Ters Değişken Dönüşümü

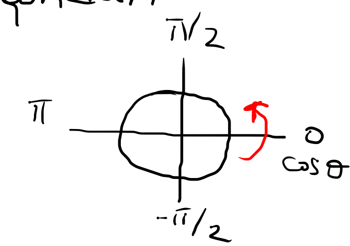
Değişken değiştirme yönteminde, integre edilen fonksiyonda yer alan bir ifadeyi tek bir değişken ile değiştirerek fonksiyonu basitleştiriyorduk. Ters değişken dönüşümünde ise integral değişkenini yeni bir değişkene sahip bir fonksiyonla değiştirerek integrali basitleştireceğiz.

Ters trigonometrik değişken dönüşümleri

1°) Ters sinüs değişken dönüşümü ($x=a\sin\theta$)

İntegralde $\sqrt{a^2-x^2}$ $(a>0)$ şeklinde terimler bulunduğunda kullanılan bir dönüşümdür.

$\sqrt{a^2-x^2}$ ifadesi ancak $a^2-x^2\geq 0 \Rightarrow -a\leq x\leq a$ ise anlamlıdır. Bu ise $x=a\sin\theta$ olduğundan θ'nın $-\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2}$ eşitsizliğini sağlamasıyla mümkündür.

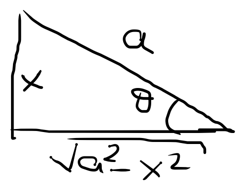


$$\sqrt{a^2-x^2}=\sqrt{a^2-a^2\sin^2\theta}=\sqrt{a^2(1-\sin^2\theta)}=\sqrt{a^2\cos^2\theta}=a\cos\theta$$

($\cos\theta\geq 0$ dır $-\pi/2\leq\theta\leq\frac{\pi}{2}$ için)

$x=a\sin\theta \Rightarrow \sin\theta=\frac{x}{a}$

$dx=a\cos\theta\, d\theta$



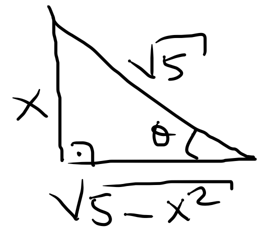
$\sin\theta=\frac{x}{a}$, $\cos\theta=\frac{\sqrt{a^2-x^2}}{a}$, $\tan\theta=\frac{x}{\sqrt{a^2-x^2}}$

Ör/ $$\int \frac{dx}{(5-x^2)^{3/2}} = \int \frac{\sqrt{5}\cos\theta\, d\theta}{\sqrt{(5-5\sin^2\theta)^3}} = \int \frac{\sqrt{5}\cos\theta\, d\theta}{\sqrt{5^3\cdot(1-\sin^2\theta)^3}} = \int \frac{\cancel{\sqrt{5}}\cos\theta\, d\theta}{5\cancel{\sqrt{5}}\sqrt{(\cos^2\theta)^3}} = \frac{1}{5}\int \frac{\cos\theta\, d\theta}{\cos^3\theta}$$

$x = \sqrt{5}\sin\theta$

$dx = \sqrt{5}\cos\theta\, d\theta$

$\sin\theta = \dfrac{x}{\sqrt{5}}$



$$= \frac{1}{5}\int \frac{d\theta}{\cos^2\theta} = \frac{1}{5}\int \sec^2\theta\, d\theta = \frac{1}{5}\tan\theta + C$$

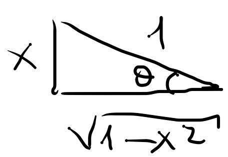
$$= \frac{1}{5}\frac{x}{\sqrt{5-x^2}} + C$$

Ör/ $$\int \frac{x^2\, dx}{\sqrt{1-x^2}} = \int \frac{\sin^2\theta\cdot\cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}} = \int \frac{\sin^2\theta\cdot\cancel{\cos\theta}}{\cancel{\cos\theta}}\, d\theta = \int \sin^2\theta\, d\theta$$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

