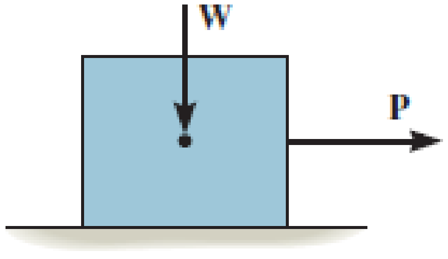
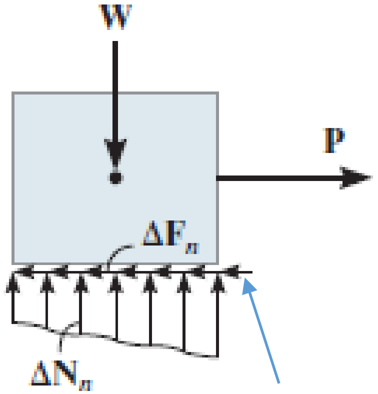
STATICS-FRICTION

Characteristics of Dry Friction

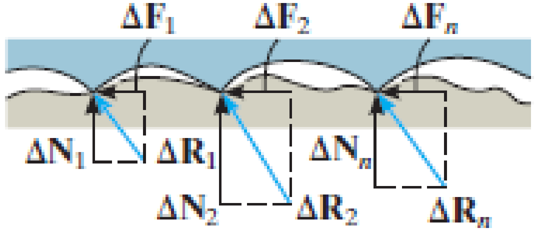
- *Friction* is a force that resists the movement of two contacting surfaces that slide relative to one another
- This force always acts *tangent* to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.



(a)



(b)



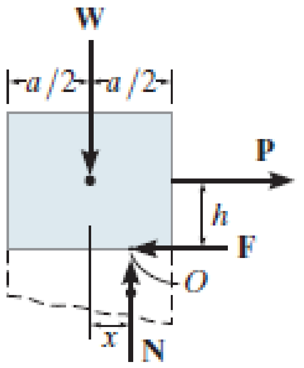
(c)

the floor exerts an uneven *distribution* of both *normal force* **N***n* and *frictional force* **F***n* along the contacting surface.

the frictional forces act to the left to prevent the applied force **P** from moving the block to the right

It can be seen that many microscopic irregularities exist between the two surfaces and, as a result,
reactive forces **R***n* are developed at each point of contact

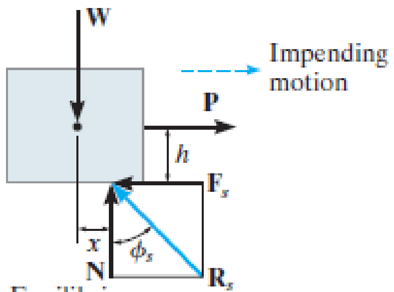
- **Equilibrium.** The effect of the *distributed* normal and frictional loadings is indicated by their *resultants* **N** and **F** on the free-body diagram.



Resultant Normal and Frictional Forces
(d)

moment equilibrium about point *O* is satisfied if
$Wx = Ph$ or $x = Ph/W$.

- **Impending Motion.** In cases where the surfaces of contact are rather "slippery," the frictional force **F** may *not* be great enough to balance **P** , and consequently the block will tend to slip. In other words, as *P* is slowly increased, *F* correspondingly increases until it attains a certain *maximum value Fs*, called the *limiting static frictional force*



Equilibrium
(e)

$$\boxed{F_s = \mu_s N}$$

$\boldsymbol{\mu_s}$ is called the *coefficient of static friction* .

$\boldsymbol{F_s}$ is called the *Limiting static friction force*.

When the block is on the *verge of sliding* , the normal force **N** and frictional force **F***s* combine to create a resultant **R***s*,

angle of static friction

$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$$

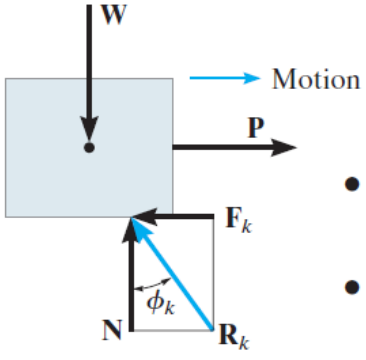
Table 8–1 Typical Values for μ_s

Contact Materials	Coefficient of Static Friction (μ_s)
Metal on ice	0.03–0.05
Wood on wood	0.30–0.70
Leather on wood	0.20–0.50
Leather on metal	0.30–0.60
Aluminum on aluminum	1.10–1.70

