be calculated as

$$h'_s = h_s + 2d_s \left[\frac{V_s}{u_s} - 1.5 \right] \quad (4-17)$$

where h_s is the physical stack height (m), V_s is the stack velocity (m/s), and u_s is the windspeed (m/s) measured or calculated at the height, h_s . The reduced stack height in this case will be less than the actual stack height. This reduced value is then used in subsequent calculations of the effective stack height due to plume rise. The actual height above the ground of the plume due to plume rise is obtained by adding the plume rise, Δh , to the reduced stack height, h'_s .

The majority of the historical equations that predict plume rise contain a momentum term and a thermal buoyancy term. The momentum term accounts for the vertical momentum of the stack gas due to its own velocity, V_s . The buoyancy term takes into account, in some manner, the difference between the stack gas temperature, T_s , and the environmental temperature, T_a .

In 1969, Carson and Moses [14] concluded that the following equation gave the best agreement with observed data based on 711 plume rise observations, regardless of stability condition.

$$\Delta h = -0.029 \frac{V_s d_s}{u_s} + 2.62 \left(\frac{(Q_h)^{1/2}}{u_s} \right) \quad (4-18)$$

where Δh is the plume rise in meters, V_s is the stack gas exit velocity in meters per second, d_s is the stack exit diameter in meters, u_s is the wind speed at the stack exit in meters per second, and Q_h is the heat emission rate in kilojoules per second.

To further clarify Equation (4-18),

$$Q_h = \dot{m} c_p (T_s - T_a)$$

where \dot{m} is the stack gas mass flow rate in kilograms per second ($\dot{m} = \pi d_s^2 V_s \rho_s / 4$), c_p is the constant pressure specific heat of the stack gas T_s is the stack gas temperature at the stack exit in degrees Kelvin, T_a is the atmospheric air temperature at stack height in degrees Kelvin, and M_w is the molecular weight of the stack gas.

In 1970, Thomas, Carpenter, and Colbaugh [15] compared the observed plume-rise data for large stacks of electric generation stations with values calculated employing 10 different equations. Several of the equations employed in reference 14 were included in the comparisons of Thomas et al. The Holland formula,

$$\Delta h = \frac{V_s d_s}{u_s} \left[1.5 + 2.68 \times 10^{-3} P d \left(\frac{T_s - T_a}{T_s} \right) \right] \quad (4-19)$$

showed fairly good agreement with observations, with a slight tendency to underestimate the plume rise. The symbols and units for the Holland equation are the same as those listed above. In addition, the pressure P , must be expressed in millibars. This equation appears to be more accurate for tall stacks. The last term in the Holland formula can be replaced by $0.0096 Q_h / V_s d$, if desired.

Moses and Kraimer [16] analyzed 17 equations for plume rise on the basis of 615 observations involving 26 different stacks. Among those equations that gave reasonably good predictions were two proposed by Concawe [10]. The original Concawe formula, based on observations in Europe, is

$$\Delta h = 2.71 \left(\frac{Q_h^{1/2}}{u_s^{3/4}} \right) \quad (4-20)$$

When optimized by Thomas et al. [15] on the basis of TVA data, the equation took the form

$$\Delta h = 4.71 \frac{Q_h^{0.444}}{u_s^{0.694}} \quad (4-21)$$

The preceding equations are examples of the prediction of the effective plume rise above the top of a stack at some distance downwind where the plume has essentially reached its maximum height. The vertical displacement, Δh , is shown in Figure 4-2, as noted earlier for a stack which has no stack tip downwash. If stack tip downwash were present, the Δh would be displaced from h_s rather than h_s . This figure shows, in addition, that the position of the maximum rise may occur considerably downwind from the stack in the x -direction. As a result, the dispersion of pollutants from the plume at a position close to the stack occurs at a height overestimated by the effective stack height, H . Past methods for estimating the effective plume rise for a specific distance downwind from the stack have shown considerable ambiguity. Consequently, most models have necessarily been based on the total effective height. More recently, models such as the ISC3 model [4] have adopted algorithms developed by Briggs [18, 19] which determine the plume rise as a function of atmospheric stability and downwind distance until the plume reaches its maximum plume rise.

The plume rise equations developed by Briggs determine the plume rise for two stability categories (unstable, or neutral, and stable conditions). For each category, a calculation is first made to determine whether the plume rise is dominated by momentum or buoyancy. A summary of the Briggs equations as presented in the User's Guide for the Industrial Source Complex (ISC3) Dispersion Models [4], is presented here. In order to determine whether the plume rise is dominated by momentum or buoyancy, it is first necessary to determine the value of the buoyancy and momentum fluxes, F_b and F_m . The value of the buoyancy flux parameter (m^4/s^3), is

$$F_b = g V_s d_s^2 \left(\frac{T_s - T_a}{4 T_s} \right) \quad (4-22)$$

where T_s is stack gas temperature (K), T_a is ambient air temperature (K), and g is the acceleration due to gravity (9.8 m/s^2).

For determining plume rise due to the momentum of the plume, the momentum flux parameter (m^4/s^2) is calculated based on the following formula:

$$F_m = V_s^2 d_s^2 \frac{T_a}{4 T_s} \quad (4-23)$$

For cases where the stack gas temperature is greater than or equal to the ambient temperature, it must be determined whether the plume rise is dominated by momentum or buoyancy. This is done by first calculating a crossover temperature difference, $(\Delta T)_c$. For unstable or neutral conditions, it is necessary to calculate a crossover temperature difference, $(\Delta T)_c$, which is dependent on the buoyancy flux. If the difference between the stack gas and ambient temperatures, ΔT , is greater than or equal to ΔT_c , plume rise is assumed to be buoyancy dominated; otherwise, plume rise is assumed to be momentum dominated [4]. For $F_b < 55 \text{ m}^4/\text{s}^3$,

$$(\Delta T)_c = 0.0297 T_s \frac{V_s^{1/3}}{d_s^{2/3}} \quad (4-24)$$

and for $F_b \geq 55$,

$$(\Delta T)_c = 0.00575 T_s \left(\frac{V_s^{2/3}}{d_s^{1/3}} \right) \quad (4-25)$$

After $(\Delta T)_c$ is determined, it is then compared to the difference in the stack and air temperatures, ΔT .

For stable conditions, the crossover temperature difference is

$$(\Delta T)_c = 0.019582 T_s V_s \sqrt{s} \quad (4-26)$$

where the stability parameter, s , is calculated from

$$s = g \frac{\partial \theta / \partial z}{T_a} \quad (4-27)$$

For stability class E, the potential temperature gradient, $\partial \theta / \partial z$, is taken as 0.020 K/m , and for class F, $\partial \theta / \partial z$ is taken as 0.035 K/m .

For unstable or neutral conditions where buoyancy is dominant (i.e., where ΔT exceeds $(\Delta T)_c$ as determined above), the downwind distance in meters to the point of final plume rise, x_f , is determined as follows. For $F_b < 55$

$$x_f = 49F_b^{5/8} \quad (4-28)$$

and the final effective plume height, H (m), is

$$H = h_s' + 21.425 \frac{F_b^{3/4}}{u_s} \quad (4-29)$$

For $F_b \geq 55$, the distance to final plume rise is

$$x_f = 119F_b^{2/5} \quad (4-30)$$

and the final effective plume height is

$$H = h_s' + 38.71 \frac{F_b^{3/5}}{u_s} \quad (4-31)$$

For unstable or neutral where the stack gas temperature is less than or equal to the ambient air temperature, the assumption is made that the plume rise is dominated by momentum. If ΔT is less than $(\Delta T)_c$ from Equation (4-24) or (4-25), the assumption is also made that the plume rise is dominated by momentum. The effective plume height is calculated as

$$H = h_s' + 3d_s \frac{V_s}{u_s} \quad (4-32)$$

Briggs [18] indicated that equation (4-32) is most applicable when V_s/u_s is greater than four.

