$$a_0 y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1} y' + a_n y = f(x) = f_1(x) + f_2(x) + \cdots + f_n(x)$$

$$y_{\ddot{o}} = y_{\ddot{o}_1} + y_{\ddot{o}_2} \cdots + y_{\ddot{o}_n}$$

Ör/ $y''' + 9y' = 18e^{3x} + 10\sin 2x + \underbrace{9x-3}$ (1. der. pol.) dif. denkleminin genel çözümünü bulunuz.

$\alpha = 3 \neq r_1 \neq r_{2,3}$

$(D^3 + 9D)y = \underbrace{18e^{3x}}_{y_{\ddot{o}1}} + \underbrace{10\sin 2x}_{y_{\ddot{o}2}} + \underbrace{9x-3}_{y_{\ddot{o}3}}$

k.D: $r^3 + 9r = 0 \Rightarrow r(r^2+9) = 0$

$\Rightarrow r_1 = 0 \quad r_{2,3} = \mp 3i$

$$\boxed{y_h = c_1 + c_2\cos 3x + c_3 \sin 3x}$$

$y_{\ddot{o}1} = Ae^{3x} \Rightarrow y'_{\ddot{o}} = 3Ae^{3x} \Rightarrow y''_{\ddot{o}} = 9Ae^{3x} \Rightarrow y'''_{\ddot{o}} = 27Ae^{3x}$

$\Rightarrow 27Ae^{3x} + 27Ae^{3x} = 18e^{3x} \Rightarrow 54A = 18$

$A = \frac{1}{3}$

$$\Rightarrow \boxed{y_{\ddot{o}1} = \frac{e^{3x}}{3}}$$

$\lambda = 2$

$r_1 \neq \mp 2i$

$r_{2,3} \neq \mp 2i$

$$y_{ö2} = A\cos 2x + B\sin 2x$$
$$y'_{ö2} = -2A\sin 2x + 2B\cos 2x$$
$$y''_{ö2} = -4A\cos 2x - 4B\sin 2x$$
$$y'''_{ö2} = 8A\sin 2x - 8B\cos 2x$$

$$\Rightarrow 8A\sin 2x - 8B\cos 2x - 18A\sin 2x + 18B\cos 2x = 10\sin 2x$$

$$-10A\sin 2x + 10B\cos 2x = 10\sin 2x$$

$$\Rightarrow \quad -10A = 10 \qquad 10B = 0$$
$$A = -1 \qquad B = 0$$

$$\Rightarrow \boxed{y_{ö2} = -\cos 2x}$$

$r_1 = 0$ old. yazdık

$f(x)$'de 1. der. polinom

$$y_{ö3} = x\,.\,(ax+b) = ax^2 + bx$$
$$y'_{ö3} = 2ax + b$$
$$y''_{ö3} = 2a$$
$$y'''_{ö3} = 0$$

$$\Rightarrow 0 + 18ax + 9b = 9x - 3$$

$$\Rightarrow 18a = 9 \qquad 9b = -3$$
$$a = \frac{1}{2} \qquad b = -\frac{1}{3} \Rightarrow \boxed{y_{ö3} = \frac{x^2}{2} - \frac{x}{3}}$$

$$\boxed{y_{G.Ç} = y_h + y_{ö1} + y_{ö2} + y_{ö3} = c_1 + c_2\cos 3x + c_3\sin 3x + \frac{e^{3x}}{3} - \cos 2x + \frac{x^2}{2} - \frac{x}{3}}$$

Uygulama

1) $y'' - (y')^2 + xy = 0$ dif. denkleminin mertebesi ve derecesi nedir?

2. mertebe, 1. derece

2) $y = (x^3 + c).e^{-3x}$ eğri ailesinin dif. denklemini kurunuz.

$$y' = 3x^2 e^{-3x} - 3(x^3+c)e^{-3x}$$

$$x^3 + c = y.e^{3x} \Rightarrow y' = 3x^2 e^{-3x} - 3y \Rightarrow \boxed{y' + 3y = 3x^2 e^{-3x}}$$

3) $(6xy^3 + \cos y)dx + (kx^2y^2 - x\sin y)dy = 0$ denkleminin tam dif. denklem olması için k ne olmalıdır?