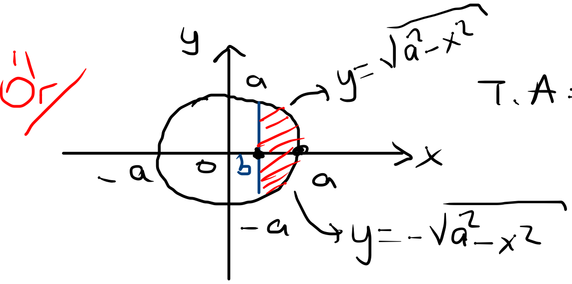
$\theta = \arcsin x$



$$= \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + C$$

$$= \frac{\theta}{2} - \frac{1}{\cancel{4}}\cdot\cancel{2}\sin\theta\cos\theta + C$$

$$= \frac{1}{2}\arcsin x - \frac{1}{2}\cdot x\cdot\sqrt{1-x^2} + C$$



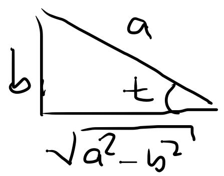
T.A = ?

$$x = a\sin\theta \Rightarrow dx = a\cos\theta\, d\theta$$

$$x = b \Rightarrow \sin\theta = \frac{b}{a} \Rightarrow \theta = \arcsin\frac{b}{a}$$

$$x = a \Rightarrow \sin\theta = 1 \Rightarrow \theta = \pi/2$$

$$T.A = 2\cdot\int_b^a \sqrt{a^2-x^2}\,dx = 2\int_{\arcsin\frac{b}{a}}^{\pi/2} \sqrt{\underbrace{a^2-a^2\sin^2\theta}_{a^2\cos^2\theta}}\cdot a\cos\theta\, d\theta$$



$$= 2a^2\int_{\arcsin\frac{b}{a}}^{\pi/2} \cos^2\theta\, d\theta = \cancel{2}a^2\int_{\arcsin\frac{b}{a}}^{\pi/2}\left(\frac{1+\cos 2\theta}{\cancel{2}}\right)d\theta = a^2\int_{\arcsin\frac{b}{a}}^{\pi/2} d\theta + a^2\int_{\arcsin\frac{b}{a}}^{\pi/2} \cos 2\theta\, d\theta$$

$$\arcsin\frac{b}{a} = t$$

$$\frac{b}{a} = \sin t$$

$$= a^2\left(\theta + \frac{1}{2}\sin 2\theta \Big|_{\arcsin\frac{b}{a}}^{\pi/2}\right)$$

$$= a^2\left[\left(\frac{\pi}{2} + \frac{1}{2}\cancel{\sin\pi}^{\,0}\right) - \left(\arcsin\frac{b}{a} + \frac{1}{2}\underbrace{\sin 2\left(\arcsin\frac{b}{a}\right)}_{2\sin(\arcsin\frac{b}{a})\cos(\arcsin\frac{b}{a})}\right)\right]$$

$$= a^2\left(\frac{\pi}{2} - \arcsin\frac{b}{a} - \frac{b}{a}\cdot\frac{\sqrt{a^2-b^2}}{a}\right)$$

$$= a^2\frac{\pi}{2} - a^2\arcsin\frac{b}{a} - b\sqrt{a^2-b^2} \quad br^2$$

Ters tanjant değişken dönüşümü ($x=a\tan\theta$)

$\sqrt{a^2+x^2}$ veya $\frac{1}{a^2+x^2}$ $(a>0)$ şeklinde ifadeler içeren integrallerde kullanılır.