For stable atmospheric conditions where buoyancy is dominant, the distance to final plume rise, x_f , is

$$x_f = 2.0715 \frac{u_s}{\sqrt{s}} \quad (4-33)$$

and the effective plume height is

$$H = h_s' + 2.6 \left(\frac{F_b}{u_s s} \right)^{1/3} \quad (4-34)$$

For the condition in which the stack gas temperature is less than or equal to the ambient air temperature, or if ΔT is less than $(\Delta T)_c$ as determined by Equation (4-16), the assumption is made that the plume rise is dominated by momentum. The effective plume height is calculated by the following equation

$$H = h_s' + 1.5 \left(\frac{F_m}{u_s \sqrt{s}} \right)^{1/3} \quad (4-35)$$

or by using Equation (4-32), whichever yields the lowest effective plume height.

The above equations are summarized in Table 4-6, which provides a road map for their use as a function of atmospheric stability and as to whether momentum or

TABLE 4-6 Equations for Calculating Final Plume Rise

Unstable or Neutral Atmospheric Conditions				Stable Atmospheric Conditions	
for $F_b < 55 \frac{\text{m}^4}{\text{s}^3}$, $(\Delta T)_c = 0.0297 T_s \frac{V_s^{1/3}}{d_s^{2/3}}$		for $F_b \geq 55 \frac{\text{m}^4}{\text{s}^3}$, $(\Delta T)_c = 0.00575 T_s \frac{V_s^{2/3}}{d_s^{1/3}}$		$(\Delta T)_c = 0.019582 T_s V_s \sqrt{s}$	
if $\Delta T < (\Delta T)_c$, momentum dominated	if $\Delta T \geq (\Delta T)_c$, buoyancy dominated	if $\Delta T < (\Delta T)_c$, momentum dominated	if $\Delta T \geq (\Delta T)_c$, buoyancy dominated	if $\Delta T < (\Delta T)_c$, momentum dominated	if $\Delta T \geq (\Delta T)_c$, buoyancy dominated
$\Delta h = 3d_s \frac{V_s}{u}$	$\Delta h = 21.425 \frac{F_b^{3/4}}{u}$	$\Delta h = 3d_s \frac{V_s}{u}$	$\Delta h = 38.71 \frac{F_b^{3/5}}{u}$	$\Delta h = 1.5 \left(\frac{F_m}{u \sqrt{s}} \right)^{1/3}$ or $\Delta h = 3d_s \frac{V_s}{u}$ whichever is lower	$\Delta h = 2.6 \left(\frac{F_b}{u s} \right)^{1/3}$

buoyancy is dominant [20]. While the above equations are used to calculate the final plume rise or final effective plume height, it is often desirable to calculate the plume rise at an intermediate distance, x , nearer to the stack. Equations used to predict plume rise at intermediate distances can be found in the ISC User's Guide [4].

It is evident from the above equations that the plume rise and associated effective stack height are functions of the size of the stacks, the momentum and buoyancy of the plumes and the atmospheric stability. While early equations only considered the final rise, the more recently developed equations provide estimates of plume rise and effective stack height that are a function of the downwind distance until the final plume rise is reached. As shown in Figures 4-2 and 4-3, the Gaussian dispersion model treats the plume as if it were emitted from the effective height immediately above the stack. As the distance downwind is increased, the effective emission height increases until it reaches the final rise.

EXAMPLE 4-5

The heat-emission rate associated with a stack gas is 4800 kJ/s, the wind speed and stack gas velocity are 5 and 15 m/s, respectively, and the inside stack diameter at the top is 2 m. Estimate the plume rise by means of the Moses and Carson general equation, the Holland formula, and the optimized Concawe formula.

Solution

In summary, the basic equations to be used are:

$$\Delta h = -0.029 \frac{V_s d}{u_s} + 2.62 \frac{(Qh)^{1/2}}{u_s} \quad (\text{Carson and Moses})$$

$$\Delta h = \frac{V_s d}{u_s} \left(1.5 + 0.0096 \frac{Qh}{V_s d} \right) \quad (\text{Holland})$$

$$\Delta h = 4.71 \frac{(Qh)^{0.444}}{(u_s)^{0.694}} \quad (\text{Concawe})$$

Substitution of the known data indicates that

$$\begin{aligned}\Delta h &= -0.029 \left[\frac{15(2)}{5} \right] + 2.62 \left[\frac{(4800)^{1/2}}{5} \right] = -0.1 + 36.3 \\ &= 36.2 \text{ m} \quad (\text{Carson and Moses})\end{aligned}$$

$$\begin{aligned}\Delta h &= \frac{15(2)}{5} \left[1.5 + 0.0096 \frac{4800}{15(2)} \right] = 6(1.5 + 1.54) \\ &= 18.2 \text{ m} \quad (\text{Holland})\end{aligned}$$

$$\begin{aligned}\Delta h &= 4.71 \left[\frac{(4800)^{0.44}}{(5)^{0.694}} \right] = 4.71 \left(\frac{43.1}{3.09} \right) \\ &= 66.3 \text{ m} \quad (\text{Concawe})\end{aligned}$$

The spread in the answers determined above is not usual.

EXAMPLE 4-6

The wind speed at the top of a stack and the stack gas velocity are 3 and 6 m/s, respectively, and the stack diameter is 2 m with a 40 m physical stack height. The atmospheric stability condition is neutral with a temperature of 300 K, and the stack temperature is 440 K. The flue gas has a molecular weight of 29 kg/kg-mole. Estimate the distance downwind to the point of maximum or final plume rise and the final effective plume height at that distance in meters by the appropriate Briggs equation, and by the optimized Concawe formula.

Solution

An initial check reveals that $V_s/u_s = 2.0$, therefore stack tip downwash is negligible and $h'_s = h_s$. Under neutral stability the first step is to determine the buoyancy flux, F_b , from Equation (4-22), after which the crossover temperature difference is calculated to determine whether buoyancy or momentum dominates. The buoyancy flux is as follows:

$$F_b = g V_s d_s^2 \left(\frac{T_s - T_a}{4 T_s} \right)$$

Therefore

$$F_b = (9.8) (6) (2)^2 \left(\frac{440 - 300}{(4) (440)} \right) = 18.7 \text{ m}^4/\text{s}^3$$

For neutral stability and $F_b < 55 \text{ m}^4/\text{s}^3$,

$$(\Delta T)_c = 0.0297 T_s \frac{V_s^{1/3}}{d_s^{2/3}} = 0.0297 (440) \frac{(6)^{1/3}}{(2)^{2/3}} = 14.8 \text{ K}$$

Since $\Delta T = T_s - T_a = 140$ K, then $\Delta T \gg \Delta T_c$, and the plume rise is dominated by buoyancy. Therefore, the distance downwind at which the maximum occurs is found from Equation (4-28) as

$$x_f = 49F_b^{5/8} = (49)(18.7)^{5/8} = 306 \text{ m}$$

The effective height is

$$H = h_s' + 21.425 \frac{F_b^{3/4}}{u_s} = 40 \text{ m} + (21.425) \frac{(18.7)^{3/4}}{(3)} = 40 + 64.2 = 104.2 \text{ m}$$

Note that in this case, the plume rise of 64.2 meters is greater than the original stack height.

To use the optimized Concawe equation, it is first necessary to calculate the heat emission rate, Q_h . Assuming that the pressure is 1 atm,

$$Q_h = mC_p (T_s - T_a) = \left(\frac{\pi d_s^2 V_s P M_w}{4RT_s} \right) C_p (T_s - T_a)$$

$$Q_h = \left(\frac{(3.14)(2)^2(6)(1)(29)}{(4)(0.08206)(440)} \right) \left(\frac{1.005 \text{ Kj}}{\text{kg } ^\circ\text{K}} \right) (440 - 300) = 2129 \text{ Kj/s}$$

Using the Concawe equation,

$$\Delta h = 4.71 \left(\frac{Q_h^{0.444}}{u_s^{0.694}} \right) = 4.71 \left(\frac{(2129)^{0.444}}{(3)^{0.694}} \right) = 66 \text{ m}$$

While this predicted value of Δh is in reasonable agreement with the value of 64.2 m calculated using the Briggs equation, the agreement may not be so good for other conditions.

4-8 Some Other Considerations Regarding Gaseous Dispersion

In preceding sections we developed specific equations for the dispersion of gaseous pollutants from ground and elevated sources and illustrated some general methods for solutions. In this section we consider some other ramifications of the dispersion problem.

