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Chapter 4
TRANSIENT HEAT CONDUCTION

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Objectives

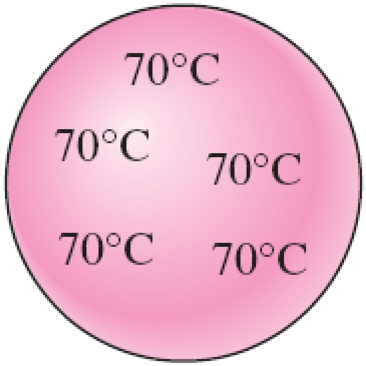
- Assess when the spatial variation of temperature is negligible, and temperature varies nearly uniformly with time, making the simplified lumped system analysis applicable
- Obtain analytical solutions for transient one-dimensional conduction problems in rectangular, cylindrical, and spherical geometries using the method of separation of variables, and understand why a one-term solution is usually a reasonable approximation
- Solve the transient conduction problem in large mediums using the similarity variable, and predict the variation of temperature with time and distance from the exposed surface
- Construct solutions for multi-dimensional transient conduction problems using the product solution approach.

LUMPED SYSTEM ANALYSIS

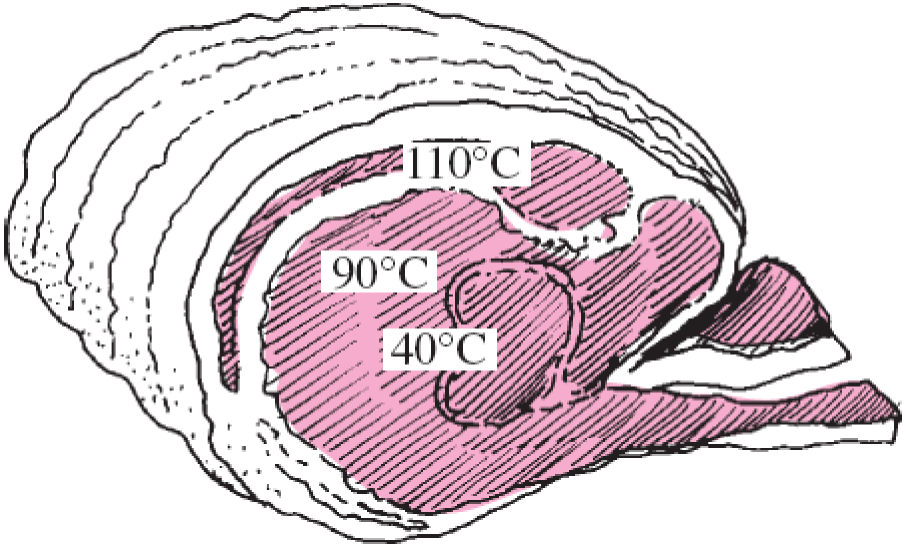
Interior temperature of some bodies remains essentially uniform at all times during a heat transfer process.

The temperature of such bodies can be taken to be a function of time only, $T(t)$.

Heat transfer analysis that utilizes this idealization is known as **lumped system analysis**.



(a) Copper ball



(b) Roast beef

A small copper ball can be modeled as a lumped system, but a roast beef cannot.

$$\begin{pmatrix}\text{Heat transfer into the body}\\ \text{during } dt\end{pmatrix} = \begin{pmatrix}\text{The increase in the}\\ \text{energy of the body}\\ \text{during } dt\end{pmatrix}$$

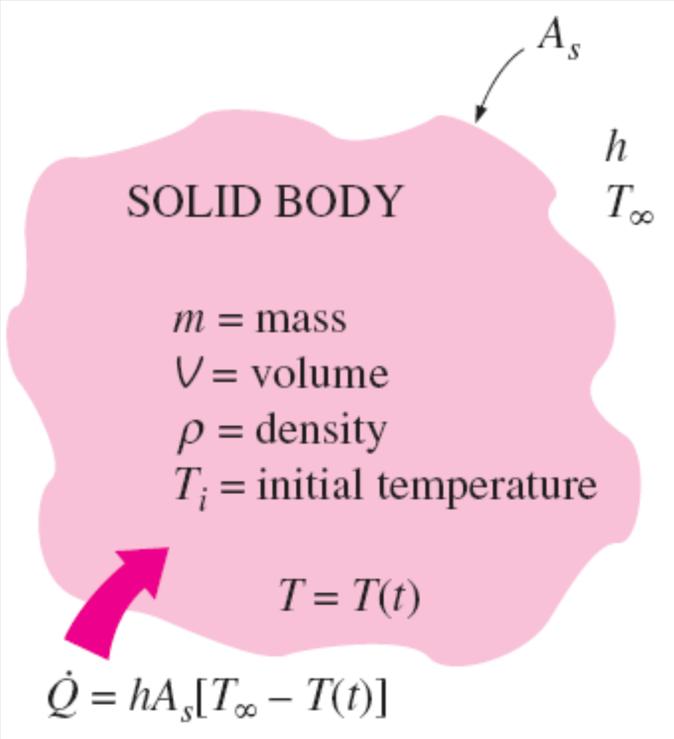
$$hA_s(T_\infty - T)\, dt = mc_p\, dT$$

$$m = \rho V \qquad dT = d(T - T_\infty)$$

$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p}\, dt$$

Integrating with
$T = T_i$ at $t = 0$
$T = T(t)$ at $t = t$

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}\, t$$



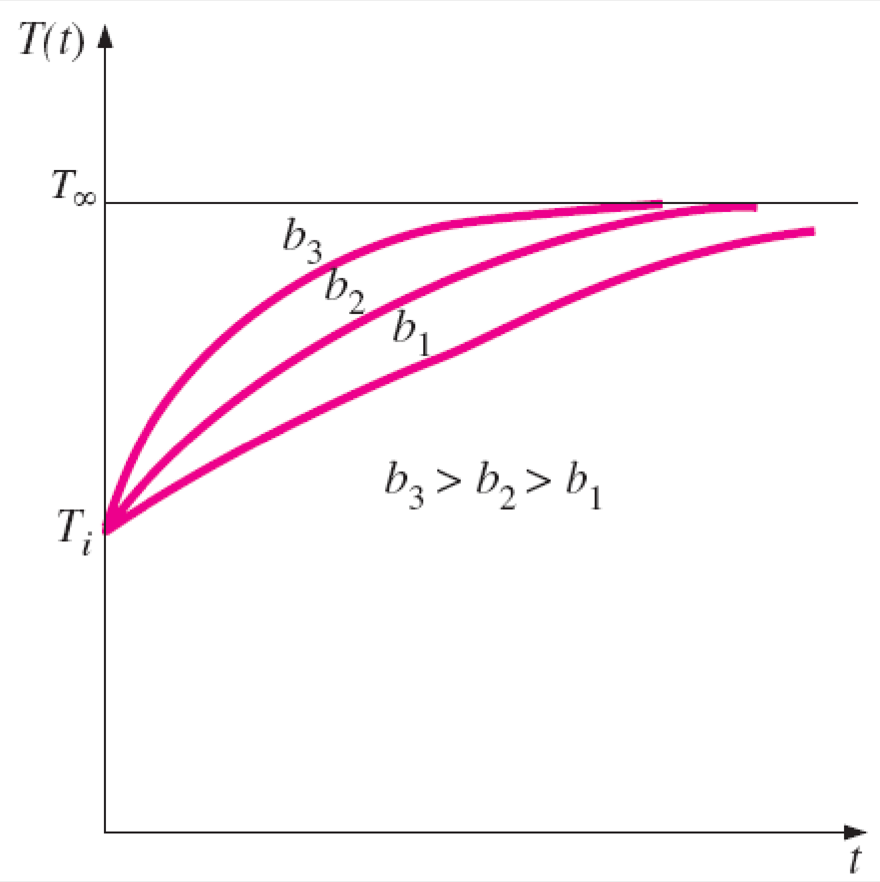
The geometry and parameters involved in the lumped system analysis.

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad b = \frac{hA_s}{\rho V c_p} \quad (1/\text{s})$$

time constant

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{hA_s}{\rho V c_p}$$



The temperature of a lumped system approaches the environment temperature as time gets larger.

- This equation enables us to determine the temperature $T(t)$ of a body at time t, or alternatively, the time t required for the temperature to reach a specified value $T(t)$.
- The temperature of a body approaches the ambient temperature T_∞ exponentially.
- The temperature of the body changes rapidly at the beginning, but rather slowly later on. A large value of b indicates that the body approaches the environment temperature in a short time

$$\dot{Q}(t) = hA_s[T(t) - T_\infty] \qquad \text{(W)}$$

The *rate* of convection heat transfer between the body and its environment at time *t*

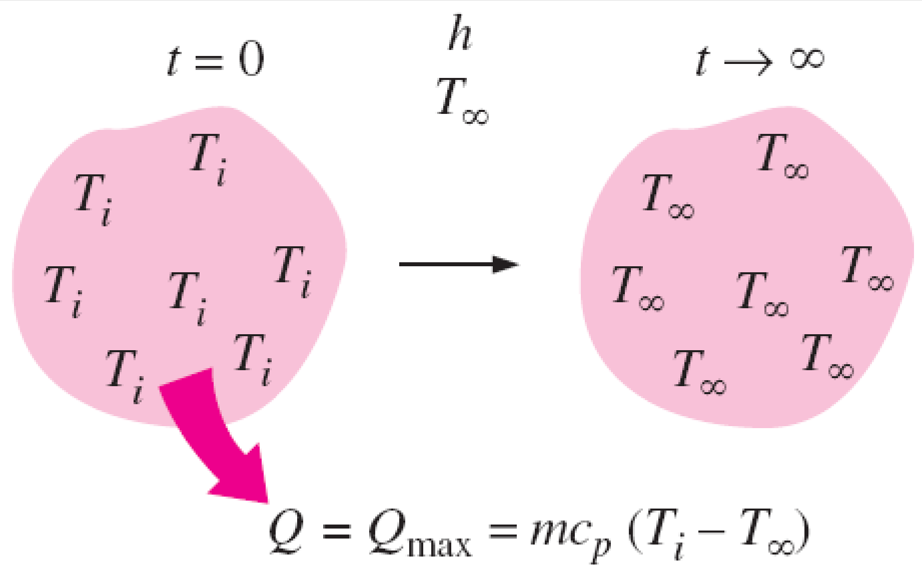
$$Q = mc_p[T(t) - T_i] \qquad \text{(kJ)}$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to t

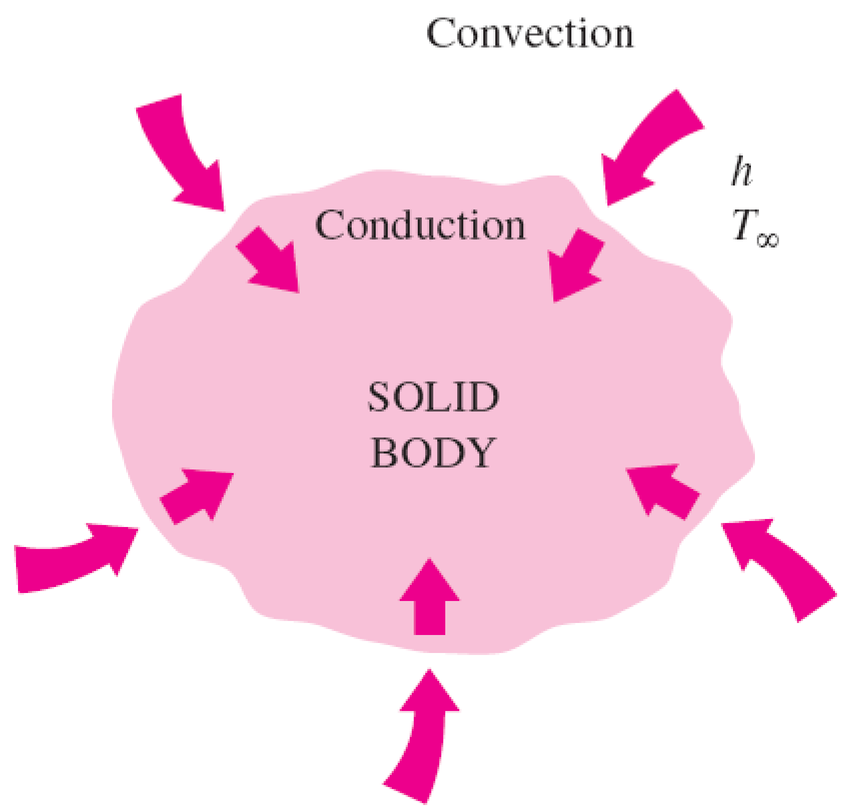
$$Q_{\max} = mc_p(T_\infty - T_i) \qquad \text{(kJ)}$$

The *maximum* heat transfer between the body and its surroundings

Heat transfer to or from a body reaches its maximum value when the body reaches the environment temperature.