$$\frac{\partial}{\partial y}(6xy^3 + \cos y) = \frac{\partial}{\partial x}(kx^2y^2 - x\sin y) \text{ olmalıdır.}$$

$$18xy^2 - \sin y = 2kxy^2 - \sin y \Rightarrow 2k = 18 \Rightarrow \boxed{k=9}$$

4) $(1+x^2+y^2+x^2y^2)\,dy = y^2 dx$ dif. denkleminin $y(0)=1$ koşuluna uygun çözümünü bulunuz.

$$\left[(1+x^2)+y^2(1+x^2)\right]dy = y^2 dx$$

$$\frac{\cancel{(1+x^2)}(1+y^2)\,dy}{y^2\,\cancel{(1+x^2)}} = \frac{\cancel{y^2}\,dx}{\cancel{y^2}(1+x^2)} \qquad (y>0)$$

$$\int \frac{1+y^2}{y^2}\,dy = \int \frac{dx}{1+x^2} \Rightarrow -\frac{1}{y} + y = \arctan x + c$$

$$\left.\begin{array}{l} x=0 \\ y=1 \end{array}\right\} \Rightarrow -1+1 = \arctan 0 + c$$

$$\boxed{c=0} \Rightarrow \boxed{\frac{y^2-1}{y} = \arctan x}$$

5) $\underbrace{(5y-6x)}_{P}dx + \underbrace{x}_{Q}dy = 0$ dif. denklemi tam dif. denklem midir? Değilse integrasyon çarpanı nedir?

$$\frac{\partial}{\partial y}(5y-6x) \overset{?}{=} \frac{\partial}{\partial x}(x)$$

$5 \neq 1$ Tam dif. denk. değildir.

$$\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} = 5-1=4$$

$\underline{\lambda=\lambda(x)}$

$$\ln\lambda = \int \frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}dx = \int \frac{4}{x}dx$$

$$\lambda = x^4$$

1

$$\underbrace{x^4(5y-6x)}_{P_1}dx + \underbrace{x^5}_{Q_1}dy = 0 = du(x,y) = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

2

$$\frac{\partial P_1}{\partial y} = 5x^4 = \frac{\partial Q_1}{\partial x} = 5x^4$$

1'den

$$\frac{\partial u}{\partial x} = 5x^4y-6x^5 \Rightarrow \int \partial u(x,y) = \int(5x^4y-6x^5)\,dx$$

$$\frac{\partial u}{\partial y} = x^5$$

$$u(x,y) = x^5y - x^6 + h(y)$$

$$\frac{\partial u}{\partial y} = x^5 + \frac{dh}{dy}$$

$$x^5 + \frac{dh}{dy} = x^5 \Rightarrow \frac{dh}{dy} = 0 \Rightarrow h(y) = c_1$$

$$\Rightarrow u(x,y) = x^5 y - x^6 + c_1$$

$$\Rightarrow x^5 y - x^6 + c_1 = c \qquad \boxed{c - c_1 = k}$$

$$\Rightarrow \boxed{x^5 y - x^6 = k} \text{ G.Ç.}$$

② 'den $du(x,y) = 0$

$$u(x,y) = c$$

6°) $xy' - y = x e^{y/x}$ dif. denk'nin genel çöz. bulunuz.

$$y' - \frac{y}{x} = e^{y/x}$$

$$y' = \frac{y}{x} + e^{y/x} = f\left(\frac{y}{x}\right) \quad \text{Homojen dif. denk.}$$

$$\frac{y}{x} = u \Rightarrow y = ux \Rightarrow y' = u'x + u$$

$$\Rightarrow u'x + \cancel{u} = \cancel{u} + e^u \Rightarrow x.\frac{du}{dx} = e^u \Rightarrow \int e^{-u} du = \int \frac{dx}{x}$$

$$\Rightarrow -e^{-u} = \ln|x| + c \Rightarrow \boxed{-e^{-\frac{y}{x}} = \ln x + c}$$

7°) $y' - 2xy = 2xe^{x^2}\sqrt{y}$ dif. denkleminin $y(0)=1$ koşuluna uygun çözümünü bulunuz.