4-8-A Concentration Estimates for Various Sampling Times

Most correlations of σ -data for atmospheric dispersion estimates, as typified by the work of Turner [2], lead to concentrations averaged over a 10-min time interval. If the concentration is to be modified for some other time interval, it is generally necessary to correct the results predicted by a dispersion model. Information to date indicates that the effects of sampling time are exceedingly complex. One equation which has been used to estimate concentrations from a single source for time intervals greater than a few minutes is

$$C_2 = C_1 \left(\frac{t_1}{t_2} \right)^q \quad (4-36)$$

where C_2 is the desired concentration, C_1 is the concentration calculated by the dispersion equation, t_2 is the new sampling time period in minutes, t_1 is 10 min, and q has a value between 0.17 and 0.20.

The value of the exponent q is substantiated by Nonhebel [21], who based his work on dispersion coefficients rather than sampling results. His base point was a 3-min sampling interval, also, rather than a 10-min period. Nevertheless, he suggested a concentration-time relationship which varied inversely to the 0.17 power.

Although the original σ -data presented by Turner were based on a 10-minute time interval, the models that have been developed and approved by U.S. EPA as regulatory models assume that the results predicted using the rural Pasquill-Gifford curves and the urban McElroy-Pooler curves represent a maximum one-hour concentration. This is a conservative assumption that leads to a higher predicted value than would be obtained by correcting the 10-minute concentration to a one-hour concentration using Equation (4-36). For purposes of consistency with current regulatory applications of the Gaussian model, it is assumed in this text that the dispersion modeling predictions represent a one-hour concentration, unless otherwise stated. In the models such as the Screen2 model [22], the one-hour concentration is multiplied by the following factors to obtain an approximate estimate of concentrations for longer averaging periods:

Time Period	Multiply 1 hr value by
3-hr	0.9
8-hr	0.7
24-hr	0.4
Annual	0.03–0.08

While an annual value is provided, it is reported in the Screen model [22] that the value of 0.08 is considered to be very conservative and that the estimation of an annual value from a one-hour predicted value is not recommended. Long-term concentrations are more appropriately predicted by running the actual meteorology for the time period of interest. For example, the ISC Long-Term model (ISCLT2) utilizes a full year of meteorological data to estimate an annual average. Similarly, a more appropriate estimate of the 24-hr concentration can be obtained by predicting hourly concentrations (using hourly meteorological values) for each of the 24 hours in question. It should be noted that the above factors are comparable to the values of 0.83, 0.70 and 0.58 that would have been obtained for the 3-, 8-, and 24-hour time periods if Equation (4-36) had been used with a q -value of 0.17, and the base time, t_1 , was defined as one hour.

EXAMPLE 4-7

The ground-level concentration of SO_2 was predicted to be $80 \mu\text{gm}^3$ using the Gaussian dispersion equation and σ -values obtained from Figures 4-6 and 4-7. Assuming that this value represents a maximum one-hour value (per U.S. EPA guidance), estimate the maximum three-hour value of the concentration.

Solution

Using U.S. EPA guidance to correct from a one-hour averaging time to a three-hour averaging time,

$$C_{\text{three-hour}} = C_{\text{one-hour}} \times 0.9 = 72 \mu\text{g}/\text{m}^3$$

4-8-B Effect of a Plume Inversion Trap

The presence of an elevated inversion will increase the ground-level concentration downwind from a stack, since the inversion acts as a giant lid on the upward dispersion of pollutant gases. This situation is often modeled as a gas passing downwind between two reflecting surfaces—the ground and the bottom of the elevated inversion layer. In Section 4-4-B we pointed out that ground reflection can be modeled by a virtual image at distance $-H$ below the earth's surface. With the addition of an inversion layer, additional reflections are considered at heights L and $-L$, where L is the distance to the bottom of the inversion layer. An accounting of all of the stable layer and ground reflections can be made through a summation of terms. The end result is a center-line expression of the form

$$C = \frac{Q}{2\pi\sigma_y\sigma_z u} \sum \left\{ \exp\left[\frac{-(z-H+2jL)^2}{2\sigma_z^2}\right] + \exp\left[\frac{-(z+H+2jL)^2}{2\sigma_z^2}\right] \right\}$$

where the summation is carried out from $j = -\infty$ to $+\infty$. This series usually converges rapidly, requiring only the first few terms, for example, values of j up to ± 2 or ± 3 .

A good approximation to this equation may be made by assuming that the inversion layer has no effect on the vertical dispersion until a downwind distance x_L for which $\sigma_z = 0.47(L-H)$. If we know the height of the inversion layer, we can use this relation to estimate x_L through the use of Figure 4-7. The effect of reflections from the stable layer and the ground beyond the distance x_L is such that uniform vertical mixing has taken place by the downwind distance $2x_L$. Beyond $2x_L$ the appropriate equation is

$$C(>2x_L, y, z) = \frac{Q}{(2\pi)^{1/2} \sigma_y L u} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \quad (4-37)$$

Note that this expression contains only x and y as variables. For distances between x_L and $2x_L$ Turner [2] suggests that ground-level center-line concentrations be read from a straight line drawn between the concentrations for points x_L and $2x_L$ on a log-log plot of ground-level center-line concentration versus distance.

EXAMPLE 4-8

Sulfur dioxide is emitted at a rate of 160 g/s from a stack with an effective height of 60 m. The wind speed at stack height is 6 m/s, and the atmospheric stability class is C . Estimate the distance at which reflection from the stable layer just begins to occur, in meters, for an inversion layer 150 m above ground level. Also find the concentration at a distance of $2x_L$.

Solution

As an estimate, the value of x_L occurs when

$$\sigma_z = 0.47(L - H) = 0.47(150 - 60) = 42.3 \text{ m}$$

From Figure 4-7, for stability class *C* we find that reflection just begins to occur at a downwind distance of 680 m, approximately. Consequently, the value of $2x_L$ is 1360 m. From Figure 4-6, the value of σ_y is 140 m at this latter distance. For the center-line concentration at a distance of 1360 m Equation (4-37) yields

$$C(2x_L, 0, z) = \frac{Q}{(2\pi)^{1/2} \sigma_y L u} = \frac{160(10^6)}{2.51(140)(150)(6)} = 505 \text{ } \mu/\text{m}^3$$

This value persists, of course, at all heights up to 150 m. ◆

4-8-C Line Sources

In some situations, such as a series of industries located along a river or harbor, or heavy traffic along a straight stretch of highway, the pollution problem may be modeled as a continuous emitting infinite line source. When the wind direction is normal to the line of emission, the ground-level concentration downwind is given by

$$C(x, 0) = \frac{2q}{(2\pi)^{1/2} \sigma_z u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \quad (4-38)$$

where q is the source strength per unit distance. For example, q might be expressed in terms of $\text{g/s} \cdot \text{m}$. The horizontal standard deviation, σ_y , is absent from the equation, since crosswind diffusion from various portions of the emitted gases should be self-compensating. Note also that y does not appear in Equation (4-38) since the concentration should be uniform in the y -direction at a given x -distance. When the wind direction is not perpendicular to the line source, Turner [1] suggests that Equation (4-38) be divided by $(\sin \phi)$, where ϕ is the angle between the line source and the wind direction. This correction should not be used when ϕ is less than 45 degrees.

When the continuously emitting line source is reasonably short in length, we must account for the edge effects caused by the two ends of the source. These edge effects become more important, in the sense that they extend to greater crosswind distances, as the distance downwind from the source increases. If the line source is perpendicular to the wind direction, then it is convenient to define the x -axis in the direction of the wind and also passing through the sampling point downwind. The ends of the line source then are at two positions in the crosswind direction, y_1 and y_2 , where y_1 is less than y_2 . The concentration along the x -axis at ground level is then given by the expression

$$C(x, 0, 0) = \frac{2q}{(2\pi)^{1/2} \sigma_z u} \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \int_{y_1}^{y_2} \frac{1}{(2\pi)^{1/2}} \exp(-5p^2) dp$$

where $p_1 = y_1/\sigma_y$ and $p_2 = y_2/\sigma_y$. Once the limits of integration are established, the value of the integral may be determined from standard statistical tables.

EXAMPLE 4-9

Estimate the total hydrocarbon concentration at a point 300 m downwind from an expressway at 5:30 P.M. on an overcast day. The wind is perpendicular to the highway and has a speed of 4 m/s. The traffic density along the highway is 8000 vehicles per hour, and the average vehicle speed is 40 mi/hr. The average vehicle emission rate of hydrocarbons is 2×10^{-2} g/s.