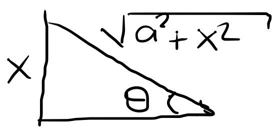
Bu ifadelerde x herhangi bir değeri alabileceğinden θ'nın değişim aralığı tanjantın π periyodlu olmasına bağlı olarak $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$ olacaktır. $x=a\tan\theta$

$\sec\theta=\frac{1}{\cos\theta}>0 \quad \left(-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right)$

$$\sqrt{a^2+x^2}=\sqrt{a^2+a^2\tan^2\theta}=\sqrt{a^2(1+\tan^2\theta)}=\sqrt{a^2\sec^2\theta}=a\sec\theta$$

$$x=a\tan\theta \Rightarrow dx=a\sec^2\theta\, d\theta$$

$$\tan\theta=\frac{x}{a} \Rightarrow \theta=\arctan\frac{x}{a}$$



$$\sin\theta=\frac{x}{\sqrt{a^2+x^2}}$$

$$\cos\theta=\frac{a}{\sqrt{a^2+x^2}}$$

Ör)

$$\int\frac{dx}{\sqrt{4+x^2}}=\int\frac{2\sec^2\theta\, d\theta}{\sqrt{4+4\tan^2\theta}}=\int\frac{2\sec^2\theta\, d\theta}{\sqrt{4(1+\tan^2\theta)}}$$

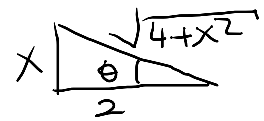
$x=2\tan\theta$

$dx=2\sec^2\theta\, d\theta$

$$=\int\frac{\cancel{2}\sec^{\cancel{2}}\theta\, d\theta}{\cancel{2}\cancel{\sec\theta}}$$

$\tan\theta=\frac{x}{2}$

$$=\int\sec\theta\, d\theta$$



$$=\ln|\sec\theta+\tan\theta|+c$$

$$=\ln\left|\frac{\sqrt{4+x^2}}{2}+\frac{x}{2}\right|+c$$

$(c_1=c-\ln 2)$

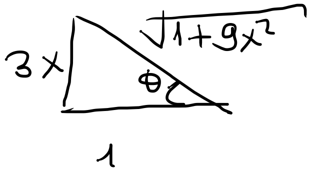
$$=\ln\left|\sqrt{4+x^2}+x\right|+c_1$$

Ör/ $\displaystyle\int \frac{dx}{(1+9x^2)^2} = \int \frac{\frac{1}{3}\sec^2\theta\, d\theta}{(1+\tan^2\theta)^2} = \frac{1}{3}\int \frac{\cancel{\sec^2\theta}\, d\theta}{\sec^{\cancel{4}\,2}\theta} = \frac{1}{3}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{3}\int \cos^2\theta\, d\theta$

$3x = \tan\theta \Rightarrow \theta = \arctan 3x$

$3\,dx = \sec^2\theta\, d\theta$

$dx = \frac{1}{3}\sec^2\theta\, d\theta$



$$= \frac{1}{3}\int \left(\frac{1+\cos 2\theta}{2}\right) d\theta = \frac{1}{6}\theta + \frac{1}{12}\sin 2\theta + C$$

$$= \frac{1}{6}\arctan(3x) + \frac{1}{6}\sin\theta\cdot\cos\theta + C$$

$$= \frac{1}{6}\arctan(3x) + \frac{1}{6}\,\frac{3x}{\sqrt{1+9x^2}}\cdot\frac{1}{\sqrt{1+9x^2}} + C$$

$$= \frac{1}{6}\arctan(3x) + \frac{x}{2(1+9x^2)} + C$$

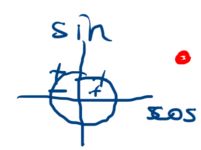
Ters sekant değişken dönüşümü ($x = a\sec\theta$)

$\sqrt{x^2-a^2}$ $(a>0)$ şeklinde terimler içeren integrallerde kullanılan bir dönüşümdür.

$\sqrt{x^2-a^2}$ ifadesi $x^2-a^2 \geq 0 \Rightarrow x \leq -a$ ve $x \geq a$ için tanımlı olduğundan $\boxed{x \leq -a \text{ için } \frac{\pi}{2} < \theta \leq \pi}$