Criteria for Lumped System Analysis



$$\text{Bi} = \frac{\text{heat convection}}{\text{heat conduction}}$$

$$L_c = \frac{V}{A_s}$$ Characteristic length

$$\text{Bi} = \frac{hL_c}{k}$$ Biot number

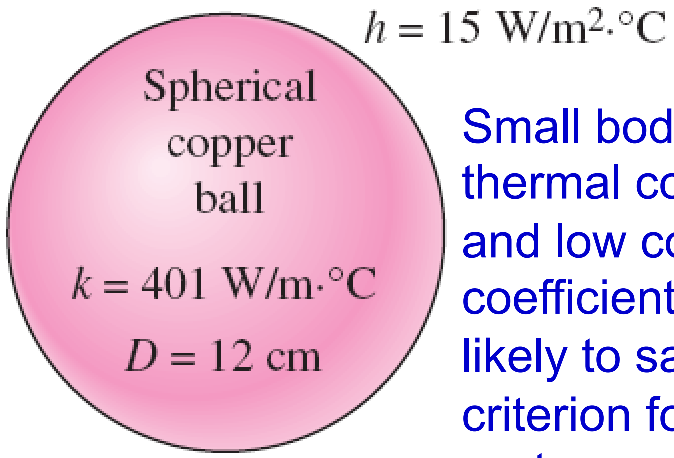
Lumped system analysis is *applicable* if

$$\text{Bi} \leq 0.1$$

When Bi $\leq$ 0.1, the temperatures within the body relative to the surroundings (i.e., $T - T_\infty$) remain within 5 percent of each other.

$$\text{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

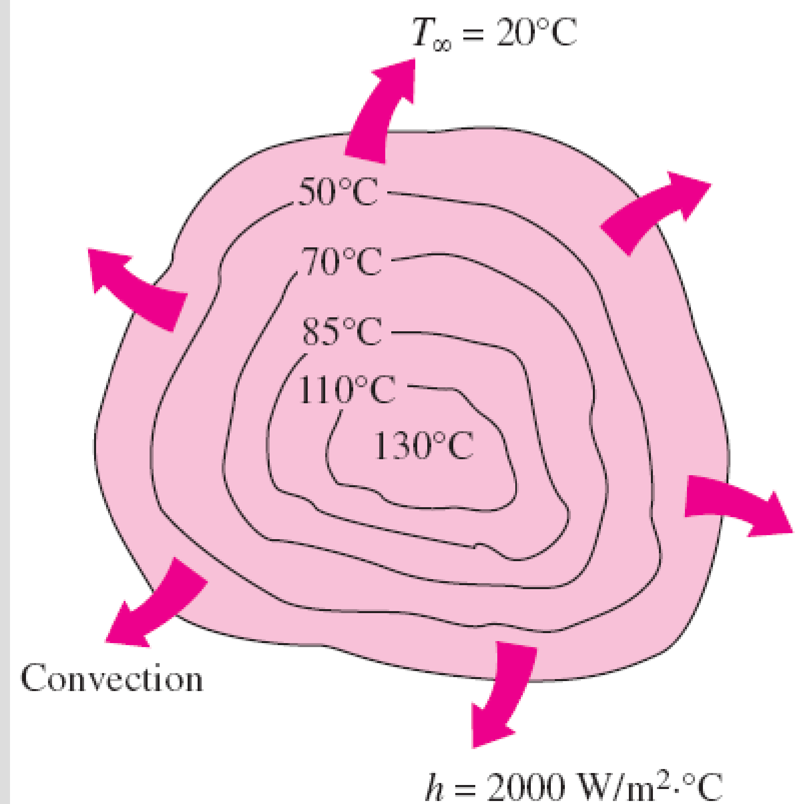
$$\text{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$



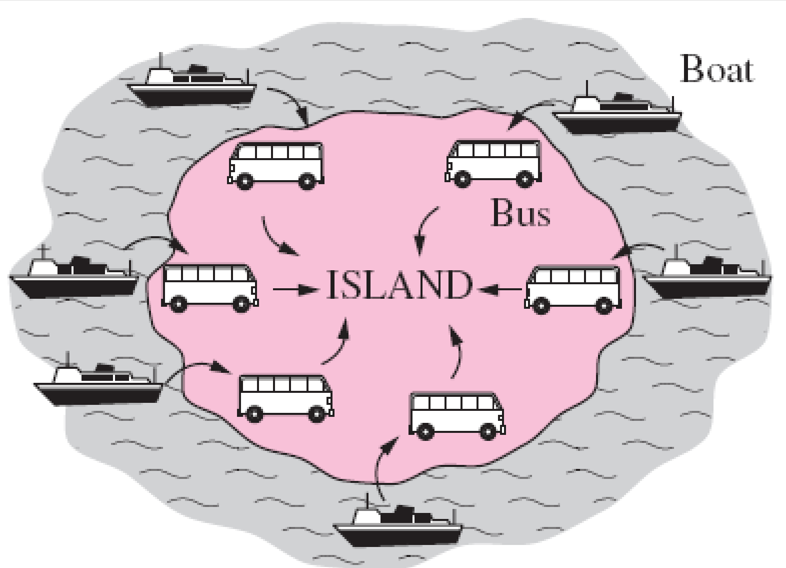
Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis.

$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = 0.02 \text{ m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$

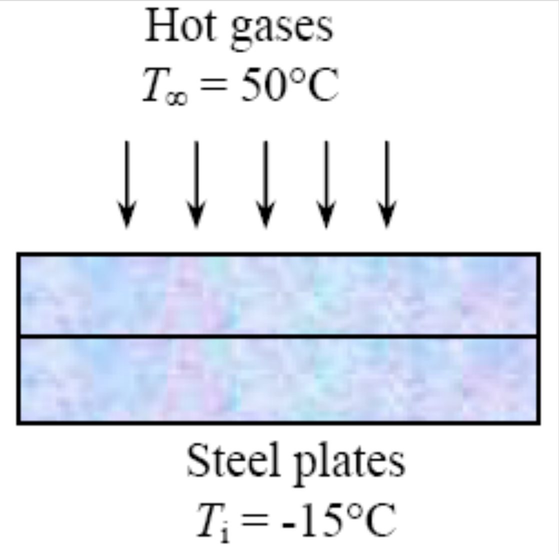


When the convection coefficient *h* is high and *k* is low, large temperature differences occur between the inner and outer regions of a large solid.



Analogy between heat transfer to a solid and passenger traffic to an island.

4–110 Consider two 2-cm-thick large steel plates (k = 43 W/m · °C and $\alpha = 1.17 \times 10^{-5}$ m²/s) that were put on top of each other while wet and left outside during a cold winter night at −15°C. The next day, a worker needs one of the plates, but the plates are stuck together because the freezing of the water between the two plates has bonded them together. In an effort to melt the ice between the plates and separate them, the worker takes a large hair dryer and blows hot air at 50°C all over the exposed surface of the plate on the top. The convection heat transfer coefficient at the top surface is estimated to be 40 W/m² · °C. Determine how long the worker must keep blowing hot air before the two plates separate.



Analysis The characteristic length of the plates and the Biot number are

$$L_c = \frac{V}{A_s} = L = 0.02 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(40 \text{ W/m}^2.°\text{C})(0.02 \text{ m})}{(43 \text{ W/m}.°\text{C})} = 0.019 < 0.1$$

Since $\text{Bi} < 0.1$, the lumped system analysis is applicable. Therefore,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{40 \text{ W/m}^2.°\text{C}}{(3.675 \times 10^6 \text{ J/m}^3.°\text{C})(0.02 \text{ m})} = 0.000544 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{0 - 50}{-15 - 50} = e^{-(0.000544 \text{ s}^{-1})t} \longrightarrow t = \mathbf{482 \text{ s} = 8.0 \text{ min}}$$

where $\rho c_p = \dfrac{k}{\alpha} = \dfrac{43 \text{ W/m}.°\text{C}}{1.17 \times 10^{-5} \text{ m}^2\text{/s}} = 3.675 \times 10^6 \text{ J/m}^3.°\text{C}$

EXAMPLE 4–2 Predicting the Time of Death

A person is found dead at 5 PM in a room whose temperature is 20°C. The temperature of the body is measured to be 25°C when found, and the heat transfer coefficient is estimated to be $h = 8\ \text{W/m}^2 \cdot °\text{C}$. Modeling the body as a 30-cm-diameter, 1.70-m-long cylinder, estimate the time of death of that person (Fig. 4–10).

Properties The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of $(37 + 25)/2 = 31°\text{C}$; $k = 0.617\ \text{W/m} \cdot °\text{C}$, $\rho = 996\ \text{kg/m}^3$, and $c_p = 4178\ \text{J/kg} \cdot °\text{C}$ (Table A–9).

Analysis The characteristic length of the body is

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.15\ \text{m})^2(1.7\ \text{m})}{2\pi(0.15\ \text{m})(1.7\ \text{m}) + 2\pi(0.15\ \text{m})^2} = 0.0689\ \text{m}$$

Then the Biot number becomes

$$\text{Bi} = \frac{hL_c}{k} = \frac{(8\ \text{W/m}^2 \cdot °\text{C})(0.0689\ \text{m})}{0.617\ \text{W/m} \cdot °\text{C}} = 0.89 > 0.1$$

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{8\ \text{W/m}^2 \cdot °\text{C}}{(996\ \text{kg/m}^3)(4178\ \text{J/kg} \cdot °\text{C})(0.0689\ \text{m})}$$
$$= 2.79 \times 10^{-5}\ \text{s}^{-1}$$

We now substitute these values into Eq. 4–4,

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25 - 20}{37 - 20} = e^{-(2.79 \times 10^{-5}\ \text{s}^{-1})t}$$

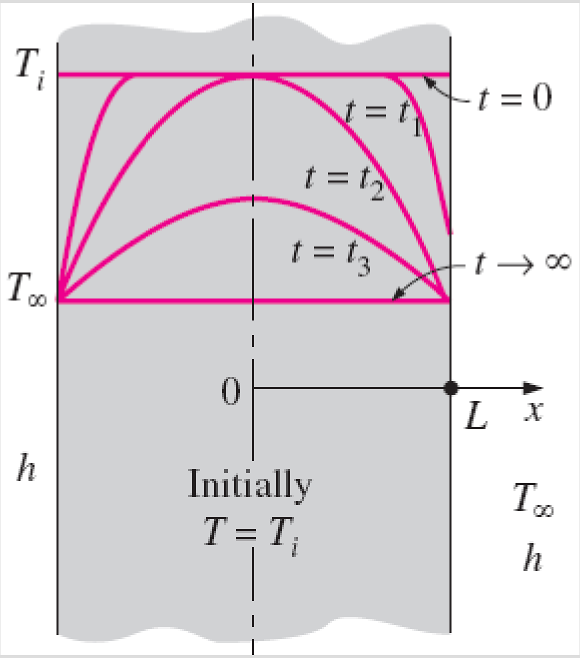
which yields

$$t = 43{,}860\ \text{s} = \mathbf{12.2\ h}$$

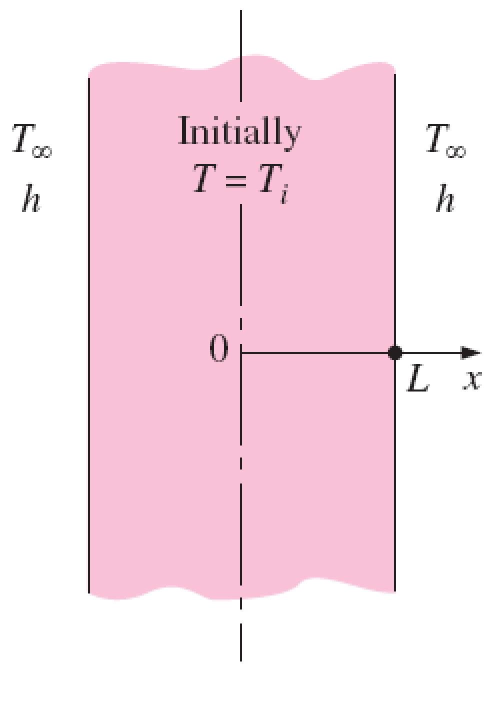
FIGURE 4–10

TRANSIENT HEAT CONDUCTION IN LARGE PLANE WALLS, LONG CYLINDERS, AND SPHERES WITH SPATIAL EFFECTS

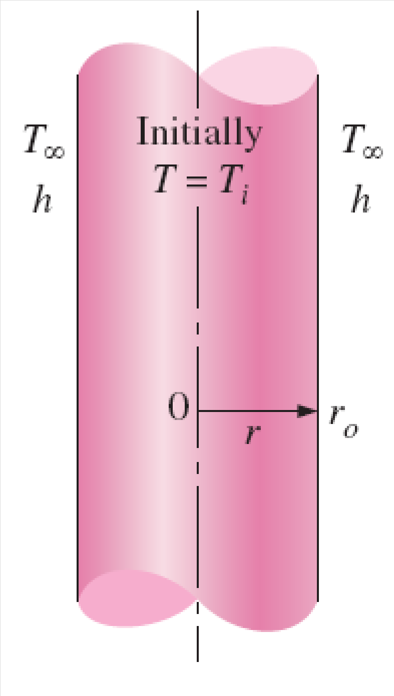
We will consider the variation of temperature with ***time*** and ***position*** in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere.



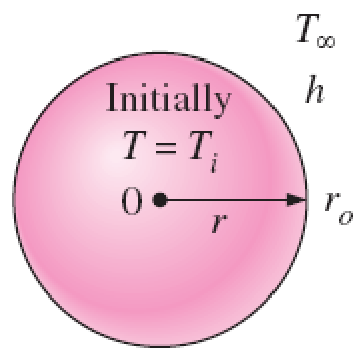
Transient temperature profiles in a plane wall exposed to convection from its surfaces for $T_i > T_\infty$.



(*a*) A large plane wall

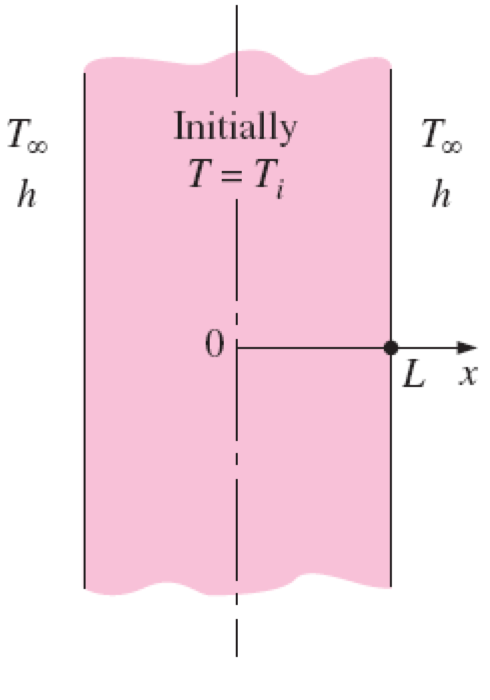


(*b*) A long cylinder



Schematic of the simple geometries in which heat transfer is one-dimensional.

Nondimensionalized One-Dimensional Transient Conduction Problem



(*a*) A large plane wall

Differential equation: $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$

Boundary conditions:

$$\frac{\partial T(0, t)}{\partial x} = 0 \quad \text{and} \quad -k\frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty]$$

Initial condition: $T(x, 0) = T_i$

$\alpha = k/\rho c_p$ $\quad X = x/L \quad$ $\theta(x, t) = [T(x, t) - T_\infty]/[T_i - T_\infty]$

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{and} \quad \frac{\partial \theta(1, t)}{\partial X} = \frac{hL}{k}\theta(1, t)$$

Dimensionless differential equation: $\dfrac{\partial^2 \theta}{\partial X^2} = \dfrac{\partial \theta}{\partial \tau}$

Dimensionless BC's: $\dfrac{\partial \theta(0, \tau)}{\partial X} = 0 \quad \text{and} \quad \dfrac{\partial \theta(1, \tau)}{\partial X} = -\text{Bi}\,\theta(1, \tau)$

Dimensionless initial condition: $\theta(X, 0) = 1$

$$\theta(X, \tau) = \frac{T(x, t) - T_i}{T_\infty - T_i}$$ *Dimensionless temperature*

$$X = \frac{x}{L}$$ *Dimensionless distance from the center*

$$\mathrm{Bi} = \frac{hL}{k}$$ *Dimensionless heat transfer coefficient* (Biot number)

$$\tau = \frac{\alpha t}{L^2} = \mathrm{Fo}$$ *Dimensionless time* (Fourier number)

(*a*) Original heat conduction problem:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad T(x, 0) = T_i$$

$$\frac{\partial T(0, t)}{\partial x} = 0, \quad -k\frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty]$$

$$T = F(x, L, t, k, \alpha, h, T_i)$$

(*b*) Nondimensionalized problem:

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}, \quad \theta(X, 0) = 1$$

$$\frac{\partial \theta(0, \tau)}{\partial X} = 0, \quad \frac{\partial \theta(1, \tau)}{\partial X} = -\mathrm{Bi}\,\theta(1, \tau)$$

$$\theta = f(X, \mathrm{Bi}, \tau)$$

Nondimensionalization reduces the number of independent variables in one-dimensional transient conduction problems from 8 to 3, offering great convenience in the presentation of results.