Solution

Assuming a reasonably straight section of highway, we will consider the pollutants emanating from a continuous infinite line source. The emission rate per unit length, q , is determined from the product of the emission rate per vehicle times the number of vehicles per unit length. This latter quantity is found by dividing the rate of vehicle travel past a point by the average speed of the vehicle. Hence,

$$\frac{\text{vehicles}}{\text{m}} = \frac{8000(\text{vehicles/hr})}{40(\text{mi/hr})} \left(\frac{\text{mi}}{1600 \text{ m}} \right) = 0.125$$

Therefore,

$$q = 0.125 \text{ vehicle/m} \times 2 \times 10^{-2} \text{ g/vehicle(s)} = 2.5 \times 10^{-3} \text{ g/s m}$$

For an overcast day, the stability class is *D*. From Figure 4-7, at a downwind distance of 300 m, the value of σ_z is 12 m. Recognizing that the exponential term in Equation (4-38) is unity for a ground-level source, we find that this equation yields

$$C(300, 0, 0) = \frac{2(2.5)(10^{-3})}{(2\pi)^{1/2}(12)(4)} = 42(10^{-6}) \text{ g/m}^3 = 42 \mu\text{g/m}^3$$

Conservatively, this is the concentration estimate for a one-hour sampling period as discussed in Section 4-8-A. ◆

EXAMPLE 4-10

An interstate highway has a traffic volume of 10,000 vehicles/hour. Based on the emission factors for vehicles (as reported by EPA in 1990), the emission of carbon monoxide (CO) is 3 g/mi per vehicle at 50 mph and 27 g/mi at 5 mph. Calculate the worst case concentration of CO at the road edge for vehicles moving at 50 mph and 5 mph, respectively. Based on knowledge of mixing in the highway corridor, it is assumed that the initial σ_z at the road edge is 3 meters. Assume that the wind speed is perpendicular to the road and has a value of 1 m/sec.

Solution

Again we will assume that the condition described can be modeled using the infinite line source Equation (4-38). Based on the emission factors for 50 mph,

$$q = \frac{3 \text{ g}}{\text{mi-veh.}} \left(\frac{1 \text{ mi}}{1600 \text{ m}} \right) \left(\frac{10,000 \text{ veh.}}{\text{hr.}} \right) \left(\frac{1 \text{ hr.}}{3600} \right) = 0.0052 \text{ g/m-s}$$

Worst case conditions for ground-level emissions are represented by using *F* stability and a wind speed of 1 m/s. Normally, one would use Figure 4-7 to determine

the value of σ_z at the appropriate downwind distance. However, Figure 4-7 is limited to distances greater than 100 m downwind. In this case, we are given that σ_z is approximately 3 m at the road edge. Using Equation (4-38), and setting the exponential term to unity for a ground-level source,

$$C(0, 0, 0) = \frac{2 (0.0052)}{(2\pi)^{1/2} (3) (1)} = 0.00138 \text{ g/m}^3$$

or $1380 \text{ } \mu\text{g/m}^3$ (approximately 1.16 ppmv). This value represents the maximum one-hour average and can be compared directly to the NAAQS of $40,000 \text{ } \mu\text{g/m}^3$ (or 35 ppm). It is obvious that this condition is well within the air quality standard.

For the condition of 5 mph, however, the emission factor is nearly nine times higher (27 g/mi). In this case, the concentration at the road edge would be 10.4 ppm, which is still only one-third of the one-hour primary air quality standard. Note, however, that if this condition persisted for eight hours it would create an eight-hour concentration (based on section 4-8-A) of

$$8 \text{ hr value} = 0.7 (1 \text{ hr value}) = 0.7(10.4) = 7.28 \text{ ppm}$$

When compared to the eight-hour primary air quality standard of 9 ppm, this condition is close to the standard. If background concentrations of 2–3 ppm existed, it could create an exceedance of the standard. Observations in the environment have shown that the eight-hour standard is generally more difficult to meet and is the cause of nonattainment more frequently than exceedances of the one-hour standard, indicating that a prolonged heavy traffic volume at low speeds is a major concern.

Computer programs are available, such as the EPA HiWay and California line source models (CALINE-3 and CAL3QHC), which have been developed to simulate a wide variety of highway configurations and conditions. These are available on the EPA TTNWeb SCRAM bulletin board and from other sources.

4-8-D Instantaneous Release of a Puff

Another condition that may need to be modeled is that of the instantaneous release of a pollutant such as might occur during an explosion or an accidental spill. In those cases where the time of release is very short compared to the time of transport, the emission can be modeled using the instantaneous puff equation. This equation, like the equations for the continuous point source and line source is based on the Gaussian distribution function. The equation for a puff contains the three-dimensional expression in x , y , and z and results in the following equation when ground-level reflection is included:

$$C = \frac{Q_p}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\left(\frac{y-y_o}{\sigma_y}\right)^2\right) \exp\left(-\frac{1}{2}\left(\frac{x-x_o}{\sigma_x}\right)^2\right) \left[\exp\left(-\frac{1}{2}\left(\frac{z-z_o}{\sigma_z}\right)^2\right) + \exp\left(-\frac{1}{2}\left(\frac{z+z_o}{\sigma_z}\right)^2\right) \right] \quad (4-39)$$

The emission strength, Q_p , is the mass that is instantaneously released (typically in grams) and x_o , y_o , and z_o define the location of the center of the puff. Unlike previous fixed location sources, the puff is transported in the x direction by the wind such that the location of the center of the puff corresponding to the peak concentration in the x direction (x_o) becomes a function of time and windspeed. The value of x_o at any time, t , after release of the puff is $x_o = ut$. For conditions in which it is desirable to predict the ground level concentration directly downwind from a ground-level release, then z_o , y_o , z , and y are all equal to zero, resulting in the most familiar form of the puff equation:

$$C = \frac{2Q_p}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\left(\frac{x-ut}{\sigma_x}\right)^2\right) \quad (4-40)$$

It is generally assumed that dispersion in the x direction is similar to that in the y direction, such that σ_y can be substituted for the σ_x value. Figure 4-9 illustrates the movement of the Gaussian plume showing the concentration of the puff as it moves in the downwind direction. The plume spreads in both the x , y , z directions, as it moves downwind, decreasing the maximum concentration at the center of the puff while, at the same time, increasing the size of the puff. The receptor at some distance x downwind would see a gradual increase in concentration until the center line of the puff passed by, followed by a decrease in concentration. The peak concentration observed by the receptor at distance, x , can be predicted by realizing that it occurs at the point at which $x = ut$. At that point, the exponential term in Equation (4-40) has a value of unity, and the maximum concentration is

$$C_{\max} = \frac{2Q_p}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \quad (4-41)$$

Determination of the actual dosage received by the receptor would require a calculation of the concentration at successive time increments, from which the exposure in ppmv-hrs could be made by determining the area under the concentration versus time curve.

The instantaneous nature of the release of a puff requires the use of σ_y , σ_x , and σ_z values that are representative of shorter averaging times than those used for continuous releases. The values listed in Table 4.7 are currently used in the

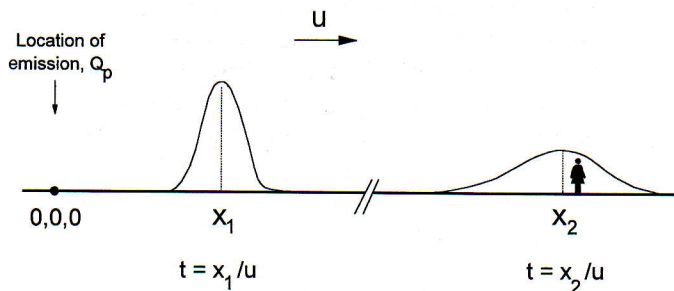


FIGURE 4-9 An instantaneous puff traveling downwind at windspeed, u .