$\boxed{x \geq a \text{ için } 0 \leq \theta < \frac{\pi}{2}}$

şeklinde olacaktır. $x = a\sec\theta$



$x \leq -a$

$x = a\sec\theta$

- $-a = a\sec\theta$

  $-1 = \sec\theta \Rightarrow \theta = \pi$

- $-\infty = a\sec\theta$

  $-\infty = \sec\theta \Rightarrow \theta = \frac{\pi}{2}$

$x \geq a$

- $a = a\sec\theta$

  $1 = \sec\theta \Rightarrow \theta = 0$

- $\infty = a\sec\theta$

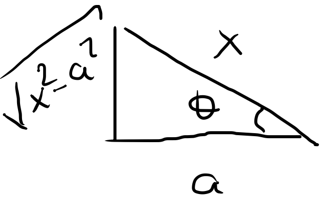
  $\infty = \sec\theta \Rightarrow \theta = \pi/2$

$$\sqrt{x^2-a^2} = \sqrt{a^2\sec^2\theta - a^2} = \sqrt{a^2(\sec^2\theta - 1)} = \sqrt{a^2\tan^2\theta} = a|\tan\theta| = \begin{cases} a\tan\theta & 0 \leq \theta < \frac{\pi}{2} \\ -a\tan\theta & \frac{\pi}{2} < \theta \leq \pi \end{cases}$$

$x = a\sec\theta \Rightarrow \sec\theta = \frac{x}{a}$

$dx = a\sec\theta\tan\theta\, d\theta$

$\Rightarrow \theta = \operatorname{arcsec}\frac{x}{a}$



$\sin\theta = \frac{\sqrt{x^2-a^2}}{x}$

$\cos\theta = \frac{a}{x}$

Or

$I = \int \frac{dx}{\sqrt{x^2-a^2}} = ?\quad (a>0)$

$x = a\sec\theta$

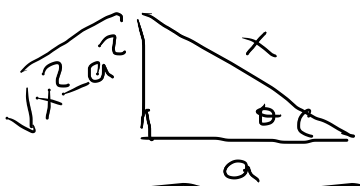
$dx = a\sec\theta \cdot \tan\theta\, d\theta$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \int \frac{a\sec\theta\tan\theta\, d\theta}{\sqrt{a^2\sec^2\theta - a^2}}$$

$$= \int \frac{a\sec\theta\tan\theta\, d\theta}{\sqrt{a^2\underbrace{(\sec^2\theta-1)}_{\tan^2\theta}}} = \int \frac{\cancel{a}\sec\theta\tan\theta\, d\theta}{\cancel{a}\cdot|\tan\theta|}$$

$$= \begin{cases} \ln|\sec\theta+\tan\theta|+c & 0\le\theta<\frac{\pi}{2} \\ -\ln|\sec\theta+\tan\theta|+c & \frac{\pi}{2}<\theta\le\pi \end{cases}$$

$$= \begin{cases} \ln\left|\frac{x}{a}+\frac{\sqrt{x^2-a^2}}{a}\right|+c & 0\le\theta<\pi/2 \\ -\ln\left|\frac{x}{a}+\frac{\sqrt{x^2-a^2}}{a}\right|+c & \frac{\pi}{2}<\theta\le\pi \end{cases}$$



$\sec\theta = \frac{x}{a}$

$x \ge a \Rightarrow I = \ln\left|\frac{x}{a}+\frac{\sqrt{x^2-a^2}}{a}\right|+c$

$dx = -du$

$x \le -a \Rightarrow \quad x = -u \Rightarrow -u \le -a \Rightarrow u \ge a$

$$I = \int \frac{-du}{\sqrt{u^2-a^2}} = -\ln\left|\frac{u}{a}+\frac{\sqrt{u^2-a^2}}{a}\right|+c$$

$$= -\ln\left|\frac{-x}{a}+\frac{\sqrt{x^2-a^2}}{a}\right|+c$$

$$= -\ln\left|\frac{-x+\sqrt{x^2-a^2}}{a}\right| + c \qquad \{c_1 = c - \ln a\}$$

$$= -\ln\left|-x+\sqrt{x^2-a^2}\right| + c_1$$

$$= \ln\left|\frac{1}{-x+\sqrt{x^2-a^2}}\right| + c_1$$

$$= \ln\left|\frac{1}{-x+\sqrt{x^2-a^2}} \cdot \frac{x+\sqrt{x^2-a^2}}{x+\sqrt{x^2-a^2}}\right| + c_1$$