Exact Solution of One-Dimensional Transient Conduction Problem

TABLE 18–1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_o and a sphere of radius r_o subjected to convention from all surfaces.*

Geometry	Solution	λ_n's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x / L)$	$\lambda_n \tan \lambda_n = \text{Bi}$
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r / r_o)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x / L)}{\lambda_n x / L}$	$1 - \lambda_n \cot \lambda_n = \text{Bi}$

*Here $\theta = (T - T_i)/(T_\infty - T_i)$ is the dimensionless temperature, $\text{Bi} = hL/k$ or hr_o/k is the Biot number, $\text{Fo} = \tau = \alpha t / L^2$ or $\alpha\tau / r_o^2$ is the Fourier number, and J_0 and J_1 are the Bessel functions of the first kind whose values are given in Table 18–3.

$$\theta_n = A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X)$$

$$A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}$$

$$\lambda_n \tan \lambda_n = \text{Bi}$$

For Bi = 5, $X = 1$, and $t = 0.2$:

n	λ_n	A_n	θ_n
1	1.3138	1.2402	0.22321
2	4.0336	−0.3442	0.00835
3	6.9096	0.1588	0.00001
4	9.8928	−0.876	0.00000

The analytical solutions of transient conduction problems typically involve infinite series, and thus the evaluation of an infinite number of terms to determine the temperature at a specified location and time.

The term in the series solution of transient conduction problems decline rapidly as n and thus λ_n increases because of the exponential decay function with the exponent $-\lambda_n \tau$.

Approximate Analytical and Graphical Solutions

The terms in the series solutions converge rapidly with increasing time, and for $\tau > 0.2$, keeping the first term and neglecting all the remaining terms in the series results in an error under 2 percent.

Solution with *one-term approximation*

Plane wall: $$\theta_{\text{wall}} = \frac{T(x, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \quad \tau > 0.2$$

Cylinder: $$\theta_{\text{cyl}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o), \quad \tau > 0.2$$

Sphere: $$\theta_{\text{sph}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}, \quad \tau > 0.2$$

Center of plane wall $(x = 0)$: $$\theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

Center of cylinder $(r = 0)$: $$\theta_{0,\text{ cyl}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

Center of sphere $(r = 0)$: $$\theta_{0,\text{ sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

TABLE 18–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

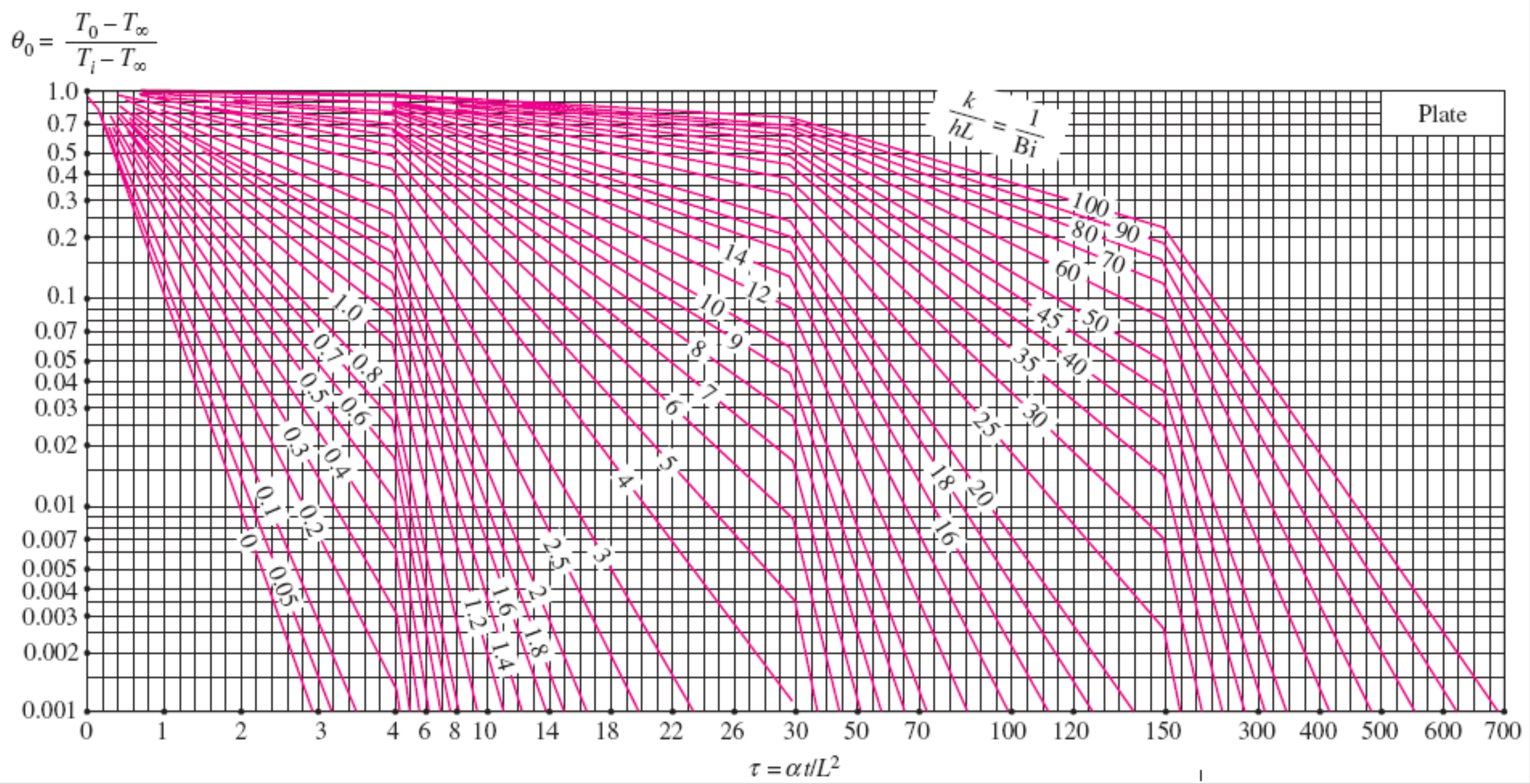
	Plane Wall		Cylinder		Sphere	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 18–3

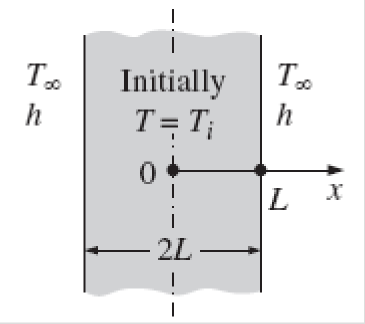
The zeroth- and first-order Bessel functions of the first kind

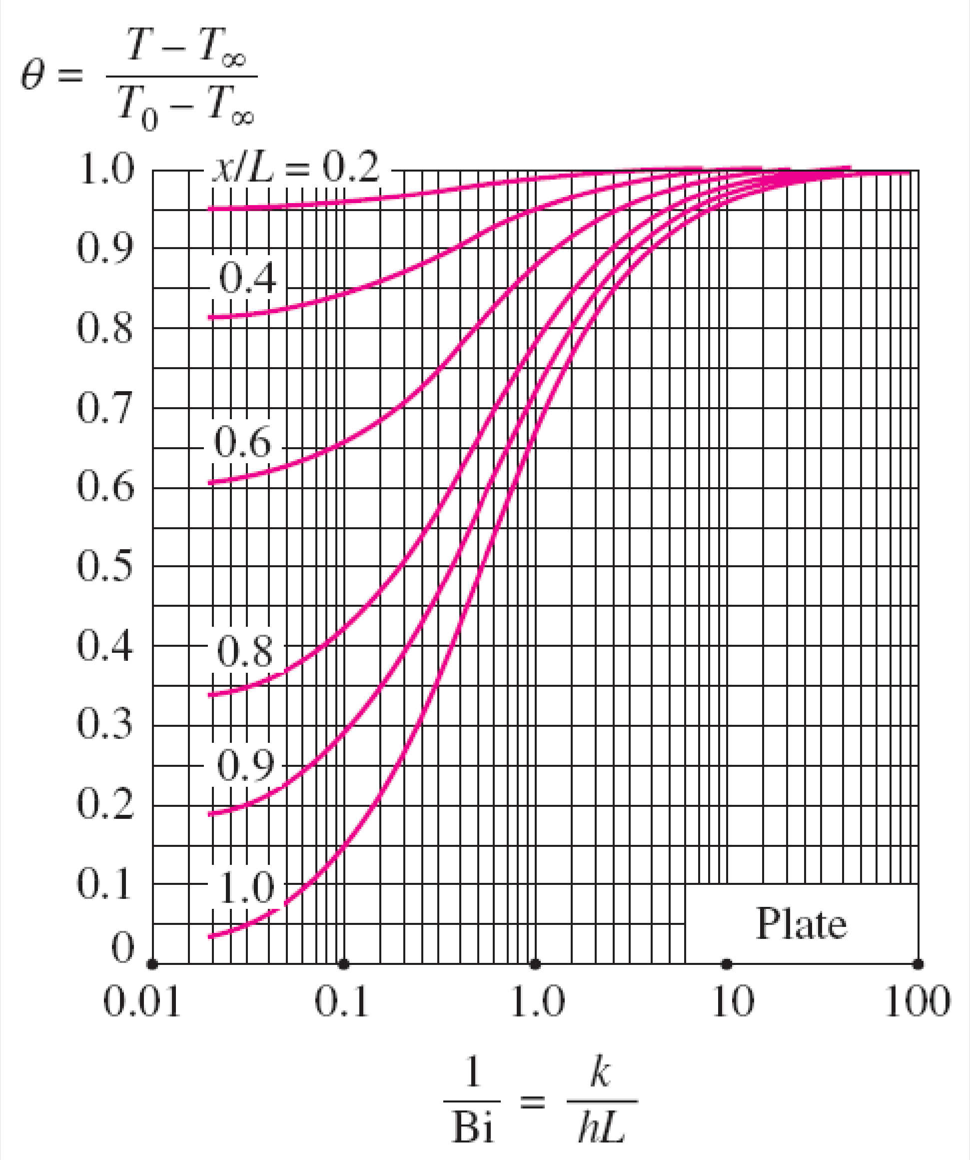
η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

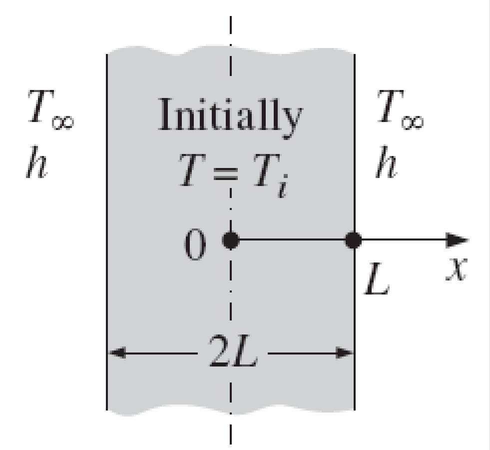
(*a*) Midplane temperature



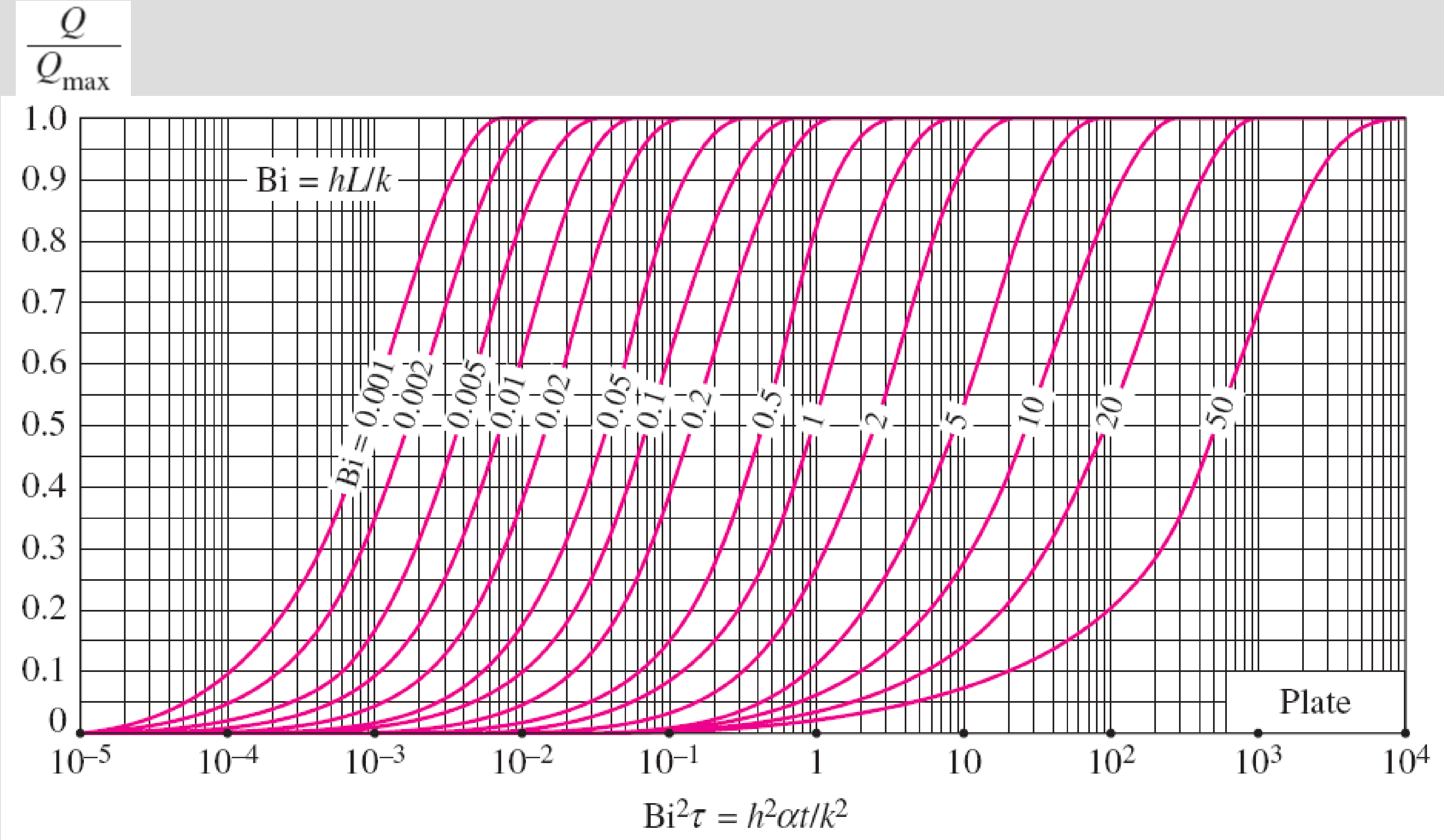
Transient temperature and heat transfer charts (Heisler and Grober charts) for a plane wall of thickness $2L$ initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of h.



