2. Operatör Yöntemi (Cramer Yöntemi)

Bu yöntemde sistemin denklemleri diferansiyel operatör ile ifade edilerek ve bilinmeyen fonksiyon-Lara göre düzenlenerek sistem Lineer denklem sistemleri gibi çözülür.

Ör/
$$\left.\begin{aligned} \frac{dy}{dx} &= -5y - z + 1 + x^2 \\ \frac{dz}{dx} &= y - 3z + e^{2x} \end{aligned}\right\}$$
denklem sistemini operatör yöntemiyle çözünüz.

$$\frac{d}{dx} = D \Rightarrow \left.\begin{aligned} Dy &= -5y - z + 1 + x^2 \\ Dz &= y - 3z + e^{2x} \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} (D+5)y + z &= 1 + x^2 \\ -y + (D+3)z &= e^{2x} \end{aligned}}$$

$$\Delta = \begin{vmatrix} D+5 & 1 \\ -1 & D+3 \end{vmatrix} = (D+5)(D+3) + 1 = D^2 + 8D + 16$$

$$y = \frac{\Delta_y}{\Delta} = \frac{\begin{vmatrix} 1+x^2 & 1 \\ e^{2x} & D+3 \end{vmatrix}}{D^2+8D+16} \Rightarrow (D^2+8D+16)y = (D+3)(1+x^2) - e^{2x}$$

$$(D^2+8D+16)y = 2x + 3 + 3x^2 - e^{2x}$$

$$(D^2+8D+16)y = 0$$

$$y_h = (c_1 + c_2 x)e^{-4x}$$

$$\boxed{(D^2+8D+16)y = 3x^2+2x+3-e^{2x}}$$

$$y_{G.C} = (c_1+c_2x)e^{-4x} + \frac{3x^2}{16} - \frac{x}{16} + \frac{25}{128} - \frac{e^{2x}}{36}$$

K.D: $r^2+8r+16=0$

$(r+4)^2=0$

$r_{1,2}=-4$

$$\boxed{y_h = (c_1+c_2x)e^{-4x}}$$

$$\left.\begin{aligned} y_{ö1} &= ax^2+bx+c \\ y'_{ö1} &= 2ax+b \\ y''_{ö1} &= 2a \end{aligned}\right\}$$

$2a+8(2ax+b)+16(ax^2+bx+c) \equiv 3x^2+2x+3$

$16ax^2+(16a+16b)x+(2a+8b+16c) \equiv 3x^2+2x+3$

$16a=3$ $\qquad$ $16a+16b=2$ $\qquad$ $2a+8b+16c=3$

$a=\frac{3}{16}$ $\qquad$ $b=\frac{-1}{16}$ $\qquad$ $\frac{6}{16}-\frac{8}{16}+16c=3$

$16c = 3+\frac{1}{8}$

$c=\frac{25}{128}$

$$\Rightarrow y_{ö1} = \frac{3x^2}{16} - \frac{x}{16} + \frac{25}{128}$$

$$\left.\begin{aligned} y_{ö2} &= Ae^{2x} \\ y'_{ö2} &= 2Ae^{2x} \\ y''_{ö2} &= 4Ae^{2x} \end{aligned}\right\} \Rightarrow 4Ae^{2x}+16Ae^{2x}+16Ae^{2x} \equiv -e^{2x}$$

$$36A \equiv -1 \Rightarrow A = \frac{-1}{36} \Rightarrow y_{ö2} = -\frac{e^{2x}}{36}$$

$$\boxed{y_{G.C} = (C_1+C_2x)e^{-4x} + \frac{3x^2}{16} - \frac{x}{16} + \frac{25}{128} - \frac{e^{2x}}{36}} \Rightarrow \frac{dy}{dx} = C_2e^{-4x} - 4.(C_1+C_2x)e^{-4x} + \frac{3x}{8} - \frac{1}{16} - \frac{e^{2x}}{18}$$

Sistemin 1. denkleminden z'i çekersek;

$$z = -\frac{dy}{dx} - 5y + 1 + x^2$$

$$= -C_2e^{-4x} + 4(C_1+C_2x)e^{-4x} - \frac{3x}{8} + \frac{1}{16} + \frac{e^{2x}}{18} - 5\left[(C_1+C_2x)e^{-4x} + \frac{3x^2}{16} - \frac{x}{16} + \frac{25}{128} - \frac{e^{2x}}{36}\right] + 1 + x^2$$

$$\boxed{z = -C_1e^{-4x} - C_2e^{-4x} - C_2xe^{-4x} + \frac{x^2}{16} - \frac{x}{16} + \frac{11}{128} + \frac{7e^{2x}}{36}}$$

2. yol

$$z = \frac{\Delta_z}{\Delta} = \frac{\begin{vmatrix} D+5 & 1+x^2 \\ -1 & e^{2x} \end{vmatrix}}{D^2+8D+16} \Rightarrow (D^2+8D+16)z = (D+5).e^{2x} + (1+x^2) = 2e^{2x} + 5e^{2x} + 1 + x^2 = 7e^{2x} + 1 + x^2$$