- **Motion.** If the magnitude of **P** acting on the block is increased so that it becomes slightly greater than *Fs*, the frictional force at the contacting surface will drop to a smaller value *Fk*, called the *kinetic frictional force* .
- The block will begin to slide with increasing speed. Typical values for μ_k are approximately 25 percent *smaller* than μ_s

Magnitude of *kinetic frictional force* is proportional to normal force

$$\boxed{F_k = \mu_k N}$$

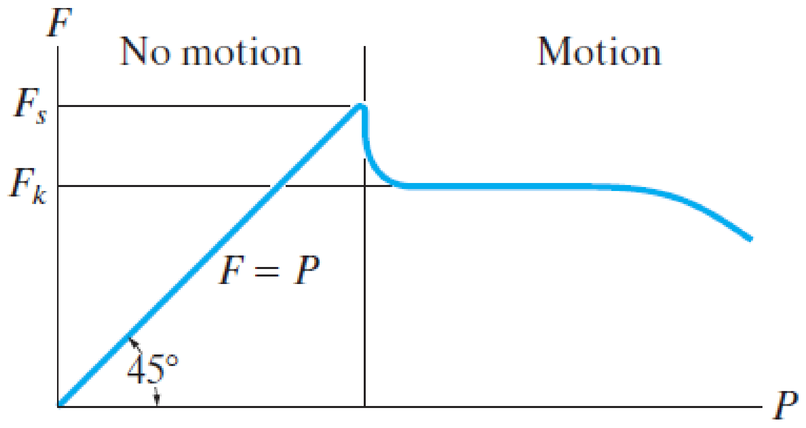


(a)

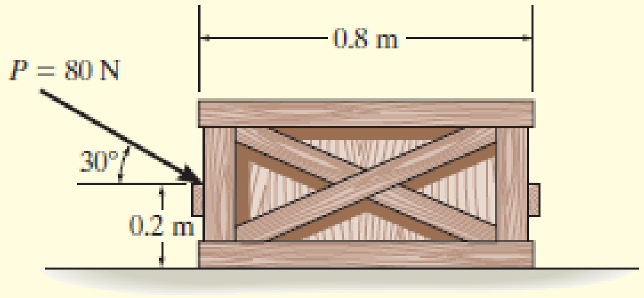
, $\phi_s \geq \phi_k$.

angle of kinetic friction

- F is a *static frictional force* if equilibrium is maintained.
- F is a *limiting static frictional force* F_s when it reaches a maximum value needed to maintain equilibrium.
- F is a *kinetic frictional force* F_k when sliding occurs at the contacting surface.

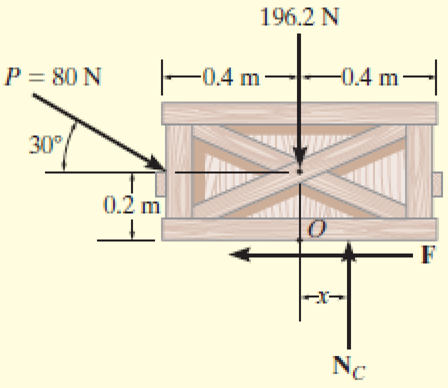


The uniform crate shown in Fig. 8–7*a* has a mass of 20 kg. If a force $P = 80$ N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

Free-Body Diagram. As shown in Fig. 8–7*b*, the *resultant* normal force $\mathbf{N}_C$ must act a distance x from the crate's center line in order to counteract the tipping effect caused by **P**. There are *three unknowns*, F, N_C, and x, which can be determined strictly from the *three* equations of equilibrium.



(b)

Equations of Equilibrium.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 80 \cos 30° \text{ N} - F = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -80 \sin 30° \text{ N} + N_C - 196.2 \text{ N} = 0$$