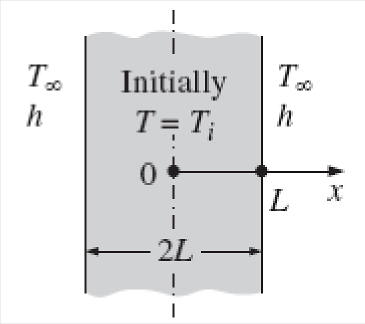




(*b*) **Temperature distribution**

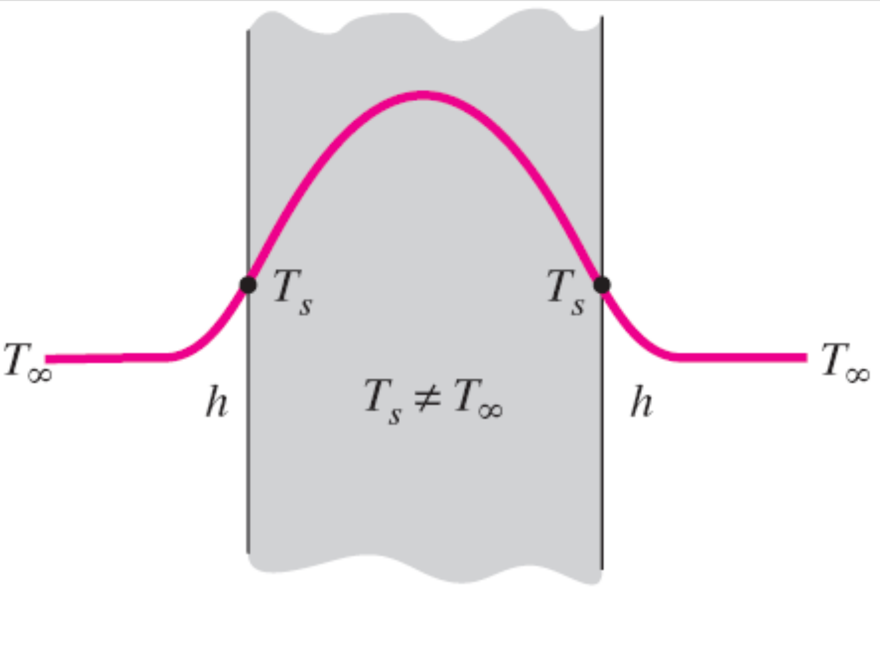


(*c*) Heat transfer

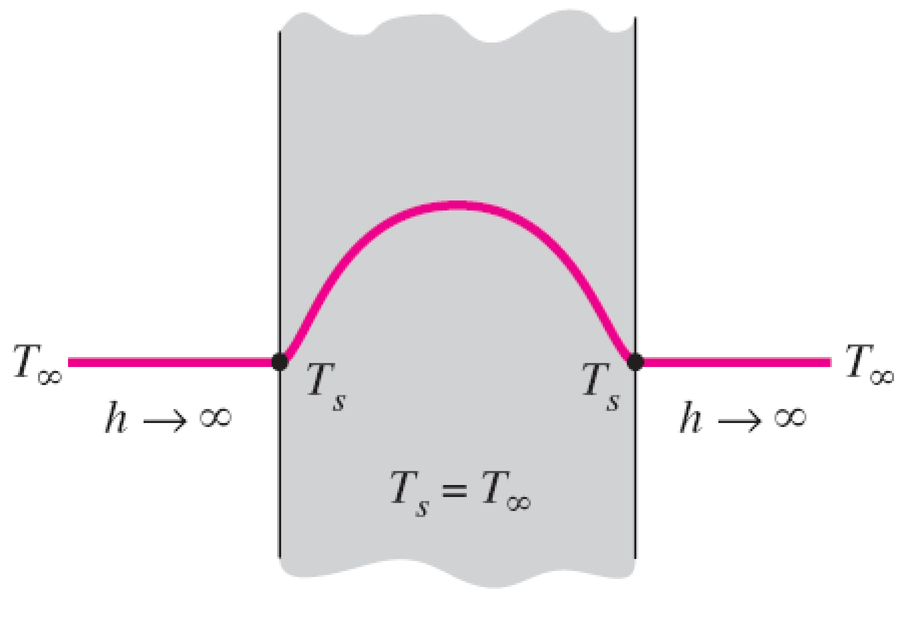


The dimensionless temperatures anywhere in a plane wall, cylinder, and sphere are related to the center temperature by

$$\frac{\theta_{\text{wall}}}{\theta_{0,\text{ wall}}} = \cos\left(\frac{\lambda_1 x}{L}\right), \quad \frac{\theta_{\text{cyl}}}{\theta_{0,\text{ cyl}}} = J_0\left(\frac{\lambda_1 r}{r_o}\right), \quad \text{and} \quad \frac{\theta_{\text{sph}}}{\theta_{0,\text{ sph}}} = \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$



(*a*) Finite convection coefficient



(*b*) Infinite convection coefficient

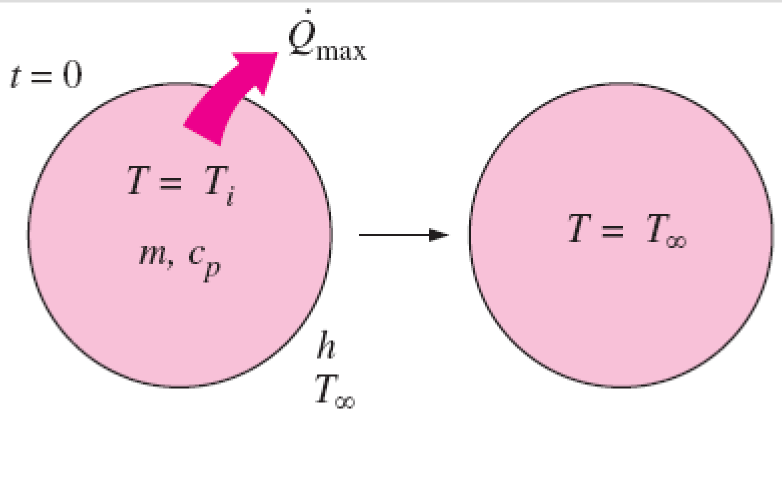
The specified surface temperature corresponds to the case of convection to an environment at T_∞ with with a convection coefficient h that is *infinite.*

$$Q_{max} = mc_p(T_\infty - T_i) = \rho V c_p(T_\infty - T_i) \qquad \text{(kJ)}$$

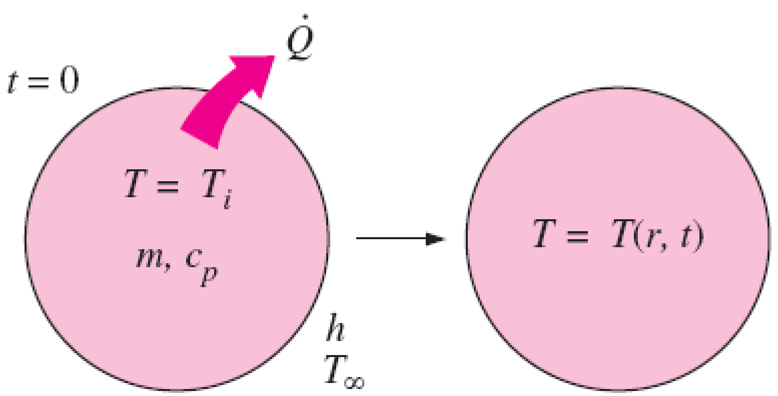
Plane wall: $$\left(\frac{Q}{Q_{max}}\right)_{wall} = 1 - \theta_{0,\,wall}\frac{\sin\lambda_1}{\lambda_1}$$

Cylinder: $$\left(\frac{Q}{Q_{max}}\right)_{cyl} = 1 - 2\theta_{0,\,cyl}\frac{J_1(\lambda_1)}{\lambda_1}$$

Sphere: $$\left(\frac{Q}{Q_{max}}\right)_{sph} = 1 - 3\theta_{0,\,sph}\frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3}$$



(a) Maximum heat transfer ($t \to \infty$)



$$\left.\begin{array}{l} \mathrm{Bi} = \ldots \\ \dfrac{h^2\alpha t}{k^2} = \mathrm{Bi}^2\tau = \ldots \end{array}\right\} \quad \frac{Q}{Q_{max}} = \ldots$$

(Gröber chart)

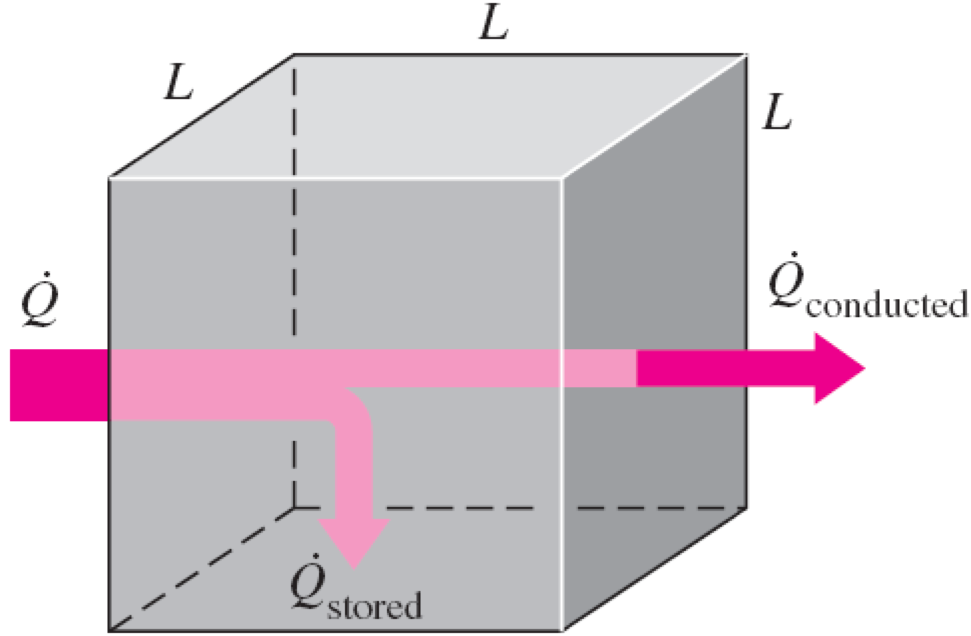
(b) Actual heat transfer for time t

The fraction of total heat transfer Q/Q_{max} up to a specified time t is determined using the Gröber charts.

The physical significance of the *Fourier number*

$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2\,(1/L)}{\rho c_p\, L^3/t}\,\frac{\Delta T}{\Delta T} = \frac{\text{The rate at which heat is } \textit{conducted} \text{ across } L \text{ of a body of volume } L^3}{\text{The rate at which heat is } \textit{stored} \text{ in a body of volume } L^3}$$

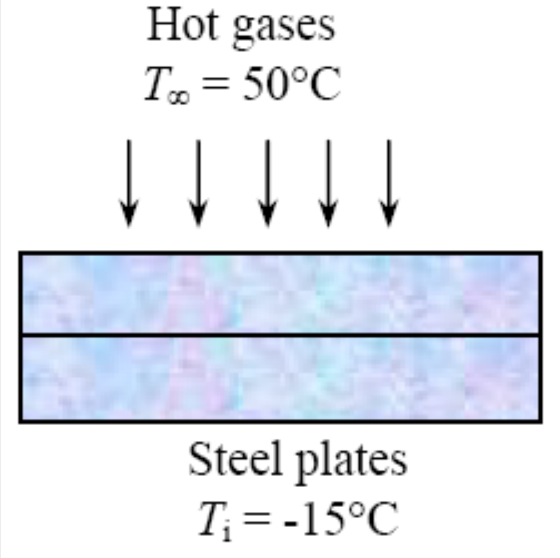
- The Fourier number is a measure of *heat conducted* through a body relative to *heat stored.*
- A large value of the Fourier number indicates faster propagation of heat through a body.



Fourier number: $\tau = \dfrac{\alpha t}{L^2} = \dfrac{\dot{Q}_{\text{conducted}}}{\dot{Q}_{\text{stored}}}$

Fourier number at time *t* can be viewed as the ratio of the rate of heat conducted to the rate of heat stored at that time.

4–110 Consider two 2-cm-thick large steel plates (k = 43 W/m · °C and $\alpha = 1.17 \times 10^{-5}$ m^2/s) that were put on top of each other while wet and left outside during a cold winter night at −15°C. The next day, a worker needs one of the plates, but the plates are stuck together because the freezing of the water between the two plates has bonded them together. In an effort to melt the ice between the plates and separate them, the worker takes a large hair dryer and blows hot air at 50°C all over the exposed surface of the plate on the top. The convection heat transfer coefficient at the top surface is estimated to be 40 W/m^2 · °C. Determine how long the worker must keep blowing hot air before the two plates separate.



$$Bi = \frac{hL_c}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.02\ \text{m})}{(43\ \text{W/m.}°\text{C})} = 0.019$$

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{1}{0.019} = 52.6 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{0-50}{-15-50} = 0.769 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 15 > 0.2$$

Then,

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(15)(0.02\ \text{m})^2}{(1.17\times 10^{-5}\ \text{m}^2/\text{s})} = \mathbf{513\ s}$$

EXAMPLE 4–3 Boiling Eggs

An ordinary egg can be approximated as a 5-cm-diameter sphere (Fig. 4–21). The egg is initially at a uniform temperature of 5°C and is dropped into boiling water at 95°C. Taking the convection heat transfer coefficient to be $h = 1200\ \text{W/m}^2 \cdot {}^\circ\text{C}$, determine how long it will take for the center of the egg to reach 70°C.