K.D: $r^2+8r+16=0$

$(r+4)^2 = 0$

$r_{1,2} = -4$

$$\boxed{z_h = (C_3+C_4x)e^{-4x}}$$

$z_{ö1} = ax^2+bx+c$

$z'_{ö1} = 2ax+b$

$z''_{ö1} = 2a$

$2a+8(2ax+b)+16(ax^2+bx+c) \equiv 1+x^2$

$16ax^2+(16a+16b)x+(2a+8b+16c) \equiv 1+x^2$

$16a=1$ $\quad 16a+16b=0$ $\quad 2a+8b+16c=1$

$a=1/16$ $\quad b=-\frac{1}{16}$ $\quad c=\frac{11}{128}$

$$z_{ö1} = \frac{x^2}{16} - \frac{x}{16} + \frac{11}{128}$$

$$z_{02}'' = Ae^{2x}, \quad z_{02}' = 2Ae^{2x}, \quad z_{02}'' = 4Ae^{2x} \quad \Bigg\} \quad 4Ae^{2x} + 16Ae^{2x} + 16Ae^{2x} \equiv 7e^{2x}$$

$$36Ae^{2x} \equiv 7e^{2x} \Rightarrow A = \frac{7}{36} \Rightarrow z_{02}'' = \frac{7e^{2x}}{36}$$

$$z_{G.Ç} = (c_3 + c_4 x)e^{-4x} + \frac{x^2}{16} - \frac{x}{16} + \frac{11}{128} + \frac{7e^{2x}}{36}$$

$$y_{GÇ} = (c_1 + c_2 x)e^{-4x} + \frac{3x^2}{16} - \frac{x}{16} + \frac{25}{128} - \frac{e^{2x}}{36}$$

Genel çözümün karakterine uygun değil. Çünkü denklemin mertebesi 2'dir ve genel çözüm 2 keyfi sabit içermelidir.

$$z = -\frac{dy}{dx} - 5y + 1 + x^2$$

$$\frac{dy}{dx} = c_2 e^{-4x} - 4(c_1 + c_2 x)e^{-4x} + \frac{3x}{8} - \frac{1}{16} - \frac{e^{2x}}{18}$$

$$(c_3 + c_4 x)e^{-4x} + \cancel{\frac{x^2}{16}} - \cancel{\frac{x}{16}} + \cancel{\frac{11}{128}} + \cancel{\frac{7e^{2x}}{36}} = -c_2 e^{-4x} + 4(c_1 + c_2 x)e^{-4x} - \cancel{\frac{3x}{8}} + \cancel{\frac{1}{16}} + \cancel{\frac{e^{2x}}{18}} - 5(c_1 + c_2 x)e^{-4x} - \cancel{\frac{15x^2}{16}} + \cancel{\frac{5x}{16}} - \cancel{\frac{125}{128}} + \cancel{\frac{5e^{2x}}{36}} + \cancel{1} + \cancel{x^2}$$

$$c_3 e^{-4x} + c_4 x e^{-4x} = -c_2 e^{-4x} + 4c_1 e^{-4x} + 4c_2 x e^{-4x} - 5c_1 e^{-4x} - 5c_2 x e^{-4x}$$

$$c_3 e^{-4x} + c_4 x e^{-4x} = (-c_1 - c_2)e^{-4x} - c_2 x e^{-4x}$$

$$\Rightarrow \boxed{c_3 = -c_1 - c_2} \quad \boxed{c_4 = -c_2}$$

Ör/ $\left.\begin{aligned} \frac{dx}{dt} &= 4y + e^{t} \\ \frac{dy}{dt} &= -4x + e^{-t} \end{aligned}\right\}$ dif. denklem sisteminin $x(t)$, $y(t)$ bilinmeyeninin 2. mertebe dif. denklemi nedir?

$\frac{d}{dt} = D \Rightarrow \left.\begin{aligned} Dx &= 4y + e^{t} \\ Dy &= -4x + e^{-t} \end{aligned}\right\} \Rightarrow \begin{aligned} Dx - 4y &= e^{t} \\ 4x + Dy &= e^{-t} \end{aligned}$

$\Delta = \begin{vmatrix} D & -4 \\ 4 & D \end{vmatrix} = D^2 + 16$

$$x(t) = \frac{\Delta_x}{\Delta} = \frac{\begin{vmatrix} e^{t} & -4 \\ e^{-t} & D \end{vmatrix}}{D^2+16} \Rightarrow (D^2+16)\,x = D.e^{t} + 4.e^{-t}$$

$$\underline{x'' + 16x = e^{t} + 4e^{-t}}$$

$$y(t) = \frac{\Delta_y}{\Delta} = \frac{\begin{vmatrix} D & e^{t} \\ 4 & e^{-t} \end{vmatrix}}{D^2+16} \Rightarrow (D^2+16)\,y = D.e^{-t} - 4e^{t}$$

$$\underline{y'' + 16y = -e^{-t} - 4e^{t}}$$

Ör/ $\left.\begin{array}{l} y' + 2y - z = 0 \\ z' - 7y - 4z = 0 \end{array}\right\}$ dif. denklem sisteminin $z(t)$ çözümünü bulunuz.

$\frac{d}{dt} = D \Rightarrow \begin{array}{l} (D+2)y - z = 0 \\ -7y + (D-4)z = 0 \end{array}$

$\Delta = \begin{vmatrix} D+2 & -1 \\ -7 & D-4 \end{vmatrix} = (D+2)(D-4) - 7$
$= D^2 - 2D - 15$

$z = \frac{\Delta_z}{\Delta} = \frac{\begin{vmatrix} D+2 & 0 \\ -7 & 0 \end{vmatrix}}{D^2 - 2D - 15} \Rightarrow (D^2 - 2D - 15)z = 0$

k.D: $r^2 - 2r - 15 = 0$
$3 \quad -5$

$(r+3)(r-5) = 0$

$r_1 = -3 \quad r_2 = 5 \Rightarrow \boxed{z = c_1 e^{-3t} + c_2 e^{5t}}$