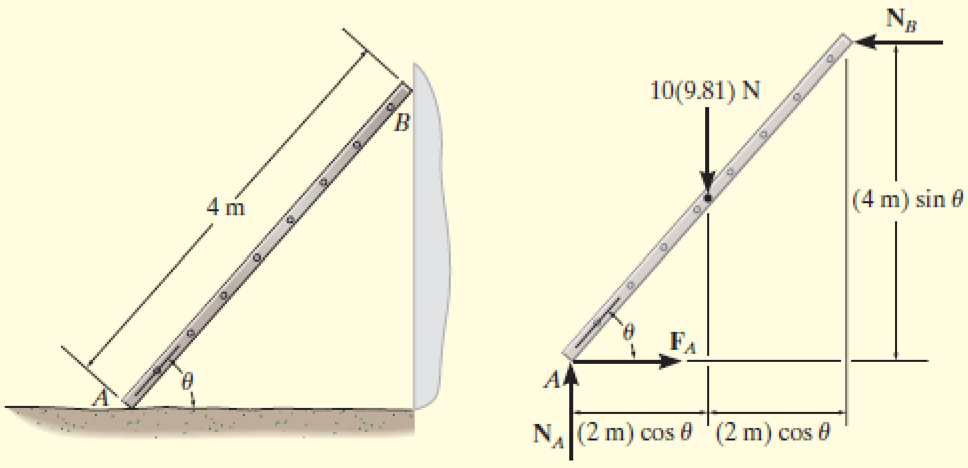
$$\circlearrowleft +\Sigma M_O = 0; \quad 80 \sin 30° \text{ N}(0.4 \text{ m}) - 80 \cos 30° \text{ N}(0.2 \text{ m}) + N_C(x) = 0$$

Solving,

$$F = 69.3 \text{ N}$$
$$N_C = 236.2 \text{ N}$$
$$x = -0.00908 \text{ m} = -9.08 \text{ mm}$$

Since x is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since $x < 0.4$ m. Also, the *maximum* frictional force which can be developed at the surface of contact is $F_{max} = \mu_s N_C = 0.3(236.2 \text{ N}) = 70.9$ N. Since $F = 69.3 \text{ N} < 70.9$ N, the crate will *not slip*, although it is very close to doing so.

The uniform 10-kg ladder in Fig. 8–9*a* rests against the smooth wall at *B*, and the end *A* rests on the rough horizontal plane for which the coefficient of static friction is $\mu_s = 0.3$. Determine the angle of inclination θ of the ladder and the normal reaction at *B* if the ladder is on the verge of slipping.



(a) (b)

SOLUTION

Free-Body Diagram. As shown on the free-body diagram, Fig. 8–9*b*, the frictional force $\mathbf{F}_A$ must act to the right since impending motion at *A* is to the left.

Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then $F_A = \mu_s N_A = 0.3N_A$. By inspection, N_A can be obtained directly.

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 10(9.81)\text{ N} = 0 \qquad N_A = 98.1\text{ N}$$

Using this result, $F_A = 0.3(98.1\text{ N}) = 29.43\text{ N}$. Now N_B can be found.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 29.43\text{ N} - N_B = 0$$

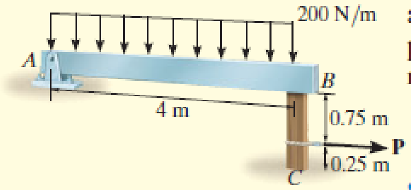
$$N_B = 29.43\text{ N} = 29.4\text{ N} \qquad \textit{Ans.}$$

Finally, the angle θ can be determined by summing moments about point *A*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad (29.43\text{ N})(4\text{ m})\sin\theta - [10(9.81)\text{ N}](2\text{ m})\cos\theta = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = 1.6667$$

$$\theta = 59.04^\circ = 59.0^\circ \qquad \textit{Ans.}$$



(a)

Beam AB is subjected to a uniform load of 200 N/m and is supported at B by post BC, Fig. 8–10a. If the coefficients of static friction at B and C are $\mu_B = 0.2$ and $\mu_C = 0.5$, determine the force $\mathbf{P}$ needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.

SOLUTION

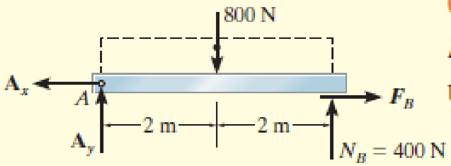
Free-Body Diagrams. The free-body diagram of the beam is shown in Fig. 8–10b. Applying $\Sigma M_A = 0$, we obtain $N_B = 400$ N. This result is shown on the free-body diagram of the post, Fig. 8–10c. Referring to this member, the *four* unknowns F_B, P, F_C, and N_C are determined from the *three* equations of equilibrium and *one* frictional equation applied either at B or C.

Equations of Equilibrium and Friction.