Analysis The temperature within the egg varies with radial distance as well as time, and the temperature at a specified location at a given time can be determined from the Heisler charts or the one-term solutions. Here we use the latter to demonstrate their use. The Biot number for this problem is

$$\text{Bi} = \frac{hr_o}{k} = \frac{(1200\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.025\ \text{m})}{0.627\ \text{W/m} \cdot {}^\circ\text{C}} = 47.8$$

which is much greater than 0.1, and thus the lumped system analysis is not applicable. The coefficients λ_1 and A_1 for a sphere corresponding to this Bi are, from Table 4–2,

$$\lambda_1 = 3.0754, \qquad A_1 = 1.9958$$

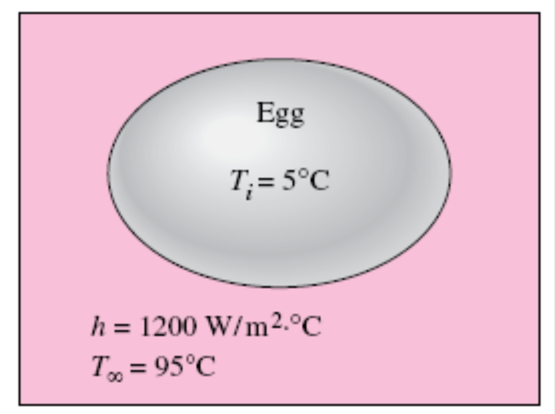
Substituting these and other values into Eq. 4–28 and solving for τ gives

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70 - 95}{5 - 95} = 1.9958 e^{-(3.0754)^2 \tau} \longrightarrow \tau = 0.209$$

which is greater than 0.2, and thus the one-term solution is applicable with an error of less than 2 percent. Then the cooking time is determined from the definition of the Fourier number to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.209)(0.025\ \text{m})^2}{0.151 \times 10^{-6}\ \text{m}^2/\text{s}} = 865\ \text{s} \approx \mathbf{14.4\ min}$$

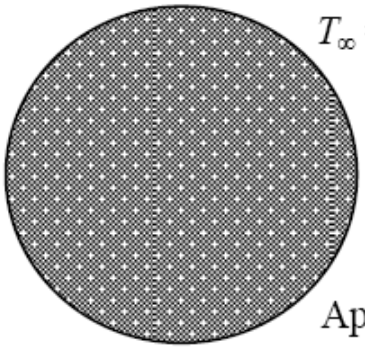
Therefore, it will take about 15 min for the center of the egg to be heated from 5°C to 70°C.



4–53 A person puts a few apples into the freezer at -15°C to cool them quickly for guests who are about to arrive. Initially, the apples are at a uniform temperature of 20°C, and the heat transfer coefficient on the surfaces is 8 W/m$^2 \cdot$ °C. Treating the apples as 9-cm-diameter spheres and taking their properties to be $\rho = 840$ kg/m^3, $c_p = 3.81$ kJ/kg · °C, $k = 0.418$ W/m · °C, and $\alpha = 1.3 \times 10^{-7}$ m^2/s, determine the center and surface temperatures of the apples in 1 h. Also, determine the amount of heat transfer from each apple.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(8\ \text{W/m}^2.°\text{C})(0.045\ \text{m})}{(0.418\ \text{W/m}.°\text{C})} = 0.861$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.476 \quad \text{and} \quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.3 \times 10^{-7}\ \text{m}^2/\text{s})(1\,\text{h} \times 3600\ \text{s/h})}{(0.045\ \text{m})^2} = 0.231 > 0.2$$

Then the temperature at the center of the apples becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - (-15)}{20 - (-15)} = (1.239)e^{-(1.476)^2(0.231)} = 0.749 \longrightarrow T_0 = \mathbf{11.2°C}$$

The temperature at the surface of the apples is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau}\,\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.239)e^{-(1.476)^2(0.231)}\,\frac{\sin(1.476\,\text{rad})}{1.476} = 0.505$$

$$\frac{T(r_o,t)-(-15)}{20-(-15)} = 0.505 \longrightarrow T(r_o,t) = \mathbf{2.7°C}$$

The maximum possible heat transfer is

$$m = \rho V = \rho\frac{4}{3}\pi r_o^3 = (840\,\text{kg/m}^3)\left[\frac{4}{3}\pi(0.045\,\text{m})^3\right] = 0.3206\,\text{kg}$$

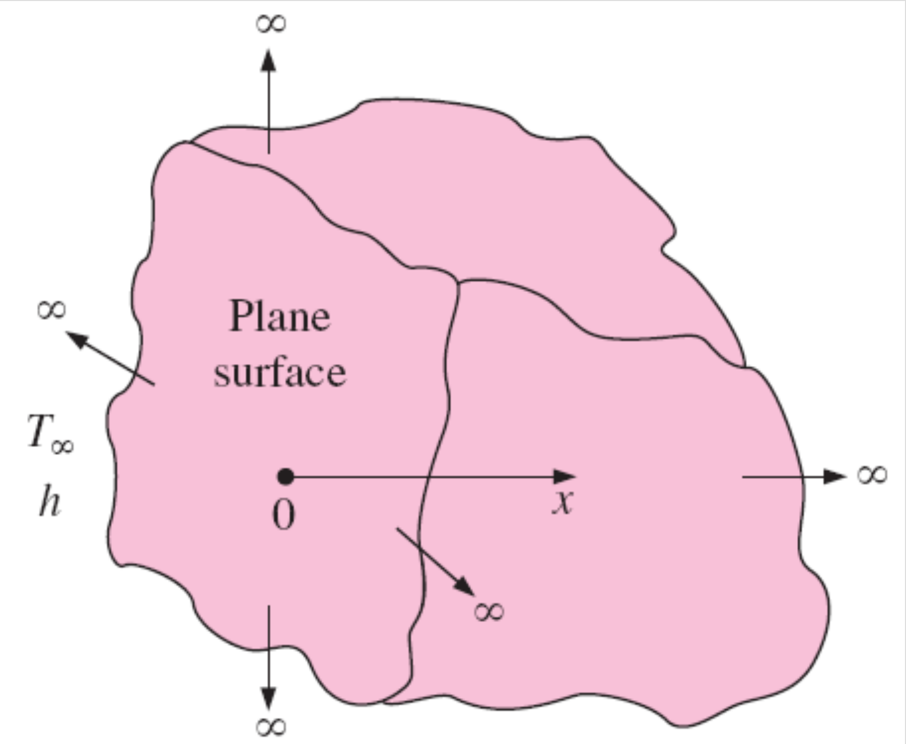
$$Q_{\max} = mc_p(T_i-T_\infty) = (0.3206\,\text{kg})(3.81\,\text{kJ/kg.°C})[20-(-15)]°\text{C} = 42.75\,\text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1-3\theta_{o,sph}\,\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1-3(0.749)\,\frac{\sin(1.476\,\text{rad})-(1.476)\cos(1.476\,\text{rad})}{(1.476)^3} = 0.402$$

$$Q = 0.402Q_{\max} = (0.402)(42.75\,\text{kJ}) = \mathbf{17.2\,kJ}$$

TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS



Schematic of a semi-infinite body.

Semi-infinite solid: An idealized body that has a *single plane surface* and extends to infinity in all directions.

The earth can be considered to be a semi-infinite medium in determining the variation of temperature near its surface.

A thick wall can be modeled as a semi-infinite medium if all we are interested in is the variation of temperature in the region near one of the surfaces, and the other surface is too far to have any impact on the region of interest during the time of observation.

For short periods of time, most bodies can be modeled as semi-infinite solids since heat does not have sufficient time to penetrate deep into the body.

Analytical solution for the case of constant temperature T_s on the surface

Differential equation: $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary conditions: $T(0, t) = T_s$ and $T(x \to \infty, t) = T_i$

Initial condition: $T(x, 0) = T_i$

Similarity variable: $$\eta = \frac{x}{\sqrt{4\alpha t}}$$

$$\frac{d^2 T}{d\eta^2} = -2\eta \frac{dT}{d\eta}$$

$T(0) = T_s$ and $T(\eta \to \infty) = T_i$

$$\frac{T - T_s}{T_i - T_s} = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-u^2} du = \operatorname{erf}(\eta) = 1 - \operatorname{erfc}(\eta)$$

$$\operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-u^2} du$$ error function

$$\operatorname{erfc}(\eta) = 1 - \frac{2}{\sqrt{\pi}} \int_0^\eta \varepsilon^{-u^2} du$$ complementary error function

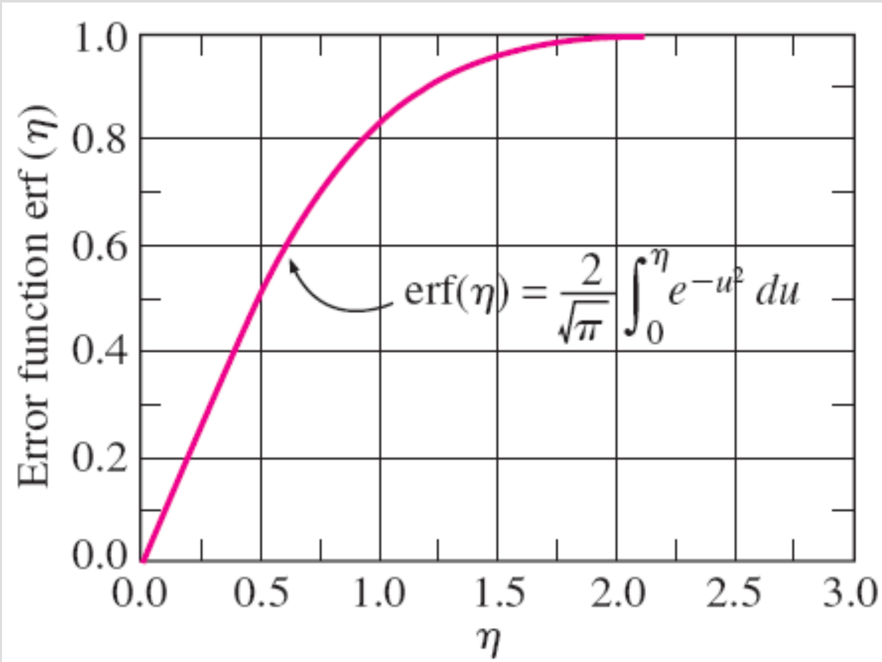
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{and} \quad \eta = \frac{x}{\sqrt{4\alpha t}}$$

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial t} = \frac{x}{2t\sqrt{4\alpha t}}\frac{dT}{d\eta}$$

$$\frac{\partial T}{\partial x} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial x} = \frac{1}{\sqrt{4\alpha t}}\frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta}\left(\frac{\partial T}{\partial x}\right)\frac{\partial \eta}{\partial x} = \frac{1}{4\alpha t}\frac{d^2 T}{d\eta^2}$$

Transformation of variables in the derivatives of the heat conduction equation by the use of chain rule.



Error function is a standard mathematical function, just like the sine and cosine functions, whose value varies between 0 and 1.

TABLE 18–4

The complementary error function*

η	erfc (η)	η	erfc (η)	η	erfc (η)
0.00	1.00000	0.38	0.5910	0.76	0.2825
0.02	0.9774	0.40	0.5716	0.78	0.2700
0.04	0.9549	0.42	0.5525	0.80	0.2579
0.06	0.9324	0.44	0.5338	0.82	0.2462
0.08	0.9099	0.46	0.5153	0.84	0.2349
0.10	0.8875	0.48	0.4973	0.86	0.2239
0.12	0.8652	0.50	0.4795	0.88	0.2133
0.14	0.8431	0.52	0.4621	0.90	0.2031
0.16	0.8210	0.54	0.4451	0.92	0.1932
0.18	0.7991	0.56	0.4284	0.94	0.1837
0.20	0.7773	0.58	0.4121	0.96	0.1746
0.22	0.7557	0.60	0.3961	0.98	0.1658
0.24	0.7343	0.62	0.3806	1.00	0.1573
0.26	0.7131	0.64	0.3654	1.02	0.1492
0.28	0.6921	0.66	0.3506	1.04	0.1413
0.30	0.6714	0.68	0.3362	1.06	0.1339
0.32	0.6509	0.70	0.3222	1.08	0.1267
0.34	0.6306	0.72	0.3086	1.10	0.1198
0.36	0.6107	0.74	0.2953	1.12	0.1132

Analytical solutions for different boundary conditions on the surface

Case 1: Specified Surface Temperature, T_s = constant

$$\frac{T(x,\ t) - T_i}{T_s - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad \text{and} \quad \dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

Case 2: Specified Surface Heat Flux, $\dot{q}_s$ = constant.