$$= \ln\left|\frac{x+\sqrt{x^2-a^2}}{\cancel{x^2}-a^2-\cancel{x^2}}\right| + c_1 = \ln|x+\sqrt{x^2-a^2}| - \ln|-a^2| + c_1$$

$$= \ln|x+\sqrt{x^2-a^2}| + c$$

Ör

$$I = \int \frac{dx}{x\sqrt{x^2-4}} \qquad (x \geq 2 \text{ için})$$

$$\left.\begin{array}{l} x = 2\sec\theta \\ dx = 2\sec\theta . \tan\theta \end{array}\right\} \; I = \int \frac{\cancel{2\sec\theta}\tan\theta\, d\theta}{\cancel{2\sec\theta}\sqrt{4\sec^2\theta - 4}}$$

$$\sec\theta = \frac{x}{2}$$

$$\theta = \operatorname{arcsec}\frac{x}{2}$$

$$= \sec^{-1}\frac{x}{2}$$

$$= \int \frac{\cancel{\tan\theta}}{2\cancel{\tan\theta}}\, d\theta$$

$$= \frac{1}{2}\theta + c$$

$$= \frac{1}{2}\operatorname{arcsec}\frac{x}{2} + c = \frac{1}{2}\sec^{-1}\frac{x}{2} + c$$

$$= \frac{1}{2}\arccos\frac{2}{x} + c = \frac{1}{2}\cos^{-1}\frac{2}{x} + c$$

Tam kareye tamamlama

integrallerde Ax^2+Bx+C şeklindeki ifadelerle karşılaşıldığında bunlar karelerin toplamı veya farkı biçiminde ifade edilebilirler.

$$Ax^2+Bx+C = A\left(x^2+\frac{B}{A}x+\frac{C}{A}\right) = A\left(x^2+\frac{B}{A}x+\frac{C}{A}+\frac{B^2}{4A^2}-\frac{B^2}{4A^2}\right)$$

$$= A\left[\left(x^2+\frac{B}{A}x+\frac{B^2}{4A^2}\right)+\left(\frac{C}{A}-\frac{B^2}{4A^2}\right)\right]$$

$$= A\left[\left(x+\frac{B}{2A}\right)^2+\left(\frac{4AC-B^2}{4A^2}\right)\right] \Rightarrow \boxed{x+\frac{B}{A}=u}$$

Ör/

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{-[x^2-2x]}} = \int \frac{dx}{\sqrt{-[(x-1)^2-1]}}$$

$$= \int \frac{dx}{\sqrt{1-(x-1)^2}} = \int \frac{dt}{\sqrt{1-t^2}} = \arcsin t + c$$

$$= \arcsin(x-1) + c$$

$x-1=t$

$dx=dt$

$x-1 = \sin u$

$dx = \cos u\, du$

$u = \arcsin(x-1)$

$$\int \frac{\cos u\, du}{\sqrt{1-\sin^2 u}} = \int \frac{\cancel{\cos u}\, du}{\cancel{\cos u}} = \int du = u + c$$

$$= \arcsin(x-1) + C$$

Ör/

$$\int \frac{x\,dx}{4x^2+12x+13} = \int \frac{x\,dx}{4\left(x^2+3x+\frac{13}{4}\right)} = \frac{1}{4}\int \frac{x\,dx}{\left(x+\frac{3}{2}\right)^2+1} = \frac{1}{4}\int \frac{\left(u-\frac{3}{2}\right)du}{u^2+1}$$

$$= \frac{1}{4}\int \frac{u\,du}{u^2+1} - \frac{3}{8}\int \frac{du}{u^2+1}$$

$x+\frac{3}{2}=u$

$dx=du$

$x=u-\frac{3}{2}$

$$= \frac{1}{8}\int \frac{2u\,du}{u^2+1} - \frac{3}{8}\arctan u$$

$$= \frac{1}{8}\ln(u^2+1) - \frac{3}{8}\arctan u + c$$

$$= \frac{1}{8}\ln\left[\left(x+\frac{3}{2}\right)^2+1\right] - \frac{3}{8}\arctan\left(x+\frac{3}{2}\right) + c$$