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_B - F_C = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_C - 400 \text{ N} = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -P(0.25 \text{ m}) + F_B(1 \text{ m}) = 0 \qquad (3)$$



(b)

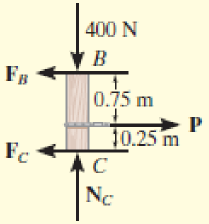
(Post Slips at B and Rotates about C.) This requires $F_C \leq \mu_C N_C$ and

$$F_B = \mu_B N_B; \qquad F_B = 0.2(400 \text{ N}) = 80 \text{ N}$$

Using this result and solving Eqs. 1 through 3, we obtain

$$P = 320 \text{ N}$$
$$F_C = 240 \text{ N}$$
$$N_C = 400 \text{ N}$$

Since $F_C = 240 \text{ N} > \mu_C N_C = 0.5(400 \text{ N}) = 200 \text{ N}$, slipping at C occurs. Thus the other case of movement must be investigated.



(c)

Fig. 8–10

(Post Slips at C and Rotates about B.) Here $F_B \leq \mu_B N_B$ and

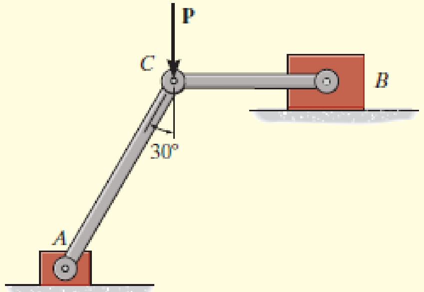
$$F_C = \mu_C N_C; \qquad F_C = 0.5 N_C \qquad (4)$$

Solving Eqs. 1 through 4 yields

$$P = 267 \text{ N} \qquad \textit{Ans.}$$
$$N_C = 400 \text{ N}$$
$$F_C = 200 \text{ N}$$
$$F_B = 66.7 \text{ N}$$

Obviously, this case occurs first since it requires a *smaller* value for P.

Blocks A and B have a mass of 3 kg and 9 kg, respectively, and are connected to the weightless links shown in Fig. 8–11a. Determine the largest vertical force $\mathbf{P}$ that can be applied at the pin C without causing any movement. The coefficient of static friction between the blocks and the contacting surfaces is $\mu_s = 0.3$.



(a)

SOLUTION

Free-Body Diagram. The links are two-force members and so the free-body diagrams of pin C and blocks A and B are shown in Fig. 8–11b. Since the horizontal component of $\mathbf{F}_{AC}$ tends to move block A to the left, $\mathbf{F}_A$ must act to the right. Similarly, $\mathbf{F}_B$ must act to the left to oppose the tendency of motion of block B to the right, caused by $\mathbf{F}_{BC}$. There are seven unknowns and six available force equilibrium equations, two for the pin and two for each block, so that *only one* frictional equation is needed.

Equations of Equilibrium and Friction. The force in links AC and BC can be related to P by considering the equilibrium of pin C.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\cos 30^\circ - P = 0; \qquad F_{AC} = 1.155P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1.155P \sin 30^\circ - F_{BC} = 0; \qquad F_{BC} = 0.5774P$$

Using the result for F_{AC}, for block A,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A - 1.155P \sin 30^\circ = 0; \quad F_A = 0.5774P \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 1.155P \cos 30^\circ - 3(9.81\text{ N}) = 0;$$

$$N_A = P + 29.43\text{ N} \qquad (2)$$

Using the result for F_{BC}, for block B,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad (0.5774P) - F_B = 0; \qquad F_B = 0.5774P \qquad (3)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 9(9.81)\text{ N} = 0; \qquad N_B = 88.29\text{ N}$$

Movement of the system may be caused by the initial slipping of *either* block A or block B. If we assume that block A slips first, then