$$\frac{y'}{\sqrt{y}} - 2x\sqrt{y} = 2xe^{x^2}$$

$$\left.\begin{array}{l} \sqrt{y} = z \\ \dfrac{y'}{2\sqrt{y}} = z' \Rightarrow \dfrac{y'}{\sqrt{y}} = 2z' \end{array}\right\} \Rightarrow 2z' - 2xz = 2xe^{x^2}$$

$$\Rightarrow \quad z' - xz = xe^{x^2} \quad \text{L.D.D.}$$

$$\left.\begin{array}{l} p(x) = -x \\ \ln\lambda = \int p(x)\,dx \\ \lambda = e^{-\int x\,dx} = e^{-\frac{x^2}{2}} \end{array}\right\} \underbrace{e^{-\frac{x^2}{2}} z' - x.e^{-\frac{x^2}{2}} z}_{\frac{d}{dx}\left(z.e^{-\frac{x^2}{2}}\right)} = x.e^{\frac{x^2}{2}}$$

$$\frac{d}{dx}\left(z.e^{-\frac{x^2}{2}}\right) = xe^{\frac{x^2}{2}}$$

$$\frac{x^2}{2} = t$$

$$x\,dx = dt$$

$$\Rightarrow \int d\left(z.e^{-\frac{x^2}{2}}\right) = \int x.e^{\frac{x^2}{2}}\,dx$$

$z.e^{-\frac{x^2}{2}} = e^{\frac{x^2}{2}} + k$

$z = 1 + k.e^{-\frac{x^2}{2}}$

$\sqrt{y} = 1 + ke^{-\frac{x^2}{2}}$

$\left.\begin{array}{l} x=0 \\ y=1 \end{array}\right\} \Rightarrow 1 = 1 + k \Rightarrow k = 0 \qquad \sqrt{y} = 1 \Rightarrow \boxed{y=1}$

8°) $y' - y^2 + y\sin x - \cos x = 0$ denk. bir özel çöz. $y_1 = \sin x$ olduğuna göre denk'in genel çöz. bulunuz.

$y' = y^2 - y\sin x + \cos x$

$y = y_1 + \frac{1}{u} = \sin x + \frac{1}{u} \Rightarrow y' = \cos x - \frac{u'}{u^2}$ $\Big\}$ $\cancel{\cos x} - \frac{u'}{u^2} = (\sin x + \frac{1}{u})^2 - \sin x(\sin x + \frac{1}{u}) + \cancel{\cos x}$

$\left.\begin{array}{l} y_1 = \sin x \\ y_1' = \cos x \end{array}\right\} \cancel{\cos x} \cancel{- \sin^2 x} + \cancel{\sin^2 x} \cancel{+ \cos x} = 0$

$-\frac{u'}{u^2} = \cancel{\sin^2 x} + \frac{2\sin x}{u} + \frac{1}{u^2} - \cancel{\sin^2 x} - \frac{\sin x}{u}$

$$-\frac{u'}{u^2}=\frac{\sin x}{u}+\frac{1}{u^2} \Rightarrow \quad u'=-u\sin x-1$$

$$\Rightarrow \quad \underline{u'+u\sin x=-1} \quad \text{Lineer.}$$

$$\Rightarrow \quad u'+u\sin x=0$$

$$\int\frac{du}{u}+\int\sin x\,dx=\int 0$$

$$\ln u-\cos x=\ln c$$

$$u=c.e^{\cos x}$$

$$c=c(x) \Rightarrow \quad u'=c'e^{\cos x}-c.\sin x.e^{\cos x}$$

$$c'e^{\cos x}-\cancel{c\sin x e^{\cos x}}+\cancel{\sin x.\,c}e^{\cos x}=-1$$

$$?\quad c'=-e^{-\cos x} \Rightarrow \quad c=-\int e^{-\cos x}dx+k$$

9°) $(D-1)^3(D^6-1)y=0$ dif denkleminin genel çöz. bulunuz.

$$k.D:\ L(r)=(r-1)^3(r^3+1)=0$$

$$(r-1)^3(r+1)(r^2-r+1)=0$$

$$r_{1,2,3}=1 \qquad r_4=-1 \qquad r_{5,6}=\frac{1\mp\sqrt{1-4}}{2}=\frac{1}{2}\mp\frac{\sqrt{3}}{2}i$$

$$y=(c_1+c_2x+c_3x^2)e^{x}+c_4e^{-x}+e^{\frac{x}{2}}\left(c_5\cos\frac{\sqrt{3}}{2}x+c_6\sin\frac{\sqrt{3}}{2}x\right)$$