TABLE 4.7 Instantaneous Values for σ_y and σ_z in meters [11]

Parameter	Stability Condition	Equation*
σ_y	Unstable	$\sigma_y = 0.14 (x)^{0.92}$
	Neutral	$\sigma_y = 0.06 (x)^{0.92}$
	Very Stable	$\sigma_y = 0.02 (x)^{0.89}$
σ_z	Unstable	$\sigma_z = 0.53 (x)^{0.75}$
	Neutral	$\sigma_z = 0.15 (x)^{0.70}$
	Very Stable	$\sigma_z = 0.05 (x)^{0.61}$

* x is the distance downwind in meters.

PUFF and TSCREEN models, available on the SCRAM BBS. TSCREEN is a screening model used to predict concentrations of toxic pollutants downwind from any one of a number of different toxic release scenarios, including puffs [23]. Additional information on modeling of toxic emissions can be found in the *Workbook of Screening Techniques for Assessing Impacts of Toxic Air Pollutants* [24].

EXAMPLE 4-11

A bottle of liquid SO_2 is burst at ground level and instantaneously releases 80 pounds of SO_2 (36,240 g). What is the approximate maximum concentration that will be observed by a person who is 1000 m directly downwind under worst-case meteorological conditions.

Solution

For ground-level sources, the worst-case meteorological conditions occur with F stability at a wind speed of 1 m/s. (This is the lowest value of u generally recommended in the continuous release Gaussian dispersion models). Based on the equations in Table 4.7, and using a stability condition of very stable, the σ_y and σ_z values at 1000 m downwind are calculated to be 9.4 m and 3.4 m, respectively. The value of σ_x is assumed to be equal to σ_y . Substituting the values into Equation (4-40) and setting the exponential term equal to unity,

$$C = \frac{(2) (36,240)}{(2\pi)^{3/2} (9.4) (9.4) (3.4)} (1.0) = 15.3 \text{ g/m}^3$$

or $1.53 \times 10^7 \text{ } \mu\text{g/m}^3$.

The primary air quality standard for SO_2 is $1300 \text{ } \mu\text{g/m}^3$ (max. 3-hr average). It is apparent that the release creates an instantaneous concentration that is significantly higher than the standard. However, the peak is short-lived and passes by the receptor relatively quickly. Since the σ_x is only 9.4 meters, then approximately 68 percent of the mass of the plume will pass by the receptor within a time equal to a distance of $2\sigma_x/u$, or approximately 19 seconds. This acute concentration is not adequately addressed by the current 3-hr air quality standard. The ACGIH TLV-C (threshold limit value not to be exceeded even instantaneously) or the TLV-STEL (short term exposure limit) would be more appropriate values for comparison, as discussed in Table 2-11. The ACGIH TLV-STEL for SO_2 is 5 ppm, or $13,300 \text{ } \mu\text{g/m}^3$, and is clearly predicted to be exceeded in this case.

4-8-E Concentration Estimates Due to Multiple Point and Area Sources

Frequently a receptor or sampling point may lie downwind from two or more continuously emitting point sources, and these sources are not directly upwind. This type of dispersion problem is usually solved by the method of superposition. That is, the total concentration at the receptor point is found by summing the concentrations coming from the individual sources. Consequently, the problem becomes a matter of correctly identifying the geometry or orientation of the receptor with respect to the wind direction and each source location, separately. Then the standard equations developed earlier for an elevated or ground source are applied, in terms of the given windspeed and stability class of interest.

Sources whose emissions tend to be distributed uniformly over an area can be modeled as an array of point sources or as a series of finite length line sources, where the points or lines are distributed uniformly over the area of interest. It should be remembered that the line-source equation is really the integration of an infinite number of point sources along the line. As in the case of multiple point sources, the concentrations coming from the sources are additive.

4-8-F Concentration Estimates Due to Decay of a Pollutant

In some cases, it is necessary to predict the decay of a pollutant as it disperses downwind. Examples include the radioactive decay of a pollutant and the reaction of the pollutant with other gases in its vicinity. If it is assumed that the pollutant's concentration is undergoing a first-order reaction, then the pollutant's concentration, C , decreases exponentially with time, t . Since the time of travel is really equal to the downwind distance, x , divided by the windspeed, then the equation is

$$C = C_o \exp\left(-\psi \frac{x}{u_s}\right) \quad (4-42)$$

where ψ is the reaction rate constant or decay constant with units of s^{-1} . As a decay constant, ψ can be defined as being equal to $0.693/T_{1/2}$ where $T_{1/2}$ is the pollutant half life in seconds. The Gaussian equation can be modified to incorporate the decay of the pollutant by multiplying the appropriate Gaussian equation by the exponential term in Equation (4-42).

EXAMPLE 4-12

In the ISC User's Guide, it is reported that the effective half life of sulfur dioxide in the atmosphere is approximately four hours due to the reaction and conversion of the sulfur dioxide to other secondary pollutants. For the data in Example 4-1, determine the concentration of sulfur dioxide at 500 m if it is assumed to have a four-hour half life.

For the half life of four hours,

$$\psi = \frac{0.693}{T_{1/2}} = \frac{(0.693)}{(4 \text{ hrs}) (3600 \text{ s/hr})} = 0.000048 \text{ s}^{-1}$$

The exponential decay term then becomes

$$\exp\left(-\psi \frac{x}{u_s}\right) = \exp\left(-0.000048 \frac{(500)}{(6)}\right) = 0.996$$

Therefore, the actual concentration of sulfur dioxide would be $0.996 \times 66 \mu\text{g}/\text{m}^3$. There is very little change in the concentration since the time of travel downwind was only 83 seconds ($500/6$) as compared to a half life of four hours. ◆

Appendix—Development of the Gaussian-Type Dispersion Equation

The following is one method for developing the Gaussian-type dispersion equation for a gas continuously released from a point source. On the basis of mass transfer theory, the mass rate of diffusion N_x of a gaseous species in the x -direction at some cross-sectional area A is given by the expression

$$N_x = -A \left(\frac{\partial(D_x C)}{\partial x} \right) \quad (\text{a})$$

where N_x is the mass transfer per unit time; D_x is the mass diffusivity, area/time, in the x -direction; C is the concentration in mass per unit volume; and A is the cross-sectional area in the x -direction.

We now wish to apply this general relationship to the diffusion of a gas, which originates continuously at a point source, through a differential volume in space. Consider the situation represented in Figure 4-A. A gaseous pollutant, carried downwind in the x -direction with a wind speed u , enters a fluid element of size $dx \, dy \, dz$. The mass rate of diffusion into the fluid element (independent of the bulk motion of the gas stream) in terms of Equation (a) would be

$$N_x = -dy \, dz \frac{\partial(D_x C)}{\partial x}$$

The rate out of the differential volume, in the x -direction, is

$$N_{x+dx} = -dy \, dz \frac{\partial(D_x C)}{\partial x} + \frac{\partial}{\partial x} \left[\left(\frac{\partial(D_x C)}{\partial x} \right) dy \, dz \right] dx$$

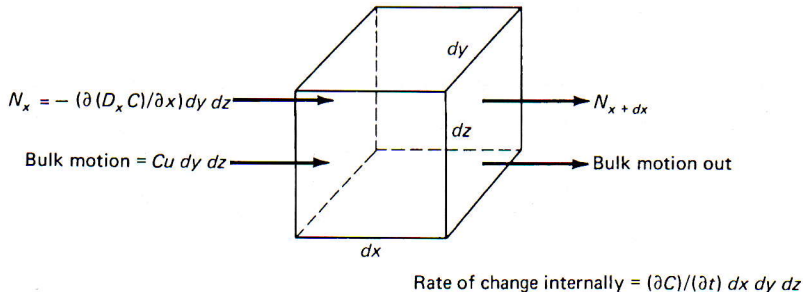


FIGURE 4-A Schematic for the development of Equation (e), which relates mass transfer due to diffusion and bulk motion in and out of a fluid element to the change in internal concentration.

The rate of change of the concentration within the differential volume, due to mass diffusion in and out in the x -direction, is the difference between N_{x+dx} and N_x . This quantity is

$$N_{x+dx} - N_x = \frac{\partial}{\partial x} \left(\frac{\partial(D_x C)}{\partial x} \right) dx dy dz \quad (b)$$

Similar expressions are valid for the y - and z -directions.