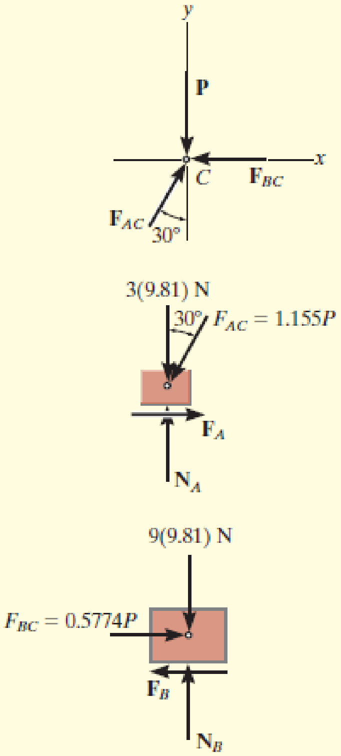
$$F_A = \mu_s N_A = 0.3N_A \qquad (4)$$

Substituting Eqs. 1 and 2 into Eq. 4,

$$0.5774P = 0.3(P + 29.43)$$

$$P = 31.8\text{ N} \qquad \textit{Ans.}$$

Substituting this result into Eq. 3, we obtain $F_B = 18.4$ N. Since the maximum static frictional force at B is $(F_B)_{max} = \mu_s N_B = 0.3(88.29\text{ N}) = 26.5\text{ N} > F_B$, block B will not slip. Thus, the above assumption is correct. Notice that if the inequality were not satisfied, we would have to assume slipping of block B and then solve for P.

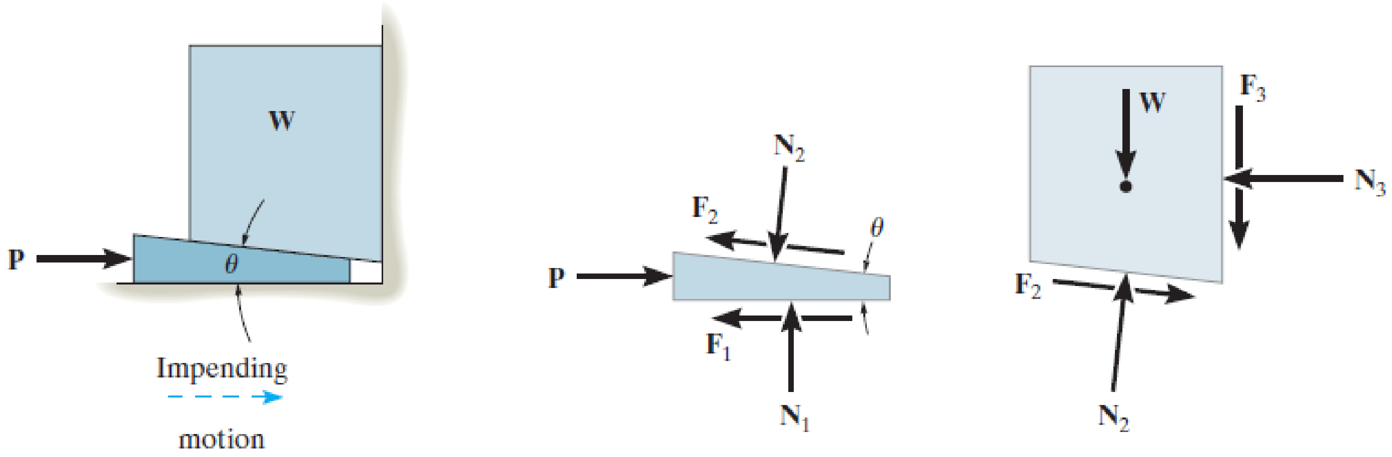


(b)

Fig. 8–11

Wedges

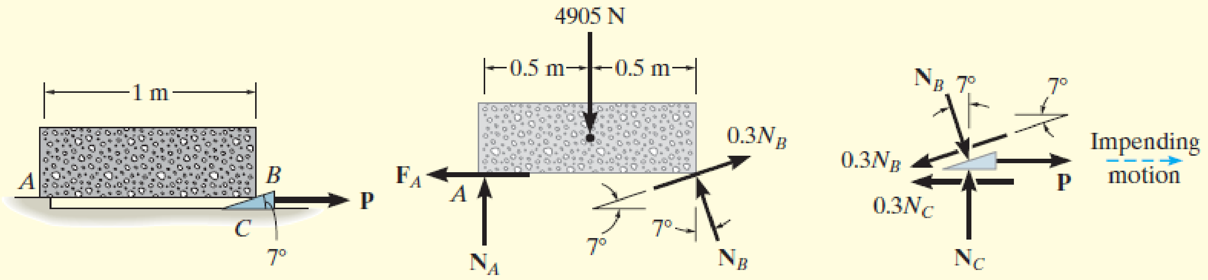
- A *wedge* is a simple machine that is often used to transform an applied force into much larger forces, directed at approximately right angles to the applied force. Wedges also can be used to slightly move or adjust heavy loads.



There are seven unknowns, consisting of the applied force P , needed to cause motion of the wedge, and six normal and frictional forces.