$\int \frac{(2x+3)\,dx}{9x^2-12x+8} = \frac{1}{9}\int \frac{9(2x+3)\,dx}{9x^2-12x+8} = \frac{1}{9}\int \frac{18x+27+12-12}{9x^2-12x+8}\,dx$

$$\boxed{\begin{aligned} 9x^2-12x+8 &= u \\ (18x-12)\,dx &= du \end{aligned}}$$

$= \frac{1}{9}\int \frac{(18x-12)}{9x^2-12x+8}\,dx + \frac{39}{9}\int \frac{dx}{9x^2-12x+8} = \frac{1}{9}\int \frac{du}{u} + \frac{39}{9}\int \frac{dx}{9(x^2-\underbrace{\frac{12}{9}}_{\frac{4}{3}}x+\frac{8}{9})}$

$= \frac{1}{9}\ln|u| + \frac{39}{81}\int \frac{dx}{(x-\frac{2}{3})^2+\frac{4}{9}}$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + c$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$

$x-\frac{2}{3} = t$

$dx = dt$

$= \frac{1}{9}\ln|9x^2-12x+8| + \frac{39}{81}\int \frac{du}{u^2+(\frac{2}{3})^2}$

$= \frac{1}{9}\ln|9x^2-12x+8| + \frac{\cancel{39}^{\,\cancel{3}\,13}}{\cancel{81}_{\,9}} \cdot \frac{\cancel{3}}{2} \cdot \arctan\left(\frac{3u}{2}\right) + c$

$= \frac{1}{9}\ln|9x^2-12x+8| + \frac{13}{18}\arctan\left(\frac{3x-2}{2}\right) + c$

$\tan\left(\frac{x}{2}\right)$ dönüşümü

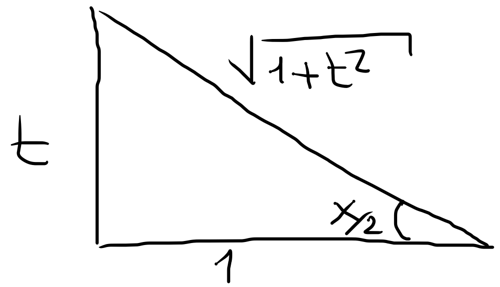
Bu dönüşüm $\sin x$ ve $\cos x$ fonksiyonlarının bir rasyonel fonksiyonu olarak verilen bir integrandı t'nin fonksiyonu olan bir rasyonel fonksiyona dönüştürür.

$$\tan\frac{x}{2} = t$$

$$\frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)dx = dt$$

$$dx = \frac{2\,dt}{1+\tan^2\frac{x}{2}} = \frac{2\,dt}{1+t^2}$$

$$\Rightarrow \boxed{dx = \frac{2\,dt}{1+t^2}}$$



$$\sin\frac{x}{2} = \frac{t}{\sqrt{1+t^2}}$$

$$\cos\frac{x}{2} = \frac{1}{\sqrt{1+t^2}}$$

$$\Rightarrow \sin x = 2\sin\frac{x}{2}.\cos\frac{x}{2} = 2.\frac{t}{\sqrt{1+t^2}}.\frac{1}{\sqrt{1+t^2}}$$

$$\Rightarrow \boxed{\sin x = \frac{2t}{1+t^2}}$$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \frac{1}{1+t^2} - \frac{t^2}{1+t^2}$$

$$\Rightarrow \boxed{\cos x = \frac{1-t^2}{1+t^2}}$$

Ör/ $\displaystyle\int \frac{d\theta}{2+\cos\theta} = \int \frac{\frac{2\,dt}{1+t^2}}{2+\frac{1-t^2}{1+t^2}} = \int \frac{\frac{2\,dt}{\cancel{1+t^2}}}{\frac{2+2t^2+1-t^2}{\cancel{1+t^2}}} = 2\int \frac{dt}{t^2+3}$