In addition, however, the movement of pollutant is also aided by the bulk motion of the fluid. The rate of influx of pollutant at position x through area $dy dz$ is found to be

$$\text{rate in (bulk motion)} = C(u) cy dz$$

and the rate out at position $(x + dx)$ is

$$\text{rate out (bulk motion)} = Cu dy dz + \frac{\partial}{\partial x} (Cu dy dz) dx$$

The net rate of change of concentration within the control volume $dx dy dz$, which results from the bulk motion of the fluid, then is

$$\text{net rate (bulk motion)} = -\frac{\partial}{\partial x} (Cu) dx dy dz \quad (c)$$

The net effect of mass diffusion and bulk motion is to change the amount of mass within the control volume. The rate of change of mass within the control volume symbolically is

$$\text{rate of change within } dx dy dz = \frac{\partial C}{\partial t} dx dy dz \quad (d)$$

Consequently, the general expression for the rate of change of mass within a differential volume $dx dy dz$, due to bulk transport as well as diffusional processes, is, from Equations (b), (c), and (d),

$$\frac{\partial C}{\partial t} = -\frac{\partial}{\partial x} (Cu) + \frac{\partial}{\partial x} \left(\frac{\partial(D_x C)}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial(D_y C)}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{\partial(D_z C)}{\partial z} \right) \quad (e)$$

In developing Equation (e), the quantity $dx dy dz$ has been canceled throughout the equation.

Some basic idealizations with regard to the stack diffusion problem at this point are the following:

1. Mass transfer due to bulk motion in the x -direction far outshadows the contribution due to mass diffusion. That is, the second term on the right side of Equation (e) is far smaller than the first term and may be dropped from the equation.

2. We are primarily interested in the steady-state solution to the dispersion of pollutants in the atmosphere. Hence the quantity $\partial C / \partial t$ is zero.

3. Even though the wind speed does vary in the three coordinate directions, the variation is relatively small. Therefore it is appropriate to assume that the wind

speed u is constant, since this leads to a simpler and more straightforward solution to the partial differential equation.

In the absence of sufficient information to the contrary, it is assumed that the mass diffusivities D_x , D_y , and D_z are constant. These four statements lead to the reduction of Equation (e) to the following form:

$$u \left(\frac{\partial C}{\partial x} \right) = D_y \left(\frac{\partial^2 C}{\partial y^2} \right) + D_z \left(\frac{\partial^2 C}{\partial z^2} \right) \quad (f)$$

The general solution to this second-order partial differential equation is

$$C = Kx^{-1} \exp \left\{ - \left[\left(\frac{y^2}{D_y} \right) + \left(\frac{z^2}{D_z} \right) \right] \frac{u}{4x} \right\} \quad (g)$$

where K is an arbitrary constant whose value is determined by the boundary conditions on the specific atmospheric problem. One boundary condition that must be satisfied is that the rate of transfer of pollutant through any vertical plane downwind from the source is a constant in steady state, and this constant must equal the emission rate of the source, Q . That is, all pollutant emitted from the source must be accounted for somewhere downwind in the y - z plane. Hence it is assumed that no chemical reactions occur downwind to remove some of the pollutant, and no other removal mechanisms such as absorption or adsorption by other media are acknowledged. In a mathematical context, this boundary condition is expressed by

$$Q = \iint uC \, dy \, dz \quad (h)$$

Generally the limits of integration on dy are minus to plus infinity. However, the limits on dz depend on the physical situation of interest.

A. Point Source at Ground Level

For a point source at ground level the limits of integration on z are taken from 0 to ∞ . With these limits and with substitution of Equation (g) into Equation (h), we find that

$$Q = \int_0^\infty \int_{-\infty}^\infty Kux^{-1} \exp \left[- \left(\frac{y^2}{D_y} + \frac{z^2}{D_z} \right) \frac{u}{4x} \right] dy \, dz$$

Now let $\bar{y} \equiv y/(D_y)^{1/2}$ and $\bar{z} \equiv z/(D_z)^{1/2}$. With this change in variables,

$$Q = Kux^{-1} (D_y)^{1/2} (D_z)^{1/2} \int_0^\infty \exp \left(- \frac{\bar{z}^2 u}{4x} \right) d\bar{z} \int_{-\infty}^\infty \exp \left(- \frac{\bar{y}^2 u}{4x} \right) d\bar{y}$$

However, from standard integral tables we find that

$$\int_0^\infty \exp(-a^2 x^2) dx = \frac{(\pi)^{1/2}}{2a}$$

and the integral of the same quantity from minus to plus infinity is twice the given value, or $\pi^{1/2}/a$. Consequently,

$$\begin{aligned} Q &= Kux^{-1}(D_y)^{1/2}(D_z)^{1/2}\left(\frac{\pi x}{u}\right)^{1/2}\left[2\left(\frac{\pi x}{u}\right)^{1/2}\right] \\ &= 2\pi K(D_y)^{1/2}(D_z)^{1/2} \end{aligned}$$

or

$$K = \frac{Q}{2\pi(D_y D_z)^{1/2}} \quad (i)$$

where Q is the strength of the emission source, that is, mass emitted per unit time. By substitution of Equation (i) into Equation (g) we find that the concentration of a pollutant emitted from a point source at ground level is modeled by the expression

$$C(x, y, z) = \frac{Q}{2\pi x(D_y D_z)^{1/2}} \exp\left[-\left(\frac{y^2}{D_y} + \frac{z^2}{D_z}\right)\frac{u}{4x}\right] \quad (j)$$

This equation has the format of the double Gaussian or normal distribution as expressed by Equation (4-4). Since for a ground-level source the maximum concentration in the y - and z -directions should occur along the center line at ground level, the values of y_0 and z_0 in Equation (4-4) are zero for this physical situation. Hence Equation (4-4) reduces to the form

$$f(y, z) = \frac{1}{2\pi\sigma_y\sigma_z} \exp\left(\frac{-y^2}{2\sigma_y^2} + \frac{-z^2}{2\sigma_z^2}\right)$$

It has been found convenient to reorganize Equation (j) into a form similar to the above expression. In order to do this we make the following definitions:

$$\sigma_y^2 \equiv \frac{2D_y x}{u} \quad \text{and} \quad \sigma_z^2 \equiv \frac{2D_z x}{u} \quad (k)$$

Substitution of these two definitions into Equation (j) leads to the following relationship for the concentration downwind from a ground-level point source:

$$C(x, y, z) = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2}\right)\right] \quad (l)$$

When Equation (l) is rearranged so that the left side is equal to $Cu/2Q$, then the right side will have the identical format of $f(y, z)$ described above, which is a double Gaussian type. The units on the gaseous concentration C are determined by the units used to express the quantities Q , u , σ_y , and σ_z . In the technical literature σ_y and σ_z usually are given in meters, and u in meters per second. If C is desired in micrograms per cubic meter, then the emission rate Q must be expressed in micrograms per second.

B. Point Source at Elevation H Above the Ground Level

For a point source at an elevation H above the ground, the limits of integration on z in Equation (h) are taken to be from minus to plus infinity. The mathematical limit of minus infinity is physically meaningful in the following sense. Even if the ground were permeable to the diffusion of a pollutant gas, the Gaussian distribution is of such a nature that the majority of the pollutant would exist between the ground and the height H . Hence adding on to Q the integration from ground level to minus infinity leads to a reasonable small error, but makes the mathematics much more tractable.

The effect of this change in the lower limit of integration for z is to halve the value of K found previously for the point source at ground level.

That is, in the present case

$$K = \frac{Q}{4\pi(D_y D_z)^{1/2}} \quad (m)$$

This relationship for K is now substituted into Equation (g), and the quantities D_y and D_z are replaced by σ_y and σ_z in terms of Equation (l). As a result,

$$C = \frac{Qx^{-1}}{4\pi(D_y D_z)^{1/2}} \exp\left[-\left(\frac{y^2}{D_y} + \frac{z^2}{D_z}\right)\frac{u}{4x}\right]$$

$$C = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2}\right)\right] \quad (n)$$

It is this formulation for a point source above ground level that leads to Equation (4-8) shown in Section 4-4-A.