The seven available equations consist of four force equilibrium equations, $\sum Fx= 0$, $\sum Fy= 0$ applied to the wedge and block, and three frictional equations, $F = \mu N$, applied at each surface of contact.

The uniform stone in Fig. 8–13*a* has a mass of 500 kg and is held in the horizontal position using a wedge at B. If the coefficient of static friction is $\mu_s = 0.3$ at the surfaces of contact, determine the minimum force P needed to remove the wedge. Assume that the stone does not slip at A.



SOLUTION

The minimum force P requires $F = \mu_s N$ at the surfaces of contact with the wedge. The free-body diagrams of the stone and wedge are shown in Fig. 8–13*b*. On the wedge the friction force opposes the impending motion, and on the stone at A, $F_A \leq \mu_s N_A$, since slipping does not occur there. There are five unknowns. Three equilibrium equations for the stone and two for the wedge are available for solution. From the free-body diagram of the stone,

$$\circlearrowleft + \Sigma M_A = 0; \quad -4905\text{ N}(0.5\text{ m}) + (N_B \cos 7^\circ\text{ N})(1\text{ m}) + (0.3N_B \sin 7^\circ\text{ N})(1\text{ m}) = 0$$

$$N_B = 2383.1\text{ N}$$

Using this result for the wedge, we have

$$+\uparrow \Sigma F_y = 0; \quad N_C - 2383.1 \cos 7^\circ\text{ N} - 0.3(2383.1 \sin 7^\circ\text{ N}) = 0$$

$$N_C = 2452.5\text{ N}$$

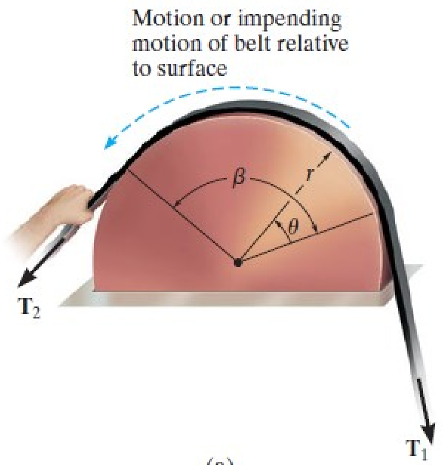
$$\xrightarrow{+} \Sigma F_x = 0; \quad 2383.1 \sin 7^\circ\text{ N} - 0.3(2383.1 \cos 7^\circ\text{ N}) + P - 0.3(2452.5\text{ N}) = 0$$

$$P = 1154.9\text{ N} = 1.15\text{ kN} \qquad \textit{Ans.}$$

NOTE: Since P is positive, indeed the wedge must be pulled out. If P were zero, the wedge would remain in place (self-locking) and the frictional forces developed at B and C would satisfy $F_B < \mu_s N_B$ and $F_C < \mu_s N_C$.

Frictional Forces on Flat Belts

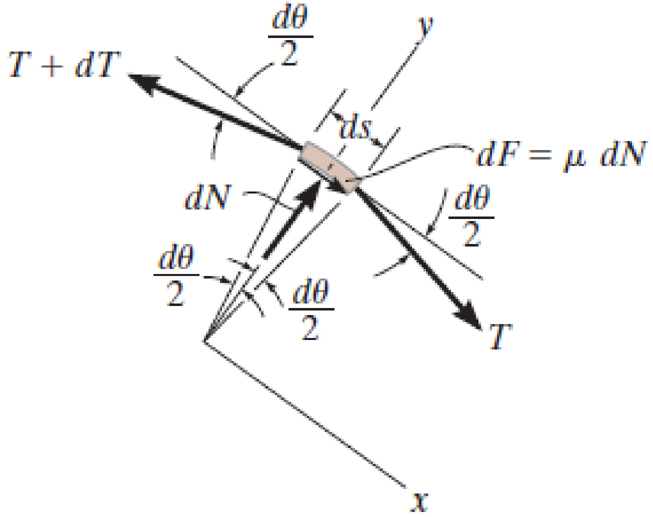
- Consider the flat belt shown in Figure which passes over a fixed curved surface. The total angle of belt to surface contact in radians is , β and the coefficient of friction between the two surfaces is μ.
- We wish to determine the tension *T*2 in the belt, which is needed to pull the belt counterclockwise over the surface, and thereby overcome both the frictional forces at the surface of contact and the tension *T*1 in the other end of the belt. Obviously, $T2 > T1$.