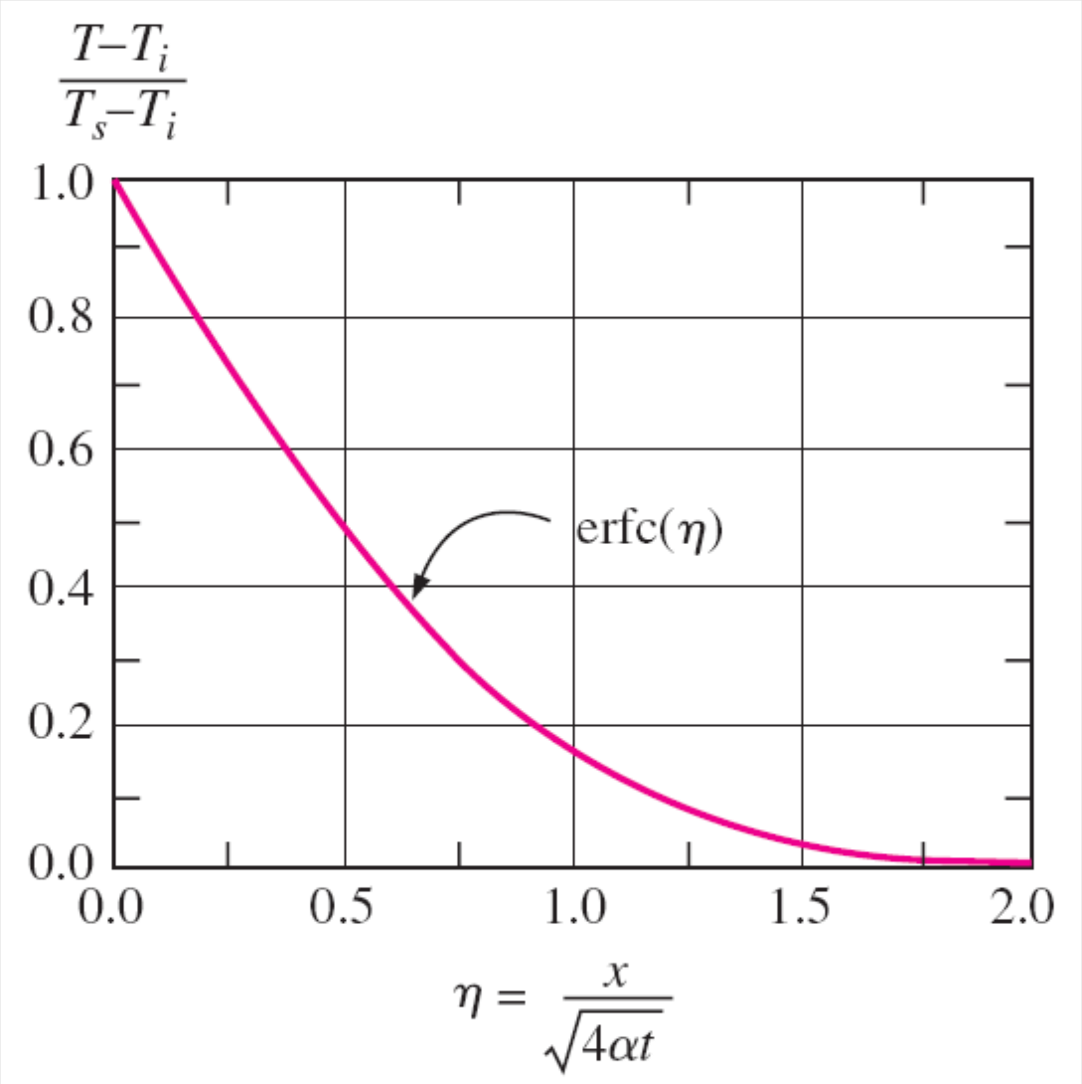
$$T(x,\ t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(-\frac{x^2}{4\alpha t}\right) - x\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

Case 3: Convection on the Surface, $\dot{q}_s(t) = h[T_\infty - T(0, t)]$.

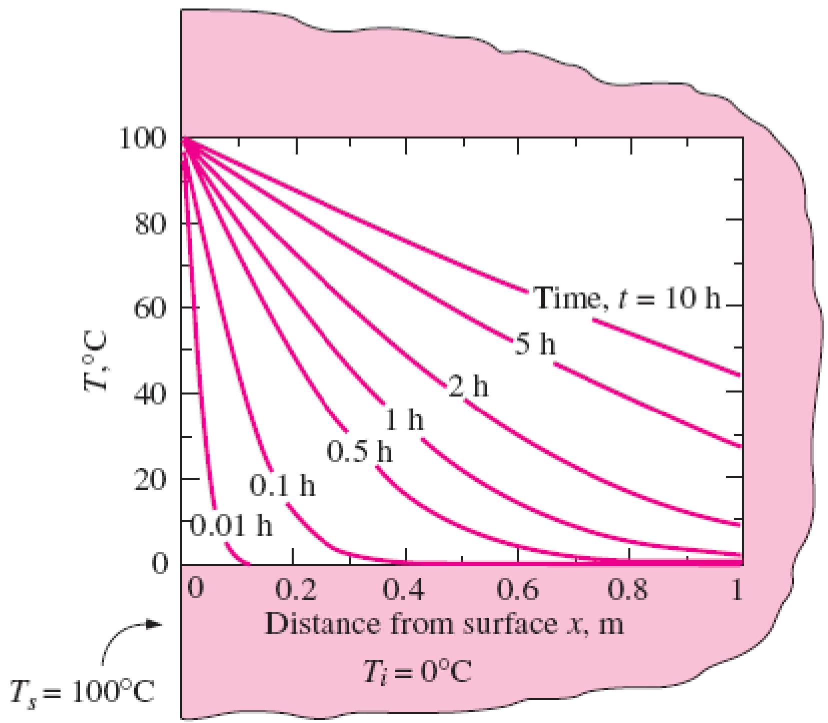
$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

Case 4: Energy Pulse at Surface, e_s = constant.

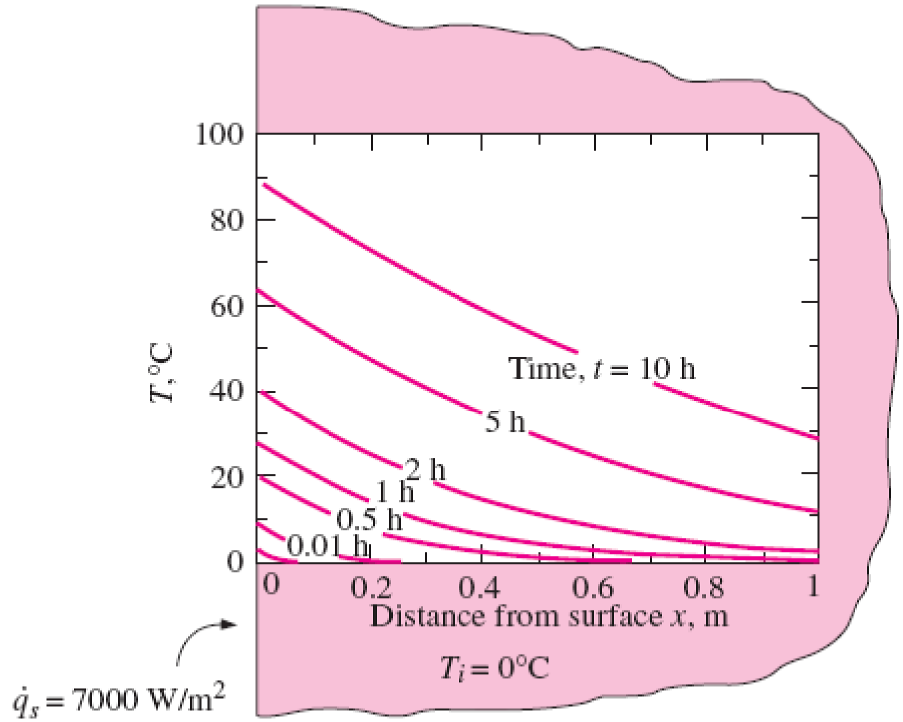
$$T(x,\ t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}}\exp\left(-\frac{x^2}{4\alpha t}\right)$$



Dimensionless temperature distribution for transient conduction in a semi-infinite solid whose surface is maintained at a constant temperature T_s.

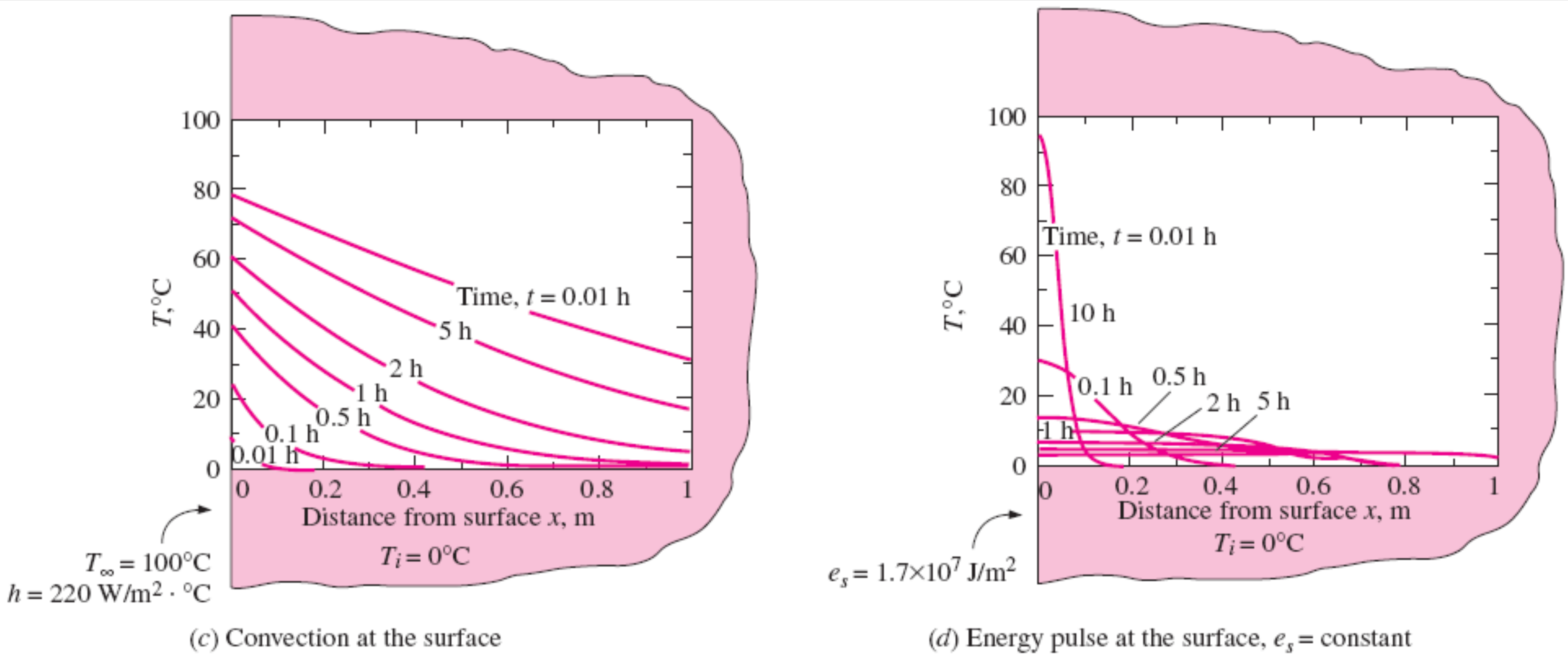


(*a*) Specified surface temperature, T_s = constant.

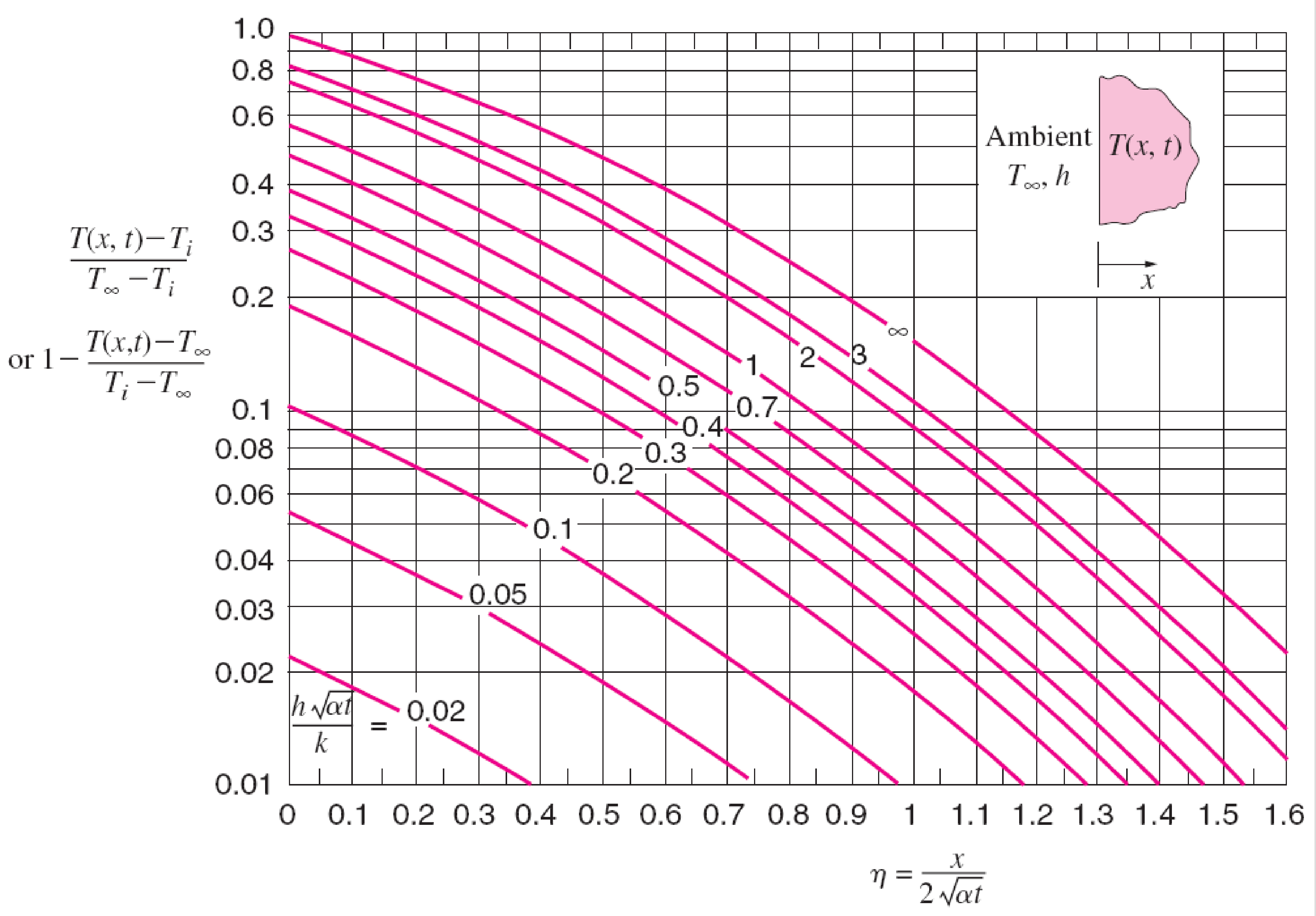


(*b*) Specified surface heat flux, $\dot{q}_s$ = constant.

Variations of temperature with position and time in a large cast iron block ($\alpha = 2.31 \times 10^{-5}\ m^2/s$, $k = 80.2\ W/m \cdot °C$) initially at 0 °C under different thermal conditions on the surface.



Variations of temperature with position and time in a large cast iron block ($\alpha = 2.31 \times 10^{-5}$ m^2/s, $k = 80.2$ W/m · °C) initially at 0 °C under different thermal conditions on the surface.