QUESTIONS

1. What two physical phenomena account for the dispersal of pollutants in the atmosphere as a gas stream leaves a stack?
2. What are the deficiencies of the eddy diffusion model as represented by Equation (4-2)?
3. What is the format of a function $f(x)$ which is said to be normally distributed?
4. Why is a double-Gaussian format for a dispersion equation necessary in atmospheric studies?
5. What is the major difference in the development of a dispersion formulation for a ground source as opposed to an elevated source?
6. At what elevation is the wind velocity u evaluated for use in Equation (4-9)? Does the actual wind velocity vary with elevation across the plume? How would one take this factor into account?
7. It may be convenient to label the curves of Figures 4-6 and 4-7 according to atmospheric stability ranging from very unstable to very stable. Do this.
8. What data are usually required to determine the maximum ground-level concentration?
9. What three sets of parameters control the phenomenon of a gaseous plume injected from a stack into the atmosphere?

10. Most equations for predicting plume rise contain two terms that account for different physical reasons for the rise. What is the nature of these two terms?
11. Is it justifiable to use a constant value for the effective plume rise for a given situation, or should one, in reality, adjust for the distance downwind from the stack? Discuss.
12. What is the general effect of sampling time on the expected concentration at a given location?
13. What is the sampling time limitation when employing Turner's data for σ -values? How does it differ from the interpretation used by EPA models?
14. How does one modify the usual dispersion equation from a stack to account for the presence of an inversion trap?
15. In what manner does one express the source emission rate for a line source, such as cars along a highway?
16. By what general mathematical technique does one attack the problem of estimating ground-level concentrations resulting from several point sources?
17. How does one handle the change in concentration of a pollutant in air due to reaction and/or deposition with time?
18. Discuss the concentration as a function of time as observed by a receptor located downwind from a continuous versus an instantaneous release point source.

PROBLEMS

- 4-1. Plot $f(x)$ versus x for a normally distributed (Gaussian) function for x values up to ± 5 (if necessary) and σ values of (a) 0.5, (b) 1.0, and (c) 2.0, for the case where $x_0 = 0$. Show table of computations for each case.
- 4-2. Consider a normally distributed variable x . For values of σ of (a) 1.0 and (b) 2.0, determine the value of x for which the ratio given by $f(x)/f(x=0)$ is (1) 0.05, (2), 0.02, and (3) 0.01. secondly, for the specified conditions, determine the value of x/σ in each case.
- 4-3. Consider a normally distributed variable x and σ values of (a) 1.0, and (b) 2.0. determine the values $f(x)/f(x=0)$ when the values of x/σ is (1) 2.5, (2) 3.0, and (3) 3.5.
- 4-4. Sulfur dioxide is being emitted in a rural area at a rate of 0.90 kg/s from a stack with an effective height of 220 m. The average wind speed at stack height is 4.8 m/s, and the stability category is B. Determine the short-time period, downwind, center-line concentration in micrograms per cubic meter at ground-level distances from the stack of (a) 0.6, (b) 0.8, (c) 1.0 (d) 1.2, (e) 1.6, (f) 2.0, (g) 3.0, and (h) 4.0 km. Plot C versus the logarithm of the distance.
- 4-5. What is the expected short-time period ground-level concentration at (a) 150 m and (b) 250 m away from the downwind center line for the conditions of Problem 4-4 for cases (b) through (h)?
- 4-6. From the results of Problems 4-4 and 4-5, plot the isopleths (lines of constant concentration) for SO_2 concentrations of 50, 150, 250, 400, and 550 $\mu\text{g}/\text{m}^3$ on an x - y diagram. Plot x from 0 to 4 km and plot y from 0 to 400 m, full scale.
- 4-7. Consider the data of Problem 4-4. Estimate the distance downwind on the center line in kilometers at which the maximum concentration will occur at ground level, and estimate what that concentration will be in micrograms per cubic meter employing Figure 4-8.
- 4-8. Check the values of σ_y and σ_z that were used in Problem 4-4, using the basic equations in Tables 4-1 and 4-2.

- 4-9.** Check the value of C_{\max} determined in Problem 4-7, using the method discussed in the text where σ_y/σ_z is a constant. Also use this method to determine x_{\max} in kilometers.
- 4-10.** Derive expressions for x_{\max} and C_{\max} for the situation where $\sigma_y = ax^p$ and $\sigma_z = bx^q$.
- 4-11.** Evaluate C_{\max} and x_{\max} from the equations developed in Problem 4-10, using the σ_y and σ_z data in Tables 4-1 and 4-2, for the data of Problem 4-4.
- 4-12.** Rework Problem 4-4 for conditions of an urban area. Compare the results with those of Problem 4-4.
- 4-13.** Reconsider Problem 4-4 for rural conditions. Change the stability class to C , and determine the downwind, center-line concentrations in micrograms per cubic meter at ground-level distances from the tack of (a) 1.2, (b) 1.6, (c) 2.0, (d) 2.5, (e) 3.0, (f) 5.0, (g) 10, and (h) 20 km. Plot C versus the logarithm of the distance.
- 4-14.** What is the expected ground-level concentrations at (a) 300 m, and (b) 500 m crosswind for the conditions of Problem 4-13 for cases (b) through (h)?
- 4-15.** From the results of Problems 4-13 and 4-14, plot the isopleths for SO_2 concentrations of 150, 200, 300, 400, and $450 \mu\text{g}/\text{m}^3$ on an x - y diagram. Plot x from 0 to 10 km and plot y from 0 to 600 m, full scale.
- 4-16.** Consider the data of Problem 4-13. Estimate the distance downwind on the center line in kilometers at which the maximum concentration will occur at ground level, and estimate what that concentration will be in micrograms per cubic meter employing Figure 4-8.
- 4-17.** Check the values of σ_y and σ_z that were used in Problem 4-13, using the basic equations and data of Tables 4-1 and 4-2.
- 4-18.** Check the value of C_{\max} determined in Problem 4-16, using the method discussed in the text where σ_y/σ_z is a constant. Also use this method to determine x_{\max} in kilometers.
- 4-19.** A new municipal waste combustor burns 300 tons/day of waste and produces 100 dscf of emissions per lb of waste burned. The plant just meets the NSPS emission standards in Table 2-4(c) and has an effective stack height of 100 m and is located in a rural area. Under C stability and a windspeed of 4 m/s measured at 10 m, determine C_{\max} and x_{\max} for (a) PM, (b) Pb in micrograms per cubic meter, and m, respectively.
- 4-20.** Rework Problem 4-19 for urban conditions.
- 4-21.** It is convenient to know the distance y_p where the concentration has dropped to p percent of its value on the plume axis. Consider the horizontal spread in pollutant concentration at ground level. Making use of Equation (4-9), prove that the value of y_p at any distance x is found simply from the expression, $y_p = [2\sigma_y^2 \ln(100/p)]^{1/2}$.
- 4-22.** Hydrogen sulfide is vented from a stack in a rural area that has an effective height of 50 m. The wind speed is 2.5 m/s on an overcast night. For an emission rate of 0.06 g/s, (a) determine the maximum ground-level concentration on the plume center line downwind from the stack, and (b) plot the ground-level concentration as a function of y -distance from the center line at the x -location determined in part (a) for y values of 50, 100, 200, and 300 m.
- 4-23.** The odor threshold for H_2S is 0.00047 ppm. Using the emission conditions given in Problem 4-22 in terms of x - and y -coordinates, estimate the region in which an average person could detect the hydrogen sulfide by smell.
- 4-24.** The ground-level concentration of SO_2 downwind from a stack is to be limited to $80 \mu\text{g}/\text{m}^3$. The wind speed is 4 m/s on a clear day, and the emission rate is 50 g/s. What

is the minimum required effective stack height, in meters? Use rural conditions and assume windspeed is at H .