(a)

(b)

A free-body diagram of the belt segment in contact with the surface is shown



$$\searrow + \Sigma F_x = 0; \qquad T\cos\left(\frac{d\theta}{2}\right) + \mu\, dN - (T + dT)\cos\left(\frac{d\theta}{2}\right) = 0 \qquad (1)$$

$$+\nearrow \Sigma F_y = 0; \qquad dN - (T + dT)\sin\left(\frac{d\theta}{2}\right) - T\sin\left(\frac{d\theta}{2}\right) = 0 \qquad (2)$$

Since *d*u is of *infinitesimal size* , sin($d\theta/2$) = $d\theta/2$ and cos($d\theta/2$) = 1 . Also, the *product* of the two infinitesimals *dT* and $d\theta/2$ may be neglected

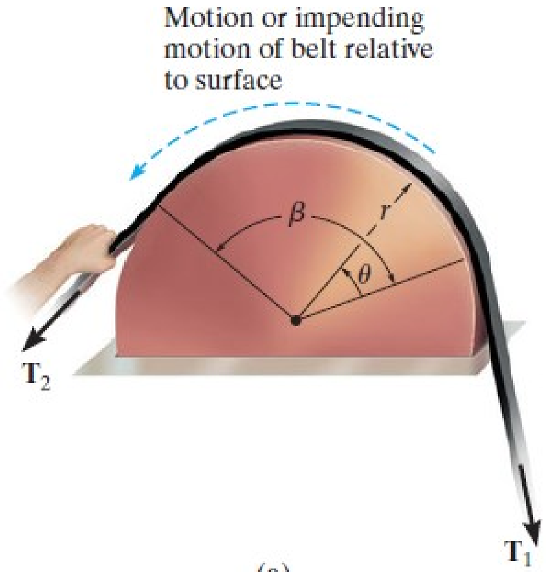
$$\mu\, dN = dT \qquad (3)$$

$$dN = T\, d\theta \qquad (4)$$

Eliminating dN yields

$$\frac{dT}{T} = \mu\, d\theta \qquad (5)$$

$$\frac{dT}{T} = \mu \, d\theta$$



noting that $T = T_1$ at $\theta = 0$ and $T = T_2$ at $\theta = \beta$, yields

$$\int_{T_1}^{T_2} \frac{dT}{T} = \mu \int_0^{\beta} d\theta$$

$$\ln \frac{T_2}{T_1} = \mu\beta$$

Solving for T_2, we obtain

$$\boxed{T_2 = T_1 e^{\mu\beta}}$$

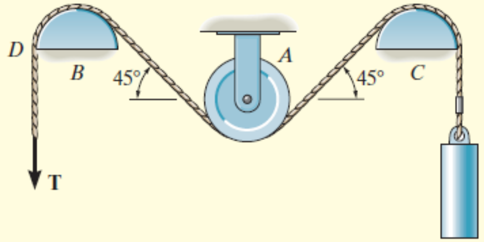
T_2, T_1 = belt tensions; T_1 opposes the direction of motion (or impending motion) of the belt measured relative to the surface, while T_2 acts in the direction of the relative belt motion (or impending motion); because of friction, $T_2 > T_1$

μ = coefficient of static or kinetic friction between the belt and the surface of contact

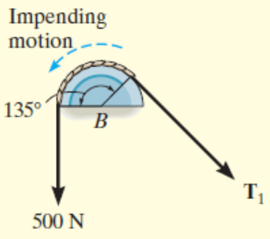
β = angle of belt to surface contact, measured in radians

e = 2.718 . . . , base of the natural logarithm

The maximum tension that can be developed in the cord shown in Fig. 8–19*a* is 500 N. If the pulley at A is free to rotate and the coefficient of static friction at the fixed drums B and C is $\mu_s = 0.25$, determine the largest mass of the cylinder that can be lifted by the cord.