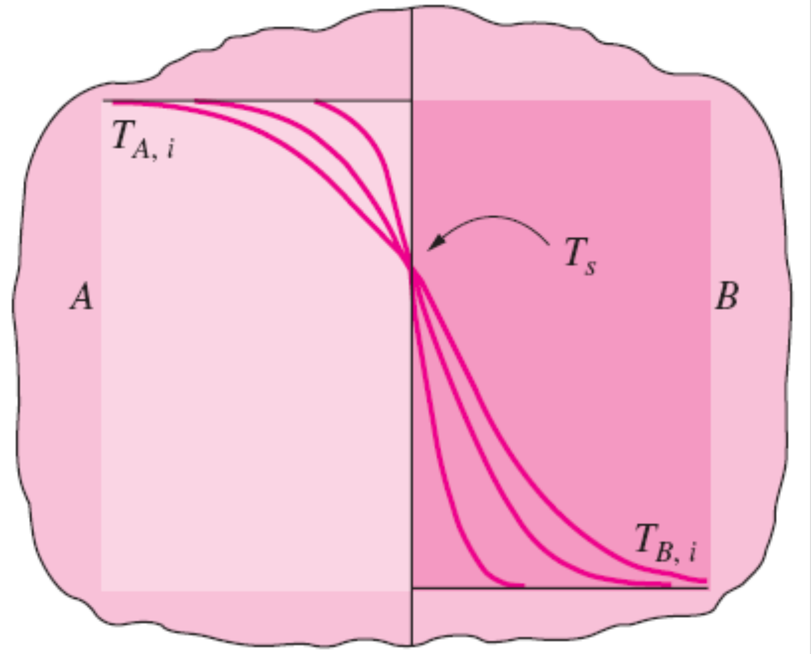
Variation of temperature with position and time in a semi-infinite solid initially at temperature T_i subjected to convection to an environment at T_∞ with a convection heat transfer coefficient of h.

Contact of Two Semi-Infinite Solids

When two large bodies *A* and *B*, initially at uniform temperatures $T_{A,i}$ and $T_{B,i}$ are brought into contact, they instantly achieve temperature equality at the contact surface.

If the two bodies are of the same material, the contact surface temperature is the arithmetic average, $T_s = (T_{A,i}+T_{B,i})/2$.

If the bodies are of different materials, the surface temperature T_s will be different than the arithmetic average.



Contact of two semi-infinite solids of different initial temperatures.

$$\dot{q}_{s,A} = \dot{q}_{s,B} \rightarrow -\frac{k_A(T_s - T_{A,i})}{\sqrt{\pi\alpha_A t}} = \frac{k_B(T_s - T_{B,i})}{\sqrt{\pi\alpha_B t}} \rightarrow \frac{T_{A,i} - T_s}{T_s - T_{B,i}} = \sqrt{\frac{(k\rho c_p)_B}{(k\rho c_p)_A}}$$

$$T_s = \frac{\sqrt{(k\rho c_p)_A}T_{A,i} + \sqrt{(k\rho c_p)_B}T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}}$$

The interface temperature of two bodies brought into contact is dominated by the body with the larger $k\rho c_p$.

EXAMPLE: When a person with a skin temperature of 35°C touches an aluminum block and then a wood block both at 15°C, the contact surface temperature will be 15.9°C in the case of aluminum and 30°C in the case of wood.

4–111 Consider a curing kiln whose walls are made of 30-cm-thick concrete whose properties are $k = 0.9$ W/m · °C and $\alpha = 0.23 \times 10^{-5}$ m²/s. Initially, the kiln and its walls are in equilibrium with the surroundings at 6°C. Then all the doors are closed and the kiln is heated by steam so that the temperature of the inner surface of the walls is raised to 42°C and is maintained at that level for 2.5 h. The curing kiln is then opened and exposed to the atmospheric air after the stream flow is turned off. If the outer surfaces of the walls of the kiln were insulated, would it save any energy that day during the period the kiln was used for curing for 2.5 h only, or would it make no difference? Base your answer on calculations.

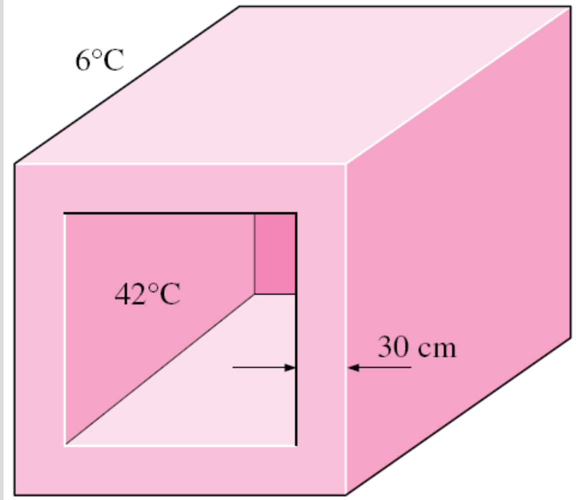


FIGURE P4–111

Analysis We determine the temperature at a depth of $x = 0.3$ m in 2.5 h using the analytical solution,

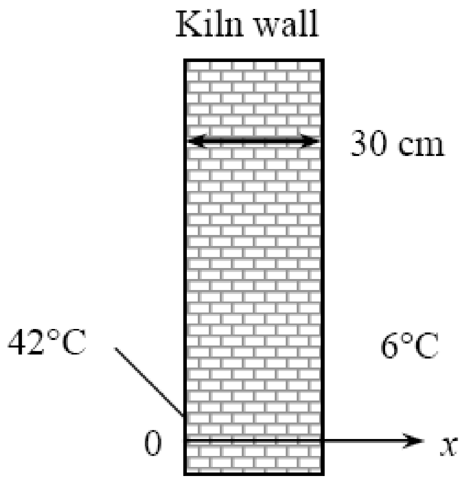
$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Substituting,

$$\frac{T(x,t)-6}{42-6} = erfc\left(\frac{0.3\text{ m}}{2\sqrt{(0.23\times10^{-5}\text{ m}^2/\text{s})(2.5\text{ h}\times3600\text{ s/h})}}\right)$$

$$= erfc(1.043) = 0.1402$$

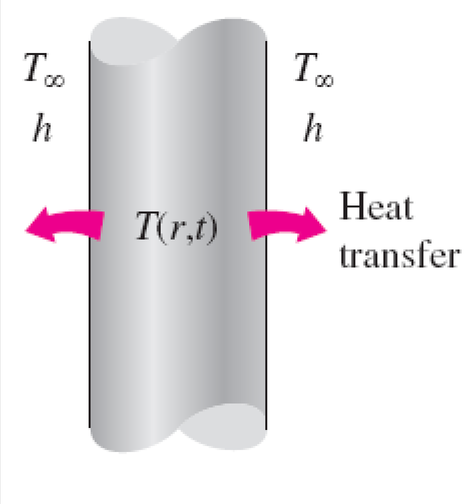
$$T(x,t) = \mathbf{11.0\,°C}$$



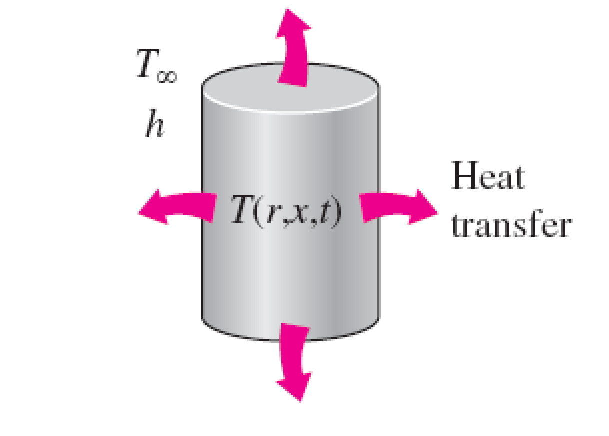
which is greater than the initial temperature of 6°C. Therefore, heat will propagate through the 0.3 m thick wall in 2.5 h, and thus it may be desirable to insulate the outer surface of the wall to save energy.

TRANSIENT HEAT CONDUCTION IN MULTIDIMENSIONAL SYSTEMS

- Using a superposition approach called the **product solution**, the transient temperature charts and solutions can be used to construct solutions for the *two-dimensional* and *three-dimensional* transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, a rectangular prism or a semi-infinite rectangular bar, provided that *all* surfaces of the solid are subjected to convection to the *same* fluid at temperature T_∞, with the *same* heat transfer coefficient h, and the body involves no heat generation.
- The solution in such multidimensional geometries can be expressed as the *product* of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.



(*a*) Long cylinder



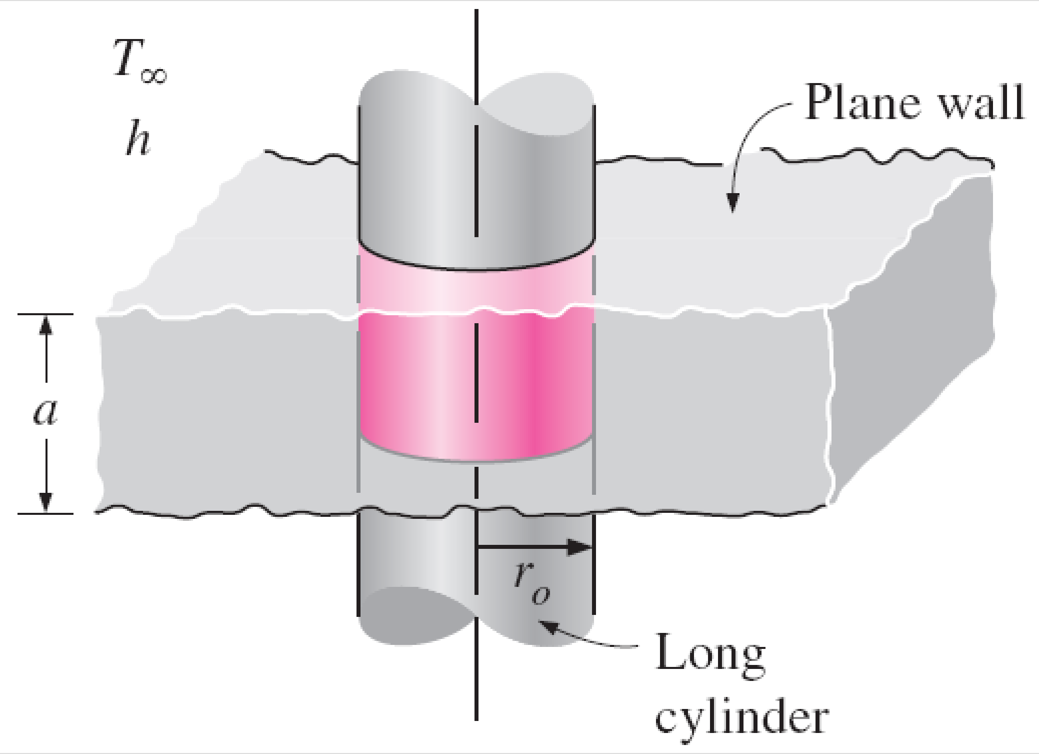
(*b*) Short cylinder (two-dimensional)

The temperature in a short cylinder exposed to convection from all surfaces varies in both the radial and axial directions, and thus heat is transferred in both directions.

The solution for a multidimensional geometry is the product of the solutions of the one-dimensional geometries whose intersection is the multidimensional body.

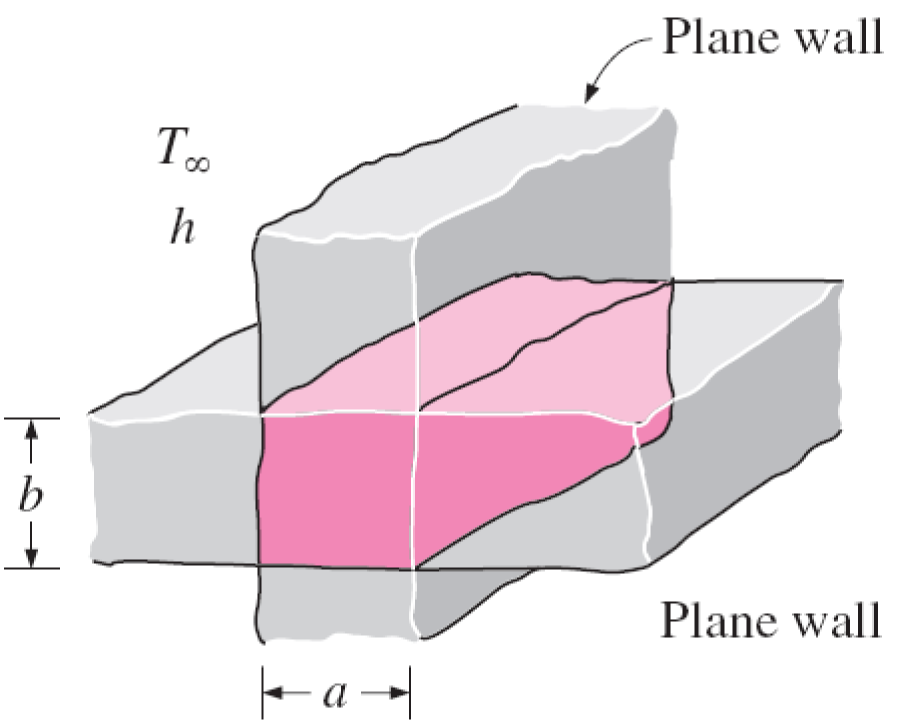
The solution for the two-dimensional short cylinder of height *a* and radius r_o is equal to the *product* of the nondimensionalized solutions for the one-dimensional plane wall of thickness *a* and the long cylinder of radius r_o.

$$\left(\frac{T(r,x,t)-T_\infty}{T_i-T_\infty}\right)_{\substack{\text{short}\\ \text{cylinder}}} = \left(\frac{T(x,t)-T_\infty}{T_i-T_\infty}\right)_{\substack{\text{plane}\\ \text{wall}}} \left(\frac{T(r,t)-T_\infty}{T_i-T_\infty}\right)_{\substack{\text{infinite}\\ \text{cylinder}}}$$



A short cylinder of radius r_o and height *a* is the *intersection* of a long cylinder of radius r_o and a plane wall of thickness *a.*

$$\left(\frac{T(x, y, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{rectangular}\\\text{bar}}} = \theta_{\text{wall}}(x, t)\theta_{\text{wall}}(y, t)$$



$$\theta_{\text{wall}}(x, t) = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{plane}\\\text{wall}}}$$

$$\theta_{\text{cyl}}(r, t) = \left(\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{infinite}\\\text{cylinder}}}$$

$$\theta_{\text{semi-inf}}(x, t) = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{semi-infinite}\\\text{solid}}}$$