- 4-25. For the data given in Example 4-2, determine the crosswind distance in meters at which the concentration will be (a) 30 percent, (b) 20 percent, and (c) 10 percent of the center-line concentration.
- 4-26. The rate of emission of SO_2 from the stack of a power plant is 126.1 g/s. The effective height of the stack is 46 m. Calculate the SO_2 concentration in parts per million at a parking lot located 900 m downwind from the stack on a sunny October day when the wind velocity is 4 m/s. Use class *C* stability and rural conditions.
- 4-27. How many meters downwind from the stack does the maximum ground-level concentration of SO_2 occur for Problem 4-26? What is the value of the concentration in parts per million at that location?
- 4-28. You are located downwind from two oil-burning power plants in a rural area. One is located 0.3 km NNE of your location and burns 1400 kg of 0.5 percent sulfur oil per hour. The second plant is located 0.5 km NNW of you and burns 1600 kg/hr of fuel oil containing 0.75 percent sulfur. Assume that both plant stacks have an effective height of 40 m. The wind is blowing from the north at 3.3 m/s. For a class *B* stability condition, what is the SO_2 concentration at your location at ground level, in micrograms per cubic meter? The wind speed is at the standard height of 10 m.
- 4-29. A plant is to be constructed that will emit 3.5 metric tons of hydrogen sulfide per day. One of the design criteria is that the concentration 1 km downwind from the stack must not exceed $120 \mu\text{g}/\text{m}^3$, so that the odor threshold is not exceeded. For the purpose of estimation, the plume rise is neglected initially. Estimate the required stack height, in meters, for wind speeds of (a) 4 m/s and (b) 8 m/s. Use rural conditions.
- 4-30. On an overcast day with a class *C* stability, the wind velocity at 10 m is 4 m/s. The emission rate of NO is 50 g/s from a stack having an effective height of 100 m. (a) Estimate the center-line, ground-level concentration 20 km downwind from the stack, in micrograms per cubic meter. (b) Estimate the ground-level concentration 20 km downwind and 900 m from the stack center line, in micrograms per cubic meter. Use rural conditions.
- 4-31. A power plant emits SO_2 on a day with class *C* stability when the wind speed at the top of the stack is 7 m/s. The effective stack height is 282 m. If the short-time ground-level concentration downwind is not to exceed (a) $1000 \mu\text{g}/\text{m}^3$ and (b) $1300 \mu\text{g}/\text{m}^3$, what is the maximum permissible SO_2 emission rate, in grams per second? Use rural conditions.
- 4-32. An existing rural power plant has been found to produce an SO_2 concentration of $20 \mu\text{g}/\text{m}^3$ at a distance of 800 m directly downwind from the stack when the wind is 4 m/s from the north during a class *C* stability situation. At a later date, another plant is built 200 m to the west of the original plant. It burns 4000 lb/hr of fuel oil that contains 0.5 percent sulfur. The second plant has an effective stack height of 60 m, and it has no SO_2 emission controls. For the same atmospheric conditions listed above, estimate the percentage of increase in SO_2 concentration at the downwind site due to the second plant.
- 4-33. The concentration of hydrogen sulfide, H_2S , is 55 ppb at a location 150 m downwind from an abandoned oil well. What is the rate of H_2S emission from the well if the winds are 2.7 m/s on a sunny June afternoon, in grams per second? Assume ground-level emission and rural conditions.
- 4-34. A fire burning at ground level is emitting nitric oxide at a rate of 3.6 b/s. The fire is assumed to be a point source with no effective rise of the plume. Determine the

concentration of NO directly downwind at a distance of 2.5 km under the following atmospheric conditions: (a) overcast night, 6 m/s wind speed; (b) clear night, 3 m/s wind speed; and (c) partly cloudy afternoon, 4 m/s wind speed. Assume rural conditions.

- 4-35.** Sulfur dioxide is emitted at a rate of 0.9 kg/s into a class *B* stability atmosphere which has a potential temperature gradient of $-0.010^{\circ}\text{C}/\text{m}$. the SO_2 concentration in the stack gas is $4.0 \text{ g}/\text{m}^3$, and the stack gas temperature and pressure are 175°C and 980 mbar, respectively. The wind speed is 4.8 m/s and its temperature is 18°C . The stack diameter at the top is 5.5 m, and it is assumed that other properties of the stack gas are the same as for air. Determine the plume rise above the stack in meters, using: (a) the Holland equation, (4-19); (b) the Briggs equations, (Table 4-6); and (c) the modified Concawe equation, (4-21).
- 4-36.** Reconsider Problem 4-35(a) and (c), with the following changes. The stability class is *C* and the potential temperature gradient is $0.010^{\circ}\text{C}/\text{m}$.
- 4-37.** Prove that the last term of Equation (4-19) can be replaced by the quantity $0.096 Q_h / V_s d$.
- 4-38.** In Example 4-6 the effective plume rise is calculated for a neutral atmosphere by use of two empirical formulas. Use these formulas to compute the plume rise for the same conditions, except that the atmosphere is (a) moderately stable with a potential temperature gradient of $0.003 \text{ K}/\text{m}$, and (b) stable with a potential temperature gradient of $0.008 \text{ K}/\text{m}$.
- 4-39.** Using Equations (4-18), (4-19), and (4-21), estimate the effective correction to stack height, Δh , in meters, for the following conditions: Q_h equal to 114,000 kJ/s; stack height of 250 m; gas exit velocity of 14.65 m/s; stack diameter of 9.13 m; wind velocity at stack exit of 7 m/s; atmospheric temperature of 280 K; gas exit temperature of 422 K; and atmospheric temperature gradient of $+0.534^{\circ}\text{C}/100 \text{ m}$.
- 4-40.** Two coal-fired furnaces discharge into one stack, 100 m tall. Each furnace is fired with coal at the rate of 250 tons every 24 hr. The combustion air is supplied at the rate of 10 lb for each pound of coal. The bases exit from the stack with a velocity of 20 ft/s at 350°F . The atmospheric temperature at the stack outlet is 60°F . The wind velocity is 10 mi/hr at 10 m. Assume neutral atmosphere. Calculate the plume-rise value in meters employing the (a) Holland, (b) modified Concawe, and (c) Briggs equations.
- 4-41.** Gas with the composition essentially of air exits from a stack with a velocity of 10 m/s. The gas temperature is 200°C . The stack diameter is 10 m. Determine the value of Δh for a neutral atmosphere by use of (a) Equation (4-18), (b) Equation (4-19), (c) Equation (4-20), (d) Equation (4-21), and (e) the Briggs Equations. The wind speed at the top of the stack is 5 m/s.
- 4-42.** The following data apply to a coal-burning steam power plant: stack height, 200 m; stack diameter, 9 m; coal-firing rate, $1.165 \times 10^6 \text{ kg}/\text{day}$; air supply, 12 lb air/lb coal; stack gas temperature, 150°C ; ambient air temperature at 350 m, 7°C ; heating value of coal, 5250 kJ/kg; sulfur content of coal, 3.1 percent; ash content of coal, 8 percent; ash carried up the stack, 80 percent; atmospheric conditions, cloudy, daytime; wind speed 6 m/s at 10-m height; temperature gradient, neutral. Calculate the effective stack height using (a) Equation (4-18), (b) Equation (4-19), (c) Equation (4-21), and (d) the Briggs equations.
- 4-43.** A PM_{10} concentration of $250 \text{ } \mu\text{g}/\text{m}^3$ was predicted using the Gaussian Equations and σ values shown in Figures 4-6 and 4-7 and is assumed to be a 1-hour concentration (for regulatory purposes). Determine the concentration for a (a) 3-hour, (b) 8-hour, and (c) 24-hour averaging time. (d) Does the concentration exceed the 24-hour PM_{10} standard?

- 4-44.** A long line of agricultural waste burning in a rural field may be considered an infinite line source. On a clear fall afternoon, the wind speed is 4.5 m/s. Determine the particulate concentration for the small airborne particles at 600 m downwind, if the source strength is 0.23 g/m.s.
- 4-45.** The traffic density for an urban interstate highway is 10,000 vehicles/hr and the average vehicle speed is 40 km/hr. The wind speed perpendicular to the highway is 3 m/s. The average carbon monoxide emission per vehicle is 10 g/km. (a) For an overcast day, estimate the CO concentration 0.1 km downwind from the highway. (b) Does the concentration found in part (a) represent a health hazard as indicated by the 1-hour primary air quality standard for carbon monoxide? (c) Does it represent a health hazard based on the 8-hour standard?
- 4-46.** Due to the sudden bursting of a tank located on the ground, 50 kg of a highly volatile organic compound were released as a vapor. Under F stability with a windspeed of 1 m/s and rural conditions, (a) plot a graph of the concentration as a function of time that would be experienced by a person located at a distance of 1 km downwind of the source. Use times of 5, 10, 20, 30, 40, 50, and 60 minutes. (b) what is the maximum concentration and time of occurrence as experienced by the person?

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