(a)



(b)

SOLUTION

Lifting the cylinder, which has a weight $W = mg$, causes the cord to move counterclockwise over the drums at B and C; hence, the maximum tension T_2 in the cord occurs at D. Thus, $F = T_2 = 500$ N. A section of the cord passing over the drum at B is shown in Fig. 8–19*b*. Since $180° = \pi$ rad the angle of contact between the drum and the cord is $\beta = (135°/180°)\pi = 3\pi/4$ rad. Using Eq. 8–6, we have

$$T_2 = T_1 e^{\mu_s \beta}; \qquad 500\text{ N} = T_1 e^{0.25[(3/4)\pi]}$$

Hence,

$$T_1 = \frac{500\text{ N}}{e^{0.25[(3/4)\pi]}} = \frac{500\text{ N}}{1.80} = 277.4\text{ N}$$

Since the pulley at A is free to rotate, equilibrium requires that the tension in the cord remains the *same* on both sides of the pulley.

The section of the cord passing over the drum at C is shown in Fig. 8–19*c*. The weight $W < 277.4$ N. Why? Applying Eq. 8–6, we obtain

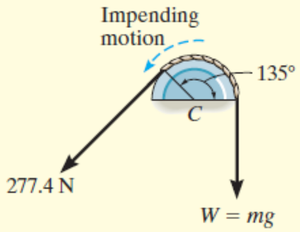
$$T_2 = T_1 e^{\mu_s \beta}; \qquad 277.4\text{ N} = W e^{0.25[(3/4)\pi]}$$

$$W = 153.9\text{ N}$$

so that

$$m = \frac{W}{g} = \frac{153.9\text{ N}}{9.81\text{ m/s}^2}$$

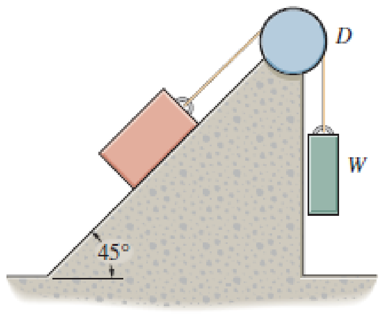
$$= 15.7\text{ kg} \qquad \textit{Ans.}$$



(c)

Fig. 8–19

Determine the maximum and the minimum values of weight W which may be applied without causing the 50-lb block to slip. The coefficient of static friction between the block and the plane is $\mu_s = 0.2$, and between the rope and the drum D $\mu_s' = 0.3$.

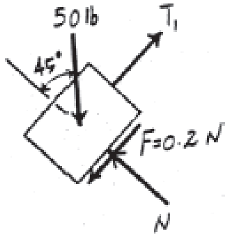


SOLUTION

Equations of Equilibrium and Friction: Since the block is on the verge of sliding up or down the plane, then, $F = \mu_s N = 0.2N$. If the block is on the verge of sliding up the plane [FBD (a)],

$\nwarrow + \Sigma F_{y'} = 0;$ $\qquad N - 50 \cos 45° = 0 \qquad N = 35.36 \text{ lb}$

$\nearrow + \Sigma F_{x'} = 0;$ $\qquad T_1 - 0.2(35.36) - 50 \sin 45° = 0 \qquad T_1 = 42.43 \text{ lb}$



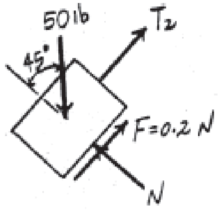
(a)

If the block is on the verge of sliding down the plane [FBD (b)],

$\nwarrow + \Sigma F_{y'} = 0;$ $\qquad N - 50 \cos 45° = 0 \qquad N = 35.36 \text{ lb}$

$\nearrow + \Sigma F_{x'} = 0;$ $\qquad T_2 + 0.2(35.36) - 50 \sin 45° = 0 \qquad T_2 = 28.28 \text{ lb}$

Frictional Force on Flat Belt: Here, $\beta = 45° + 90° = 135° = \dfrac{3\pi}{4}$ rad. If the block is on the verge of sliding up the plane, $T_1 = 42.43$ lb and $T_2 = W$.



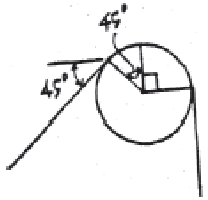
(b)

$$T_2 = T_1 e^{\mu\beta}$$

$$W = 42.43e^{0.3\left(\frac{3\pi}{4}\right)}$$

$$= 86.02 \text{ lb} = 86.0 \text{ lb}$$ **Ans.**

If the block is on the verge of sliding down the plane, $T_1 = W$ and $T_2 = 28.28$ lb.



$$T_2 = T_1 e^{\mu\beta}$$

$$28.28 = We^{0.3\left(\frac{3\pi}{4}\right)}$$

$$W = 13.95 \text{ lb} = 13.9 \text{ lb}$$ **Ans.**