A long solid bar of rectangular profile $a \times b$ is the *intersection* of two plane walls of thicknesses *a* and *b.*

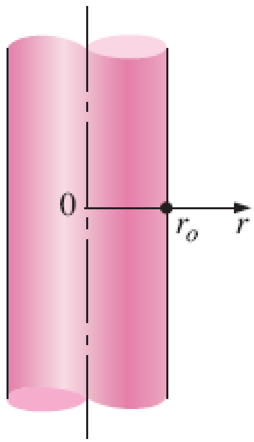
The transient heat transfer for a two-dimensional geometry formed by the intersection of two one-dimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 2D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]$$

Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is

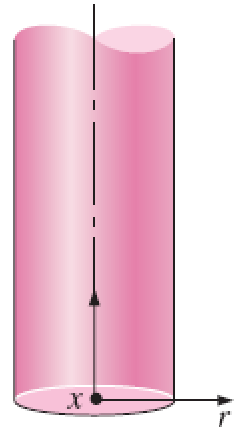
$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 3D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] + \left(\frac{Q}{Q_{\max}}\right)_3 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]\left[1 - \left(\frac{Q}{Q_{\max}}\right)_2\right]$$

Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature T_i and exposed to convection from all surfaces to a medium at T_∞



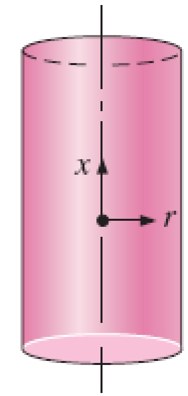
$\theta(r,t) = \theta_{cyl}(r,t)$

Infinite cylinder



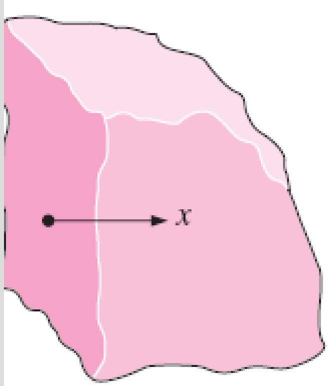
$\theta(x,r,t) = \theta_{cyl}(r,t)\,\theta_{semi\text{-}inf}(x,t)$

Semi-infinite cylinder



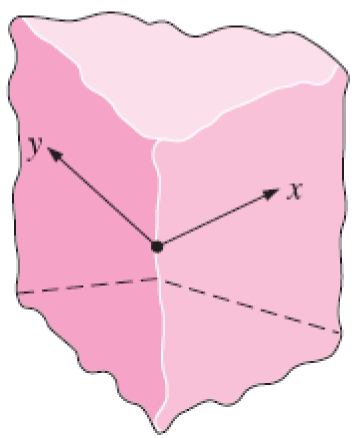
$\theta(x,r,t) = \theta_{cyl}(r,t)\,\theta_{wall}(x,t)$

Short cylinder



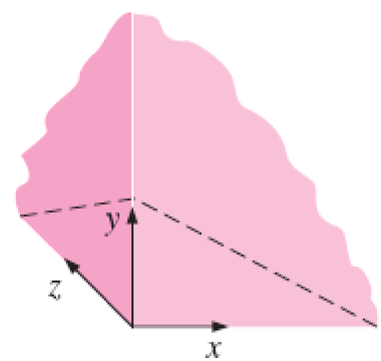
$\theta(x,t) = \theta_{semi\text{-}inf}(x,t)$

Semi-infinite medium



$\theta(x,y,t) = \theta_{semi\text{-}inf}(x,t)\,\theta_{semi\text{-}inf}(y,t)$

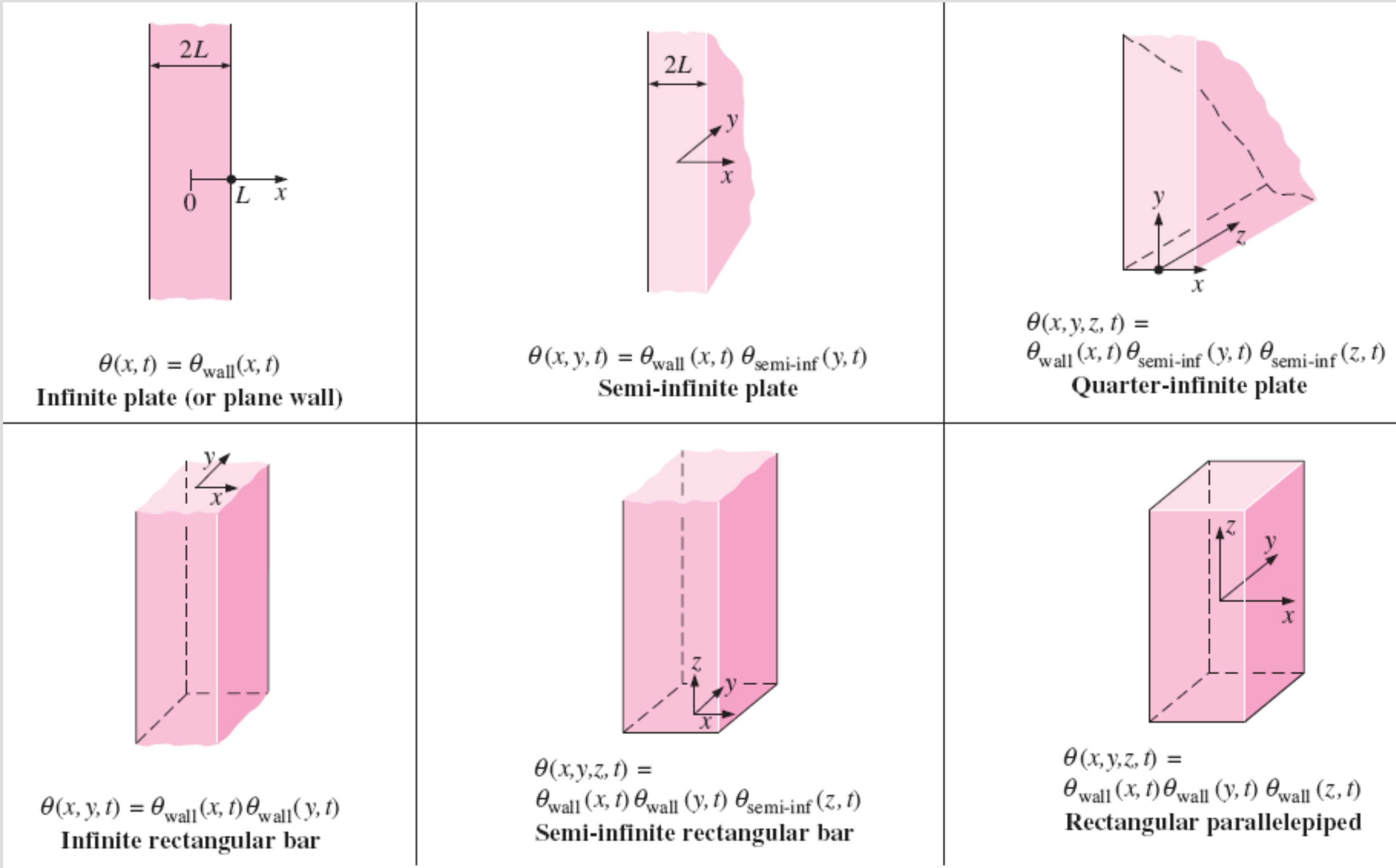
Quarter-infinite medium



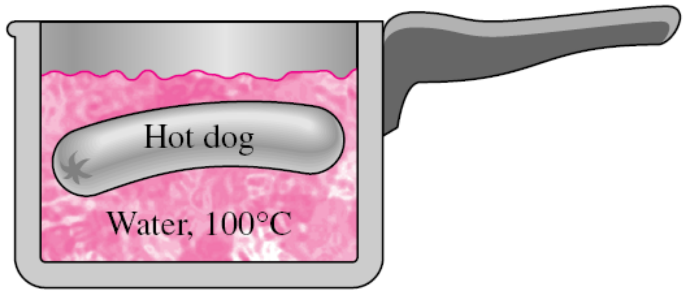
$\theta(x,y,z,t) =$
$\theta_{semi\text{-}inf}(x,t)\,\theta_{semi\text{-}inf}(y,t)\,\theta_{semi\text{-}inf}(z,t)$

Corner region of a large medium

Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature T_i and exposed to convection from all surfaces to a medium at T_∞



4–113 A hot dog can be considered to be a 12-cm-long cylinder whose diameter is 2 cm and whose properties are ρ = 980 kg/m^3, c_p = 3.9 kJ/kg · °C, k = 0.76 W/m · °C, and $\alpha = 2 \times 10^{-7}$ m^2/s. A hot dog initially at 5°C is dropped into boiling water at 100°C. The heat transfer coefficient at the surface of the hot dog is estimated to be 600 W/m^2 · °C. If the hot dog is considered cooked when its center temperature reaches 80°C, determine how long it will take to cook it in the boiling water.



$$Bi = \frac{hL}{k} = \frac{(600\ \text{W/m}^2.°\text{C})(0.06\ \text{m})}{(0.76\ \text{W/m}.°\text{C})} = 47.37 \longrightarrow \lambda_1 = 1.5380 \quad \text{and} \quad A_1 = 1.2726$$

$$Bi = \frac{hr_o}{k} = \frac{(600\ \text{W/m}^2.°\text{C})(0.01\ \text{m})}{(0.76\ \text{W/m}.°\text{C})} = 7.895 \longrightarrow \lambda_1 = 2.1249 \quad \text{and} \quad A_1 = 1.5514$$

$$\theta(0,0,t)_{block} = \theta(0,t)_{wall}\,\theta(0,t)_{cyl} = \left(A_1 e^{-\lambda_1^2\tau}\right)\left(A_1 e^{-\lambda_1^2\tau}\right)$$

$$\frac{80-100}{5-100} = \left\{(1.2726)\exp\left[-(1.5380)^2\frac{(2\times10^{-7})t}{(0.06)^2}\right]\right\}$$

$$\times\left\{(1.5514)\exp\left[-(2.1249)^2\frac{(2\times10^{-7})t}{(0.01)^2}\right]\right\} = 0.2105$$

which gives

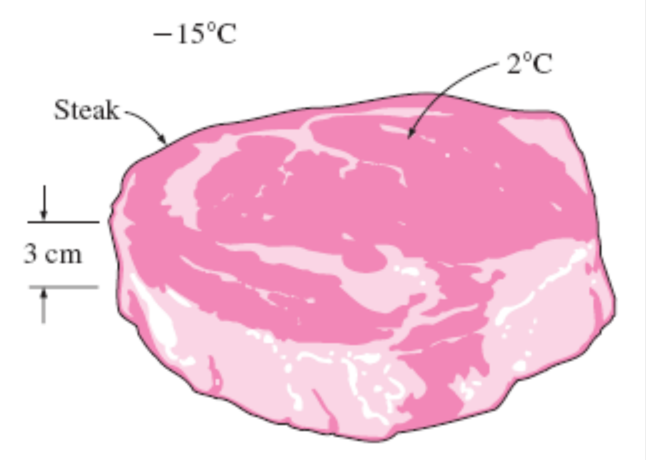
t = **244 s = 4.1 min**

This hot dog can physically be formed by the intersection of an infinite plane wall of thickness 2*L* = 12 cm, and a long cylinder of radius *ro* = *D*/2 = 1 cm.

$$\tau_{cyl} = \frac{\alpha t}{r_o^2} = \frac{(2\times10^{-7}\ \text{m}^2/\text{s})(244\ \text{s})}{(0.01\ \text{m})^2} = 0.49 > 0.2$$

EXAMPLE 4–11 Refrigerating Steaks while Avoiding Frostbite

In a meat processing plant, 3-cm-thick steaks initially at 25°C are to be cooled in the racks of a large refrigerator that is maintained at −15°C (Fig. 4–39). The steaks are placed close to each other, so that heat transfer from the 3-cm-thick edges is negligible. The entire steak is to be cooled below 8°C, but its temperature is not to drop below 2°C at any point during refrigeration to avoid "frostbite." The convection heat transfer coefficient and thus the rate of heat transfer from the steak can be controlled by varying the speed of a circulating fan inside. Determine the heat transfer coefficient h that will enable us to meet both temperature constraints while keeping the refrigeration time to a minimum. The steak can be treated as a homogeneous layer having the properties $\rho = 1200 \text{ kg/m}^3$, $c_p = 4.10 \text{ kJ/kg} \cdot °\text{C}$, $k = 0.45 \text{ W/m} \cdot °\text{C}$, and $\alpha = 9.03 \times 10^{-8} \text{ m}^2\text{/s}$.



Analysis The lowest temperature in the steak occurs at the surfaces and the highest temperature at the center at a given time, since the inner part is the last place to be cooled. In the limiting case, the surface temperature at $x = L = 1.5$ cm from the center will be 2°C, while the midplane temperature is 8°C in an environment at −15°C. Then, from Fig. 4–15*b*, we obtain

$$\left.\begin{aligned} \frac{x}{L} &= \frac{1.5 \text{ cm}}{1.5 \text{ cm}} = 1 \\ \frac{T(L,t) - T_\infty}{T_0 - T_\infty} &= \frac{2-(-15)}{8-(-15)} = 0.74 \end{aligned}\right\} \frac{1}{\text{Bi}} = \frac{k}{hL} = 1.5$$

which gives

$$h = \frac{1}{1.5}\frac{k}{L} = \frac{0.45 \text{ W/m} \cdot °\text{C}}{1.5(0.015 \text{ m})} = 20 \text{ W/m}^2 \cdot °\text{C}$$

Discussion The convection heat transfer coefficient should be kept below this value to satisfy the constraints on the temperature of the steak during refrigeration. We can also meet the constraints by using a lower heat transfer coefficient, but doing so would extend the refrigeration time unnecessarily.

Summary

- Lumped System Analysis
  - ✓ Criteria for Lumped System Analysis
- Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects
  - ✓ Nondimensionalized One-Dimensional Transient Conduction Problem
  - ✓ Exact Solution of One-Dimensional Transient Conduction Problem
  - ✓ Approximate Analytical and Graphical Solutions
- Transient Heat Conduction in Semi-Infinite Solids
  - ✓ Contact of Two Semi-Infinite Solids
- Transient Heat Conduction in Multidimensional Systems