$\displaystyle = \frac{2}{\sqrt{3}} \arctan \frac{t}{\sqrt{3}} + c$

$\displaystyle = \frac{2}{\sqrt{3}} \arctan \frac{\left(\tan\frac{\theta}{2}\right)}{\sqrt{3}} + c$

$\tan\frac{\theta}{2} = t$

$d\theta = \frac{2\,dt}{1+t^2}$

$\cos\theta = \frac{1-t^2}{1+t^2}$

Ör/ $\displaystyle\int \frac{dx}{1-\sin x} = \int \frac{\frac{2\,dt}{1+t^2}}{1-\frac{2t}{1+t^2}} = 2\int \frac{\frac{dt}{\cancel{1+t^2}}}{\frac{t^2-2t+1}{\cancel{1+t^2}}} = 2\int \frac{dt}{(t-1)^2} = -\frac{2}{t-1} + c$

$\displaystyle = \frac{-2}{\tan\frac{x}{2} - 1} + c$

$\tan\frac{x}{2} = t$

$dx = \frac{2\,dt}{1+t^2}$

$\sin x = \frac{2t}{1+t^2}$

• integralde $\sqrt{ax+b}$ varsa $\Rightarrow$ $ax+b=u^2 \Rightarrow a\,dx = 2u\,du \Rightarrow dx = \frac{2u\,du}{a}$

Ör/ $\int \frac{dx}{1+\sqrt{2x}} = \int \frac{u\,du}{1+u} = \int \frac{u+1-1}{1+u}\,du = \int\left(1-\frac{1}{1+u}\right)du$

$2x=u^2$

$2\,dx = 2u\,du$

$dx = u\,du$

$= \int du - \int \frac{du}{1+u}$

$= u - \ln|1+u| + c$

$= \sqrt{2x} - \ln|1+\sqrt{2x}| + c$

• integralde $\sqrt[n]{ax+b}$ varsa $\Rightarrow$ $ax+b=u^n \Rightarrow a\,dx = n.u^{n-1}\,du \Rightarrow dx = \frac{nu^{n-1}\,du}{a}$

Ör/ $\int_{-1/3}^{2} \frac{x\,dx}{\sqrt[3]{3x+2}} = \int_1^2 \frac{\frac{u^3-2}{3}\cdot u^{\cancel{2}}\,du}{\cancel{u}} = \frac{1}{3}\int_1^2 (u^4-2u)\,du = \frac{1}{3}\left(\frac{u^5}{5}-u^2\right)\Big|_1^2$

$3x+2=u^3 \Rightarrow x = \frac{u^3-2}{3}$

$3\,dx = 3u^2\,du$

$dx = u^2\,du$

$x=-\frac{1}{3} \Rightarrow u^3=1 \Rightarrow u=1$

$x=2 \Rightarrow u^3=8 \Rightarrow u=2$

$= \frac{1}{3}\left(\frac{32}{5}-4\right) - \frac{1}{3}\left(\frac{1}{5}-1\right)$

$= \frac{16}{15}$

- İntegral içerisinde x'in birden fazla kesirli kuvveti bulunuyorsa o zaman kesirli kuvvetlerin paydalarının en küçük ortak katı n olacak şekilde $x=u^n$ dönüşümü yapılarak integral çözülür.

Ör/ $$\int \frac{dx}{\sqrt{x}\,(1+\sqrt[3]{x})} = \int \frac{6u^{\cancel{5}\,2}\,du}{\cancel{u^3}\,(1+u^2)} = 6\int \frac{u^2+1-1}{1+u^2}\,du$$

$$= 6\int \left(1-\frac{1}{1+u^2}\right)du$$

$$= 6\,(u-\arctan u)+C$$

$$= 6\,(\sqrt[6]{x}-\arctan\sqrt[6]{x})+C$$

$n = \text{EKOK}(2,3) = 6$

$x = u^6 \Rightarrow u = \sqrt[6]{x}$

$dx = 6u^5\,du$